On Representational Content and Format in Core Numerical Cognition

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0. Abstract

Susan Carey (2009) has influentially argued that there is a system of core numerical cognition - the analog magnitude, or AM system – in which (approximate) cardinal numbers are explicitly represented in iconic format. This paper argues, on the contrary, that there can be no explicit representations having this combination of features. The argument is in two stages: first, it is argued that there can be no explicit representation of individuals in iconic format; and then it is argued that the explicit representation of (approximate) cardinal numbers requires the explicit representation of individuals; from which it is concluded that the explicit representation of (approximate) cardinal numbers requires non-iconic representational format. While the argument is couched in the terms of Carey's discussion, the considerations it adduces are perfectly general, and the conclusion should therefore be taken into consideration by all those aiming to characterize to character this analog magnitude system of core numerical cognition.

1. Introduction

Much attention has been devoted to the topic of numerical cognition in the recent philosophical and psychological literature.[1] This attention is deserved, since the experimental findings are fascinating, and they throw up a range of difficult theoretical problems. Different authors, of course, take different approaches, with different commitments, and even employing different terminology for some of the key notions: but a particularly influential account of the phenomena in this area has been that developed in Susan Carey's (2009) book, *The Origin of Concepts*; and for the sake of concrete definiteness, I focus my own discussion on hers. The issues I raise, however, are quite general in character, and should therefore be of broader relevance to the discussion of the content and format of the representations employed in our numerical cognition.

Carey's overall aim in the book is to forge a middle way between two extreme positions on the question of how our representational capacities develop. Contrary to *Classical Empiricism*, she claims that 'the innate stock of [representational] primitives [in the mind] is not limited to sensory, perceptual, or sensorimotor representations; rather, there are also innate conceptual representations' (Carey, 2011, p. 113). Carey calls the psychological processes employing such representations *core cognition*, and she adduces a range of experimental evidence in defense of the claim that such processes are real and our capacity for them is innate, so that the conceptual representations they involve are innately available. On the other hand, contrary to *Radical Nativism*, Carey argues that over the course of our individual psychological development we are able to acquire genuinely new concepts – mental representations with contents that are inexpressible, and therefore unavailable, in the initial, innate collection of representations. Finally,

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Carey articulates a psychological mechanism, which she calls *Quinean bootstrapping*, whereby she believes these novel representations, with their enriched contents, arise.

The book contains a number of case studies including, especially centrally, that of numerical cognition. Carey claims that there are three systems of core cognition that have at least some numerical content, but that none of them is as rich, representationally speaking, as the natural language numeral list representations we acquire through acculturation. I focus on just one of these three systems, however, namely the *Analog Magnitude (AM)* system; and I do not discuss the cognitive development we undergo in acquiring the ability to employ such overt systems of representation at all, though the considerations adduced here may be germane to this general topic. Instead, I concentrate on Carey's suggestion that the representations of the AM system have (i) iconic format, and (ii) (approximate) cardinal numbers as contents; and I present and consider a line of argument to the effect that no representation can have both of these features. As the discussion does not reveal any obvious errors in this reasoning, I conclude that it poses a serious challenge to Carey's views regarding the AM system of core numerical cognition; and, more generally, it imposes a constraint on an adequate characterization of the operations of this cognitive system. In the remainder of this introduction I first provide the conceptual clarifications required to understand the overall argument against Carey's position; and I then articulate the broad strokes of that argument.

1.1 Core Cognition and Representational Format

Core cognition, on Carey's view, is intermediate in character between perception and (full-blown) conception. In particular, it shares the first of the following six features with (adult, acculturated) conceptual thought, and the remaining five features with perception[2]:

> *[1] [C]ore cognition has rich integrated conceptual content.... [2] [It] is articulated in terms of representations that are created by innate perceptual input analyzers.... [3] [T]he perceptual analysis devices that identify the entities that fall under core domains continue to operate throughout life.... [4] [S]ystems of core cognition are domain-specific learning devices.... [5] [S]ome core cognition... is shared by other animals. At least some early developing cognitive systems in humans have a long evolutionary history.... [6] [T]he format of representation of core cognition is iconic rather than involving sentence-like symbol structures. (Carey, 2009, pp. 67-68).*

I summarize this by saying that, on Carey's view, core cognition employs (a) *conceptual* representations, which are produced by (b) *modular* information processing, and which have (c) *iconic* format.[3]

By calling the representations of core cognition 'conceptual', Carey means roughly that: (a1) their contents are *rich* - they cannot be defined in sensory-motor or perceptual terms; (a2) they are *integrated*, or central, interacting with the representations from other core domains; and (a3) they are *accessible* at the personal level - for instance, they 'drive voluntary action' (Carey, 2009: 67).

Features (2)-(5) of core cognition, listed above, support the claim that the information processing producing the representations of core cognition is modular, in roughly Fodor's (1983) sense. In particular, claim (2) appears to say that the processing in question is innate, rather than learned; while (4) adds that it is domain-specific, another key indicator of Fodorian modularity. Claim (3) then suggests grounds for thinking that such processing is mandatory (and perhaps that it has a neurally fixed implementation); and (5) mentions a certain kind of evidence for innateness.

Finally, iconic representations, on Carey's view, are those that obey what Fodor (2007) calls *the picture principle*: every part of the representation represents part of what the whole represents. This contrasts with symbolic representations, which have privileged decompositions into meaningful constituents, which in turn contrast with other, meaningless parts. Thus, while every part of a picture of a dog represents a part of the dog, no part of the word 'dog' does (Carey, 2009, p. 135) – for example, 'og' does not represent anything at all. Moreover, while some combinations of the (meaningful) words of the sentence 'the dog is in the park' are meaningful (such as for instance, 'the dog'), not every such combination is (for instance, 'dog is' is not). This notion of iconicity will prove important below.

1.2 Explicit Representation and Content

Two more key notions are those of *explicit representation*, and *representational content*: allow me to explain what I mean by them. To begin with, consider the representation of objects.[4] A representation might, as a matter of fact, *concern* a given object without thereby representing it *as* an object: for instance, a visual representation might represent which colours are present at which locations in the subject's environment, and there might, in fact be an object at one of those locations; yet if that object were camouflaged, matching the colour of its environment, the subject might not represent it *as* an object, distinct from its background. In this case, we can say that the representation carries information *about* the object, and that it is a representation *of* that object; but, the object is not part of the *content* of the representation, for it is not represented *as* an object. Thus, while the representation may be *explicit*, it does not *explicitly represent* the object.

Alternatively, a representation might be sensitive to the fact that an object is an object: that is, it might well distinguish the object from its background, and serve to track it independently of its environment; and in such a case we can say that the object in question is represented *as* an object, and so is itself part of the *content* of the representation. Yet the representation could do this without explicitly representing the object as being similar to other objects (and different from, say, properties) in this respect; the property of being an object might not itself be part of the content of the representation. In such a case we could say that the object itself is *explicitly represented*; but it is only *implicitly* represented *as* an object. If, by contrast, the content of a representation is *that* a given object *is* an object (as it is in a sentence of the form, 'That's an object'), the property of being an object must also be explicitly represented, and the object in question is both explicitly represented, and explicitly represented *as* an object.

Carey holds that in core cognition, objects for instance, but also (approximate) cardinal numbers, are *explicitly represented* in the above sense – that is, that they are at least implicitly represented as such (i.e. objects, or as cardinals), so that the entities themselves are parts of the contents of the representations.

1.3 The Overall Argument

We are now in a position to understand the overall objection to Carey's view of the AM system. The reasoning behind this objection has two stages. First, it is argued that there can be no iconic representation of individuals as such; or, in other words, that no representation with iconic format can *explicitly* represent an individual. Second, it is argued that the explicit representation of (even approximate) cardinal numbers requires the representation of individuals as such; from which it is concluded the explicit representation of (approximate) cardinal numbers requires non-iconic format.

In the remainder of the paper, I present each stage of the argument in turn, before providing evidence that Carey really is committed to the claim that there is iconic, but explicit representation of (approximate) cardinal numbers. I conclude by considering and rebutting a possible reply, which itself sheds general light on the issues under consideration.

2. No Explicit Iconic Representation of Individuals

This section puts forward an argument to the effect that there can be no explicit representation of individuals, whether severally or collectively, in iconic format. I begin by considering two related arguments due to Shea (2011), which aim to show that there are problems surrounding the idea that the format of the representations of core cognition is iconic;[5] although they are not ultimately unilluminating, I find them wanting for present purposes. I then present a direct argument to the effect to there can be explicit representation of objects (severally) in iconic format, before considering and rebutting a possible reply. Finally, I note that the argument can be generalized, first, to individuals other than objects (e.g. dots and tones), and second to individuals taken collectively (i.e. pluralities of individuals).

2.1 Shea's Two Arguments

Carey (2009, p.5) follows Ned Block (1986) in holding that the content of a representation is determined by two factors: (i) what the representation causally co-varies with, or carries information about; and (ii) its inferential profile. It is this which, she thinks, licenses the view that the representations of core cognition can have iconic format despite their rich content. We will examine the theoretical underpinnings of her view in more detail below, but the following quote from Shea gives its flavour:

> *Carey's idea… is that something like a perceptual image is deployed in a set of conceptual roles that gives it a content like* ***object*** *or* ***agent*** *from core cognition. The object file system may work like that. Each object file may consist of an imagistic representation of an object and its perceptible properties (size, shape, colour, etc.). It is operations on these icons… which makes them into representations of* ***objects****. (2011, p.131)*

However, Shea continues:

> *If that is right, then the representations are indeed icons, representing properties like size, shape and colour iconically, but strictly speaking… objecthood… is [not] being represented iconically, since [it is] represented only implicitly, in the inferences made using those icons. (2011, p.131)*

Shea's thought here seems to be that although an icon may explicitly represent an object, it can at best only implicitly represent the property of being an object; but then there is no explicit iconic representation of the fact that the object is an object.

Whatever the merits of this argument, it doesn't establish what's needed in the present context because it is concerned with the explicit representation of the *property* of *being an object*, whereas we are concerned with the explicit representation of *objects themselves*. As we saw above, the property of being an object need not be explicitly represented and predicated of an object for that object to be explicitly represented; indeed, it is hard to see how it could be – to what representation of the object would the predication be applied? Rather, all that is required for the explicit representation of an object – i.e. for the representation of an object as such - is that the property of being an object be implicitly represented. Accordingly, this first argument of Shea's simply concedes what is in dispute here – namely, that an icon can explicitly represent an object.

A little later Shea suggests a second argument against Carey's view that the representations of core cognition have iconic format. Considering the case of our core cognition of agency, he writes:

> *It is hard to see how there are parts of the property of* ***being an agent****, so that parts of a representation could represent parts of the property (2011, p.131).*

The argument here seems to be roughly that there are no parts of properties such as those of being an agent, or being an object, so there are no parts of the representation of the property that represent its parts, and therefore the representations of objecthood and agency are not iconic.

There is clearly something appealing about this line of thought: obviously, properties like the ones with which Shea is concerned[6] don't have *spatial* parts; and it is hard to see how something without parts can be represented iconically. In fact, I will employ a variation on this argument below. Nevertheless, I have three concerns in the current context with Shea's second argument as reconstructed here.

First, it might be maintained that while the properties in question have no spatial parts, they do, nevertheless, have *abstract, logical* parts. For instance, part (but not all) of what it is to be red is to be coloured: and maybe, then, part (but not all) of what it is to be an agent is to be an object; or, similarly, part (but not all) of what it is to be an object is to be an individual.[7] If so, then the first premise in Shea's argument is false, and the argument is unsound. Of course, it might be replied that if there are parthood relations amongst abstract properties, this is not in the same sense that there are parthood relations amongst concrete objects; but this is contentious, and I will not attempt to adjudicate this dispute here.

Second, even if this premise is true, it is not clear that the (second) inference goes through. For it is at least consistent with what the picture principle requires that some iconic representations should be atomic, in the sense that they have no (proper) parts. Thus, if iconic representations of the properties in question are atomic in this sense, then it can be true – vacuously - that all of their parts represent parts of what they represent, even if what they represent has no parts. So one needs at least a further premise to the effect that the representations in question *do* have (proper) parts.

Of course, it might seem obvious that representations of agents do have parts, but this brings us to a further concern, namely...

Third, the contention is supposed to be that agents (or objects), for instance, are represented by icons, not that agency (or objecthood) is. And, of course, agents (and objects) *do* have parts. But then, provided that the representations of them do too, there might be iconic representations of agents (or objects), just as Carey maintains. The conclusion of the reconstructed argument above appears to be beside the point.

For these reasons, Shea's two arguments against the iconicity of the representations of core cognition will not serve our present purposes. New considerations are needed.

2.2 A Direct Argument

Consider the following direct argument for the claim that no object can be explicitly represented in iconic format:

> (P1) If an object is represented as such (i.e. explicitly represented), then it is represented in a manner that its parts are not represented (in that very representation).
>
> (P2) If an object is represented in a manner that its parts are not represented (in that very representation), then the format of the representation is not iconic.

Therefore,

> (C) If an object is represented as such (i.e. explicitly represented), then the format of the representation is not iconic.

P2 follows immediately from the fact that, by definition, iconic representations are representations that respect the picture principle; it is therefore uncontroversial. Thus, the only premise which stands in need of support is P1; but it can be supported as follows.

Suppose that I drop a tea cup and the handle breaks off. I represent the cup as losing its handle. If, by contrast, the previously detached handle is placed next to the handle-less cup in such a way that the break is visually indiscernible, and is then separated again, I do not represent the cup as losing its handle. What's the difference? In the one case, prior to the separation, I represent there being just one object (the cup, including its handle), while in the other I represent there being two objects (the handle-less cup and the handle). But this difference is not a difference that is visually apparent, by hypothesis. Thus, in the first case, the cup must be represented in a manner that its handle is not (namely, as a whole, independent object); and this despite the fact that, as far as any iconic representation is concerned, the handle is represented in this case in the same way that it is in the second case (when it *is* represented as an independent whole). Before I drop the cup I represent the whole in a different manner than I represent the parts; and indeed, this is necessary for me to represent the cup as an object. But of course there is nothing special about the cup in this respect; the same holds of any object, and P1 is true. It would seem, then, that both premises of this valid argument are true, and that the direct argument is therefore sound.

2.3 Reply and Rebuttal

It might be thought that Carey has a reply available in response to the direct argument above, appealing to Block's two-factor theory of content determination, along the following lines. An icon can carry information about, or causally co-vary with features of, an object (factor one): and by virtue of its inferential role it can represent the object *as* an object (factor two); and so an icon can explicitly represent an object, in spite of the direct argument to the contrary. In fact, Block himself writes:

> *The doctrine that picturelike representations won't do for general or adult or primate concepts involves a conceptual error, one for which [Block's preferred two-factor theory, conceptual role semantics, or] CRS is a corrective. CRS tells us that to be a concept of, say, dog, a mental representation must function in a certain way. Obviously, you can't tell how a certain representation functions by confining your attention to the representation alone, or to its "resemblances" to things in the world. You must know something about how the processors that act on it treat it. Thus a pictorial representation can express quite an abstract property, so long as the processors that act on it ignore the right specificities. (1986, p.663)*

Thus, a picture, or icon, can represent an abstract concept such as objecthood, according to Block, and so (we might suppose) represent an object as such, provided that it has the right inferential role.

But Block himself lays the foundations of the response I wish to give. He writes:

> *Functional differences determine differences in the semantic (and syntactic)* ***categories*** *of representations - for example, the difference between the representational properties of* ***languagelike*** *[i.e. symbolic] and* ***picturelike*** *[i.e. iconic] representations. (1986, p.662)*

So, on Block's view, inferential role determines not only the semantic, but also the *syntactic* properties of a representation. The problem, however, is that the syntactic features of a representation are *intrinsic* to it: representations with different syntactic features are *ipso facto* different representations. Sure, *something* can be associated with, or 'have', two different sets of syntactic features without thereby differing from itself; but that thing is not a representation. For instance, one sound pattern might be associated with two different sets of syntactic features; but then the sound pattern is not itself a linguistic expression, or representation. Consider, for example, 'Visiting relatives can be boring'. This has two distinct syntactic parsings in English – that is, the sound pattern is a phonetic realization of two distinct English sentences. One of these means that relatives can be boring when they visit; the other that the activity of going to visit relatives can itself be boring. But crucially, it is not the sounds that mean these things; it is the sentences. And since there is only one sound pattern but two sentences, the sentences are not (identical to) the sound pattern. Yet it is the sentences, of course, which are the representations – it is they which have the meanings.

Similarly, then, a given brain state *b* might carry information about some object *o*: and *b* might be associated with two distinct sets of inferential roles, through one of which *b* constitutes an icon, and through the other of which it constitutes a symbol in virtue of which *o* is represented as an object; but it doesn't follow that the icon represents *o* as an object any more than it follows that one syntactic parsing of an ambiguous sound pattern represents the meaning associated with the *other* parsing!

The basic problem, then, is that even if an icon can carry information about an object, and by virtue of some inferential role of the state realizing that icon the object in question is represented as an object, it doesn't follow that the icon itself (which is realized by the state by virtue of a distinct one of its inferential roles) represents the object as such (i.e. explicitly represents the object). In particular, then, even if the two-factor theory of content determination is correct, what represents the object as such is the state *together with its <u>symbolic</u> inferential role*. No icon explicitly represents any object.

2.4 Generalizations

So far, I have put forward a direct argument that an icon cannot represent an object as such - that is, there can be no explicit representations of objects in iconic format; and I have rebutted a possible reply to that argument. But the argument itself can be generalized: first, this can be argued to be hold not just of objects, but of individuals more generally; and second, a case can be made that this is equally true of pluralities of individuals. It is to these generalizations that I now turn.

Like Carey, by 'objects' I have meant up to now (roughly) 'bounded, coherent, 3-D, separable, spatio-temporally continuous wholes' (Carey, 2009, p.97). But while the illustration I gave in support of P1 of the direct argument above appealed to (something like) the full strength of this notion, in fact much less is required, and a version of the argument would go through with 'individual' substituted for 'object' throughout.

Take, for instance, a dot. It surely does not have all of the features listed above as being possessed by objects (for instance, it is not three-dimensional): but it *is* a bounded, continuous whole; that is, there is an unbroken line such that every part of the dot is contained within the area circumscribed by that line and every sub-region of that area is occupied by some part of the dot.[8] Moreover, it is this, or some closely related fact which allows us to individuate dots, and to distinguish them from other dots; in short, it this which makes them *individuals*.

But this suffices for us to support the analog of premise P1 in the new version of the direct argument. For suppose I look at the dot, and see it as a dot (i.e. a particular kind of individual). Then the whole dot is represented in a way that its (proper) parts are not – namely, as bounded and continuous in this sense; for clearly not every part of the dot lies within a region bounded by a proper part of the dot![9] So the individual in question – the whole dot – is represented in a manner in which its (proper) parts are not. Thus, if an individual dot is represented as such, it is represented in a manner in which its (proper) parts are not (in that very representation), and the analog of P1 holds.

It is quite obvious, I think, that something similar could be said about other individuals. Tones, for instance, are clearly bounded and continuous in time in something like the above sense. More generally, whatever kind of individual one selects, one will find a feature such that if one of them is to be represented as an individual, it must be represented as having that feature while its parts are not. This will suffice to make a version of the direct argument for that type of individual.

Equally, though, it is not just the explicit representation of objects, or individuals, *severally* that is subject to such a constraint; the explicit representation of objects, or individuals, *collectively* is too. If I am to represent some individuals collectively as individuals - that is, if I am to represent a plurality of individuals as such – then again, it must be that some parts of the plurality represented, namely those that constitute whole individuals, are represented in a manner that others (e.g. pairs of individuals, or mere proper parts of individuals) are not represented in that very representation. The finding, in other words, is robust: individuals cannot be represented as such, whether severally or collectively, in iconic format.

3. No Representation of Numbers without Representation of Individuals

In this section I complete the argument to the effect that there can be no (explicit) iconic representation of (approximate) cardinal numbers, by way of a very brief history of the philosophy of number.

3.1 Numbers are Abstract

Locke famously drew a distinction between primary and secondary qualities: roughly speaking, this is the difference between, on the one hand, those (primary) qualities our ideas of which resemble them, and which are 'utterly inseparable' (1975, bk. II, ch. 8, section 9) from the things having them; and, on the other, those (secondary) qualities our ideas of which do not resemble them, and which are 'nothing in the objects themselves, but powers to produce various sensations in us' (1975, bk. II, ch. 8, section 10). Locke thought number is a primary quality (1975, bk. II, ch. 16).[10]

Berkeley rejected the primary/secondary quality distinction on the grounds that 'an idea can be like nothing but an idea' (section 8); and when it came to number, he argued (1998, section 12) that it does not exist objectively in the things numbered. Roughly speaking, he claimed that number is relative to a concept, from which he concluded that it just as subjective as the secondary qualities:

> *Number is so obviously relative and dependent on men's understanding that I find it surprising that anyone should ever have credited it with an absolute existence outside the mind. We say one book, one page, one line… yet the book contains many pages and the page contains many lines. (1998, section 12)*

Frege, however, responded to Berkeley by accepting the premise of his argument, but rejecting the inference; thus, he claimed that, although number is relative to a concept (1980, section 46), concepts are objective (1980, section 47), and so numbers are too (1980, sections 26 and 47).

For Frege (1951), a concept is a function taking individuals (he calls them 'objects') onto the two truth-values, Truth, and Falsity; thus, roughly speaking, it is a rule for determining which things it applies to. The key point here given our current concerns is that number is a more abstract notion than is, say, shape, or colour: while it is *individuals* that have shape and colour, it is *concepts* with which numbers are associated. So numbers are associated, for Frege, not with concrete individuals, as properties are, but with concepts which in turn apply to individuals; and they are, in this precise sense, *abstract*.

I think that Frege was clearly right about numbers being abstract in roughly this sense; though I am not wedded to his somewhat idiosyncratic use of the term ‘concept’ to apply to the entities (if any[11]) with which they are associated. Indeed, I will sometimes speak of properties in this connection instead; and I am equally open, at least here, to the ideas that it is aggregates,[12] pluralities, or sets of objects that numbers are associated with, despite each of these being somewhat different from Fregean concepts.[13] But since (cardinal) numbers are abstract, then their explicit representation requires the representation of individuals (collectively) as such. More fully, if the (cardinal) number of some things is to be represented as such, then those things must be represented as individuals. And this, as we have seen, requires non-iconic format. So the representation of the cardinal number of some things requires non-iconic format.

3.2 Numbers and Quantifiers

Now, Frege held not only that numbers are associated with concepts, but further that they are individuals[14] that are so associated (1980, section 57). By contrast, Russell (1919) held that numbers are quantifiers: higher-order entities (second-level concepts, in Frege’s terminology) that take properties (or first-level concepts) to yield propositions which have truth-values; they are not individuals.

I will not take a stand here on whether Frege or Russell is right about the nature of numbers. The point of drawing attention to the disagreement between Russell and Frege is rather to flag up the interpretive possibilities when it comes to characterizing our core numerical cognition. To that end, note that in addition to the well-known quantifiers *everything* and *something*, there are many others, including the sequences *none, at least one, at least two,* and so on; *none, at most one, at most two,* and so on; and *none, exactly one, exactly two,* and so on. Of these, it is the third collection of quantifiers (the exact ones) that Russell identified with the finite cardinal (i.e. natural) numbers.

We can also consider quantifiers such as *approximately sixteen*: this is the (second-order) property that a (first-order) property has just in case approximately sixteen individuals instantiate that (first-order) property; which, I take it, any (first-order) property had by, say, fifteen individuals does.[15] While these *approximate* quantifiers are in some way numerical (they concern the question, how many?), they are not plausibly regarded as the natural numbers – any more than are the *at least* and *at most* quantifiers just discussed; and crucially, they are no less abstract than (Russellian or Fregean) numbers.

3.3 Carey on the AM System

We are now in a position to see how Carey characterizes the representations of the AM system. I want to highlight three features of her account. First, Carey says:

> *In the literature on mathematical cognition, analog magnitude number representations are sometimes called “numerosity” representations, for they are representations of the cardinal values of sets of individuals, rather than fully abstract number representations. There is no evidence that animals or babies*

> *entertain thoughts about 7 (even approximately 7) in the absence of a set of entities they are attending to. Still, cardinal values of sets are numbers, which is why I speak of analog magnitude number representations rather than numerosity representations. (2009, p.136)*

It is unclear from what is said here whether Carey thinks that numbers are quantifiers, or objects: but what Carey does clearly rule out is that numbers are objects, while AM representations merely represent quantifiers; thus, whatever (finite) cardinal numbers are on Carey's view, we can say (as a first pass at least) that she thinks that AM representations represent them.[16]

Secondly, these numbers are represented *explicitly*. Carey says that in the AM system:

> *The [numerical] symbols [or rather, representations] themselves are explicit.... But much of the numerical content of this system of representation is implicit. There is no explicit representation of the axioms of arithmetic, no representation that 1–1 correspondence guarantees numerical equivalence. These principles are implicit... in the computations defined over analog magnitudes. (2009, p.135)*

Thus, (finite cardinal) numbers are explicitly represented in the AM system, though they are only implicitly represented *as* (cardinal) numbers.

Finally, third, Carey says:

> *Analog magnitude number representations are analog in [the] sense [that they obey Fodor's picture principle]: the symbol for 3 (———) contains the symbol for 2 (——). (2009, p.135)*

In short, the explicit representations of cardinal numbers in the AM system are iconic in format. Thus, Carey seems to hold that the representations have just the features which the overall argument of this paper has been concerned to show no representation *can* have: iconic format, and cardinal numbers as contents. In what follows I consider and rebut a possible reply on Carey's behalf, before concluding.

3.4 Reply and Rebuttal

It might be objected that, even if no specific fault can be found with the argument presented here, still it cannot be sound, for it has a false conclusion. In particular, it might be maintained that there are models of the AM system in which cardinal numbers are represented in iconic format.

Consider, for example, the model employed by Stanislas Dehaene and Jean-Pierre Changeux (1993), which has been very influential in this area. Dehaene and Changeux's numerosity detector module consists of a four-layer connectionist network. The first layer carries information about the (one-dimensional) sizes and locations of the various objects with which the system is presented. The second layer factors this information into two-dimensions: one dimension keeps track of whether an object is centred on a given location in one-dimensional space; the other is a measure of how large the object at that location (if any) is. The third layer throws away the information about size, simply aggregating the amount of activity in the second layer: thus, there is more activity at this third level when more things are tracked at the second level. Finally, the fourth layer has nodes that respond differentially to the amount of activity in the third level: that is, roughly speaking, its nodes respond if, and only if, the level of activity in the third layer is between a certain lower bound *l* and a certain (distinct) upper bound *u*.

It might be thought that at the third layer of this network we have an iconic representation of number. This, however, would be a mistake. The problem (recall Shea's second argument above) is that cardinal numbers don't have parts.[17] This is true whether cardinal numbers are abstract *objects*, as Frege held, or *quantifiers*, as Russell maintained. In the first case they are mereologically atomic individuals. In the second, they have sets/pluralities/properties as *members*. But even if the members of a set were parts of it – which they *aren't*, parthood being more akin to the *subset* relation (see Lewis, 1991) - none of the members of (Russell's) two (viz., the two membered sets) are members of (his) three (viz., the three membered sets)!

The point can perhaps be more directly appreciated at an intuitive level: being exactly three in number does not have being exactly two in number as an abstract, logical part; the latter feature of pluralities of objects is not entailed by the former. Of course, being *at least three* in number entails being *at least two*. But being *exactly three* also requires being *at most three*: and this entails being *at most four*; yet no one wants to say that *four* is a part of *three* – and so, for reasons of symmetry, it should not be said that *two* is a part of *three* either. Nor does it help to recognize that Carey sometimes suggests that the contents of AM system representations are not the cardinal numbers themselves, but rather *approximate* cardinal numbers: she writes, for instance, that '[a]nalog magnitudes are explicit symbols [sic (representations)] of approximate cardinal values of sets' (2009, p.135). But if approximate cardinals are the quantifiers we saw above (does anyone have a better theory?), then *they* don't have parts either. That some things are approximately sixteen in number does not entail that they are approximately four in number - in fact, it entails just the opposite!

Thus, cardinal numbers, whether approximate or exact, do not have parts: and so assuming the representations of them do, they cannot be represented in iconic format; the reason being, of course, that not every part of a representation of a number will be a representation of a part of that number! In particular, then, it cannnot be the case that the parts of the representations at the third layer of Dehaene and Changeux's numerosity detector represent parts of the number that the whole activity level of this third layer represents, and the current objection is rebutted.

Nevertheless, a question obviously remains: What should we say about the contents and formats of any representations employed in the Dehaene and Changeux network, or the AM system which it models? This is a puzzling question to which I cannot provide an adequate answer here; indeed, I can only begin to hint at the difficulties involved in doing so.

As we have seen, Carey thinks that the representations of core cognition in general, and therefore of the AM system in particular, drive voluntary action. This suggests that they have truth-conditional (or propositional) content – for instance, in the case of a given AM representation, concerning some things, that they are approximately n in number. If so, then it might be thought that the output of the modular processing which yields AM representations – in the case of Dehaene and Changeux's model, the activity on the fourth layer - is a representation with such a content. Now, it is natural to discern parts within this content – roughly, one corresponding to the things, and another to their approximate number property. If we do, however, then we surely must regard the propositional content as involving predication of the latter with respect to the former. Yet it is very hard to see how such a predication could be explicitly represented without employing a symbolic representational format. On the other hand, perhaps the parts of the proposition in question, while they exist, are not explicitly represented in the AM system at all. If they are not, then their mode of combination will not need to be represented either, and symbolic format will not be required. Which, if either, of these alternatives is correct? And what consequences might there be for the representational contents, if any, of e.g. the other layers of the Dehaene and Changeux network? Only further detailed philosophical work, informed by the best available experimental findings and models in psychology, will allow us to decide; but an adequate account should bear in mind the argument presented here to the effect that the explicit representation of (approximate) cardinal numbers requires non-iconic representational format.

4. *Conclusion*

Carey (2009) holds that the representations employed in core numerical cognition have: (i) iconic format, obeying Fodor's (2007) picture principle; and (ii) (approximate) cardinal numbers as contents. In this paper I have presented an argument which raises a difficulty for this view: in particular, it maintains that there can be no iconic representation of individuals as such, whether taken severally or collectively; and that the representation of (approximate) cardinal numbers as such requires the representation of pluralities of individuals as such; from which it is concluded that the explicit representation of (approximate) cardinal numbers requires non-iconic representational format. I have considered possible replies to the reasoning employed, but found them wanting. More specifically, I suggested that Carey cannot reject the first premise by appealing to Block's (1986) two-factor theory of content determination, since by Block's own admission the inferential role of a representation determines not only its semantic, but also its syntactic properties. And, adapting a form of argument from Shea (2011), I argued that Dehaene and Changeux's influential (1993) model of the AM system does not cast doubt on the second premise, or the conclusion, of the argument: since (approximate) cardinal numbers have no parts, then cannot be represented iconically, with parts of the representation representing parts of the content. The result is therefore a serious challenge for Carey's view that our core numerical cognition – more specifically, the analog magnitude system - involves the explicit representation of (approximate) cardinal numbers in iconic format; and it places a constraint on any attempt to characterize the operations of this system.

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6. *Notes*

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[1] See e.g. Beck (2012, 2014), Burge (2010), Carey (2009, 2011), Dehaene (2011), and references contained therein.
[2] In the book, Carey claims that '[l]ater-developing explicit knowledge differs from core cognition in every single one of these six properties' (2009, p.68). However, in the *Precis*, she corrects this error (Carey, 2011, p.114), explicitly recognizing that the first feature is shared with conception.
[3] See Shea (2011, p.131) for a similar reading of Carey's notion of core cognition.
[4] In roughly Carey's (2009, p.97) sense – see below.
[5] Shea is primarily concerned with objecthood and agency, as we shall see. He is somewhat equivocal when it comes to the representations of the analog magnitude system: he denies that 'numerosity is being represented iconically' (2011, p.131); but he also says that '[i]f analog magnitude representations are realized by some quantity in the brain, then they will indeed be iconic' (2011, p.131).
[6] Shea is overtly concerned with *being an object* and *being an agent*; but what is relevant to his argument about these properties is that they are *atomic* in the sense of being primitive, or indefinable in more basic terms. (In this sense, the property of *being red*, for instance, is atomic.) Of course, what is at issue is whether they are atomic in the sense of having (proper) parts. Note that if they do, there is no part distinct from the whole that one can add to the proper part to yield the whole: this is what makes them conceptually primitive and indefinable.

[7] See, for instance, Yablo (1992) for a view that embraces (what are, roughly speaking) parthood relations amongst both properties and events. And in support of the specific claims of parthood mentioned in the main text, see Carey's (2009) discussion of these notions.

[8] A similar account can be given of bounded continuity for three-dimensional objects: just substitute 'surface' for 'line' and 'volume' for 'area'.

[9] This is perhaps especially obvious in the case of the scattered parts of the dots, which are not continuous, and therefore have no boundaries; but it is equally true of continuous, bounded proper parts – just think of those other parts of the dot that are outside this part's boundary.

[10] Notice that this suggests (given his characterization of the primary qualities) that he thought that number representations are iconic, not in Carey's sense of obeying Fodor's picture principle, but in the sense of representing by resembling. Moreover, like Carey, Locke thought that it is only with the acquisition of number words that we have precise ideas of large numbers.

[11] It is not obvious that pluralities of objects, for instance, are entities in their own right at all. Take some things. Ontologically speaking, are they, taken collectively, anything other than those same things taken individually? It is hard to see how they could be.

[12] See Burge (2010) for the view that it is aggregates with which numbers are associated; and see his (1977) for his theory of the nature of these entities.

[13] For instance, consider pluralities. Is there an empty plurality? If not, what is it with which the number zero is associated? Or take sets. It is widely held that there is no universal set; yet the concept of self-identity is universally applicable. So is there a greatest number – the number of everything? As Boolos (1998) has noted, the answer depends on which entities numbers are associated with. Important as these differences may be for certain scientific and philosophical purposes, they need not concern us here.

[14] Roughly, Fregean objects are Aristotelian substances of which something can be predicated, but which cannot themselves be predicated. In other words, they are individuals.

[15] It is less clear whether the property *approximately sixteen* is possessed by properties instantiated by twelve things. For the sake of concreteness, I will suppose that either it is, or it isn't, though I don't know which; and similarly for other approximate cardinalities.

[16] Given that the natural numbers are the finite cardinal numbers, how can we reconcile attributing to Carey the view that the latter are explicitly represented with her claim that 'none of the [three] representational systems that underlie infants' or animals' behavior on nonlinguistic number tasks [including the AM system] represent number in the sense of natural number or positive integer' (2009, p.296)? The answer comes from the requirement that to explicitly represent the cardinals, one must represent them, at least implicitly, as cardinals. Carey thinks that the AM system does not even implicitly represent the concept *natural number*, since it does not implicitly represent the concept *successor*; thus, it does not represent the natural numbers as such.

[17] In particular, then, two is not a part of three, as Carey seems to suggest above.