Target article authors
Sam Clarke and Jacob Beck

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Commentary title
Sizes, Ratios, Approximations: on What and How the ANS Represents

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Abstract
Clarke and Beck propose that the ANS represents rational numbers. The evidence cited supports only the view that it represents ratios (and positive integers). Rational numbers are extensive magnitudes (i.e. sizes), while ratios are intensities. It is also argued that WHAT a system represents and HOW it does so are not as independent of one another as the authors assume.

Main text
Some maintain that we do not have sufficient evidence to establish the existence of the Approximate Number System (ANS) - that experimental results fail to convince that there is sensitivity (in line with Weber's Law) to the 'numerosity' of collections of individuals rather than certain potential confounds such as the total area of the disparate region covered by those individuals. Clarke and Beck (2021) effectively refute such scepticism. They point to the existence of cross-modal studies, in which e.g. the 'numerosity' of a collection of dots is compared to that of a collection of tones, and ask the pointed question - what can the alleged confound be in such a case? They also draw attention to the dumbbell effect, which provides strong evidence that the sensitivity of the ANS is to the discretely varying size of a collection of individuals – a second-order property of a given scenario – rather than a continuously varying magnitude, such as that of the area covered by those individuals. Such results (alongside myriad others) leave no grounds for reasonable doubt that the ANS exists.
But what does this 'Number Sense' represent? Clarke and Beck suggest that the ANS represents, well… numbers – and more specifically, *rational* numbers. They also hope to show that appeal to facts about imprecision in the representational capacity of the ANS does not preclude such an answer, or support an alternative one, such as Burge's (2010) view that

the cognitive system in question represents the ‘pure magnitudes’ theorized by Eudoxos in antiquity.

Peacocke (2015) has clarified that Eudoxos’ pure magnitudes are extensive, meaning that they can be added to one another: if we take an object with mass m1, and combine it with an object with mass m2, the result is an ‘object’ with mass m1 + m2. Intensities, by contrast, cannot be added. Carey (2009) discusses density, which comes in degrees. We can say how dense something is (in comparison to other things), and even measure this quantitatively. Nevertheless, the density of an ‘object’ that results from combining two objects with densities d1 and d2 cannot be assumed to be the sum d1 + d2 – it depends on the relative sizes of the two objects that are combined! (The reason, of course, is that density is ultimately a relation between two extensities, the mass of an object and its volume.)

Clarke and Beck argue that the ANS represents rational numbers, and that this suggestion has ecological validity, since it is useful to an organism to represent e.g. probabilities (which are often – though not always - determined by certain ratios). Now, rational numbers are extensive magnitudes: it makes sense to ask how much ½ + ¾ is. But as far as I can see, Clarke and Beck cite no evidence that suggests additivity here. Take the (wonderful!) lollipop experiment they discuss: infants can succeed in choosing a jar with a greater chance that a lollipop randomly selected from it will be of their preferred flavour; yet this only requires that they represent the ratios of their preferred flavour to the other flavour (or to the total). Ratios, however, are intensities: we can compare them; but it makes no sense to ask what 1:2 + 3:4 is. (Indeed, ‘one is to two plus three is to four’ is ungrammatical.) Perhaps the conclusion that rational numbers are represented (rather than ratios) is premature.

Clarke and Beck are also keen to distinguish the question of *what* the ANS represents from that of *how* it does so: but care is required in practice to do so. Their view appears to be that the ANS does not represent numbers in the abstract, as objects; rather, it attributes number properties to collections of individuals - in its approximative way. But if the ANS attributes a numerical size to a collection of objects, we can surely ask what property exactly it represents that collection as having - and it seems we can distinguish the views that it attributes *being (roughly) such and such size* (which is in fact numerical, being a size of a collection) and that it attributes *being (roughly) so numerous*.

What would answer the question? Presumably something about the processing sensitivities of the ANS - though no theorist should embrace the strong sensitivity principle Clarke and Beck articulate, for the reasons they give. And Clarke and Beck are surely right that the ANS does not represent magnitudes that are indeterminate in kind between species that vary continuously and species that vary discretely – there are no such magnitudes (even if there are ‘pure’ continuous magnitudes that are, for instance, neither spatial distances nor temporal durations). Yet it might represent numerical sizes without representing them as varying discretely. Arguably, this would be so if the only computations performed on the representations were well-defined on continuously varying magnitudes as well, such as comparison and addition/subtraction. (A system that also exhibited sensitivity to whether there is a one-one correspondence between two collections might be said to represent certain magnitudes as cardinal numbers; and one that displayed a sensitivity to the immediate successor relation might be taken to represent some magnitudes as natural numbers, if these are taken to be things related to zero by the ancestral of that relation.) Is this only a question of *how* the (numerical) magnitudes are represented?

In any case, it seems there is a difference between attributing the properties of *being eight in number* and *being roughly eight in number*: if the collection to which the property is attributed has nine items in it, the second attribution is correct, while the first is not. So this distinction would appear to concern what is represented, not how it is represented. Perhaps Clarke and Beck will say this shows instead only that cognitive episodes involving the ANS have accuracy conditions, which admit of degrees, rather than veridicality or truth conditions, which do not - and that it is indeterminate what (i.e. which property) is represented by the ANS?

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Conflicts of Interest statement
There are no conflicts of interest to declare.

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