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THE THEORY OF CONFLICT ANALYSIS: A REVIEW OF THE APPROACH BY KEITH W. HIPEL & NIALL M. FRASER

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Abstract

In the spirit of theoretical pluralism, this chapter critically illustrates an alternative game theoretic approach that extends the Nash equilibrium criterion. It is assumed that players believe in the empathic ability to anticipate other players' simultaneous and future reactions to their strategic choice. An individual's best response strategy is defined based on this projection, adding additional stability conditions to strategic choice and increasing the set of potential equilibria beyond pure Nash equilibria. Among other interesting properties the approach can thus explain the occurrence of stable outcomes that are not Nash equilibria, such as the cooperative equilibrium in the Prisoner's Dilemma, without the necessity to change the game structure. *Conflict Analysis* further enlarges flexibility as the approach requires only an ordinal preference order. As a basis for future academic debates, the assumptions of the *Conflict Analysis* approach are critically analysed and applied to a set of games, demonstrating the approaches advantages and drawbacks.

Keywords: Game Theory, Computational Methods, Non-Nash Equilibria, Dominated Strategies

AMS Subject Classification: 65C, 91A, 91-08, 74G

1. Introduction

Conflicts are an essential part of interactions between and within any species. They play an important role in interpersonal relations (Lorenz, 1974).[1] and constitute a vital part in the description and modelisation of interactions between various agents. This chapter presents an interesting and efficient alternative game theoretical approach, not only to model conflicts between players, but also to analyse games in general.

During almost half a century behavioural economics has accumulated evidence that individuals do not make choices according to the assumptions of standard rationality (for an

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[1]Konrad Lorenz is one of the founding fathers of ethology and Nobel prize laureate for physiology or medicine in 1973. In "Das sogenannte Böse" (English title: "On Aggression") he described the root of aggression both for animals and humans and its impact as a primary instinct on social life.

overview, see Kahnemann, 2011) and has also browbeat classical game theory by showing that agents frequently choose strictly dominated strategies leading to stable outcomes that are not Nash equilibria. Though various approaches have been developed in recent years (see e.g. Kahneman and Tversky, 1979; Feinberg, 2005; DellaVigna, 2009; Binmore, 2009), a unified response to incorporate these discrepancies into game theory is still lacking.

This chapter provides one alternative by focusing on the rationality of players and follows Frank Hahn's call for theoretical pluralism.[2] The approach, demonstrated in this chapter, breaks with some of the concepts of rational choice in a way that might not be accepted by all scholars. Yet, the general purpose is to illustrate an alternative perception of rationality. It is intended to give rise to academic debate and to call economists' attention to this approach as a potential step towards an augmented game theoretical framework that is able to explain the additional occurrence of non-Nash equilibria.

The approach, henceforth called "Conflict Analysis", was originally developed by Fraser and Hipel (1984) and builds on the initial work of Howard (1971). It enjoys some interesting properties that render it especially interesting: It is able to model a higher order reasoning that allows players to anticipate other players' reaction to their strategic choice. It thus expands the original stability concept and obtains equilibria that are plausible but not identified as such by the Nash criterion. Yet, this approach does not require that these "empathic" anticipations of strategy choice and of preferences to be correct. The possibility to model a game as a "hypergame" (of higher order) allows for conditions, in which players display a misperception of the rules underlying the game. Furthermore, the approach is capable of tackling larger (non-quantitative) strategy and player sets than most game theoretical approaches solvable in a closed form. Another advantage, though not unique to this approach, lies in the sole requirement of only an ordinal preference order. A cardinal order should demand detailed data on the relative weights that the conflicting parties place on the various outcomes associated to the strategy profiles. In contrast, this approach stays completely within the concept of preference relations, without the necessity of a function that attaches real numbers to each element in the outcome set, thus completely avoiding the standard notion of utility representation. A quantification of the players' preferences, which is because of the lack of precise data mostly arbitrary, can thus be avoided. In addition, the pairwise comparison of preferences over outcomes does not require transitivity of preferences.

After reviewing the theoretical basis of Fraser's and Hipel's approach, it is applied to standard one-shot games, to paradoxes and to sequential games. This chapter further analyses the requirements for the "rational" validity of Conflict Analysis and illustrates both the differences and the potentials with respect to standard game theory, discussing the advantages, as well as the drawbacks of this approach.

The first section illustrates the general theoretical basis and methodology necessary to analyse games. It constitutes a more concise and analytical representation of the solution algorithm explained in "Conflict Analysis - Models and Resolutions" (Fraser and Hipel, 1984). Section 3 exemplifies the approach via the Prisoner's Dilemma. It demonstrates how misperception of preferences can be modelled, and introduces a simple dynamic rep-

[2] Hahn states: "...[A]ll these 'certainties' and all these 'schools' which they spawn are a sure sign of ignorance ... we do not possess much certain knowledge about the economic world and ... our best chance of gaining more is to try in all sorts of directions and by all sorts of means." (Hahn, 1985, p. 8)

resentation. Section 4 shows how the approach manages certain popular dilemmas and paradoxes. Section 5 applies this approach to various sequential games and shows a way to discriminate between the equilibria obtained. Section 6 provides a critical examination of the theoretical background of Conflict Analysis, discussing eventual deficiencies that arise. Section 7 constitutes the conclusion. The appendix (section A) contains a short introduction to Metagame Theory, for the interested reader.

2. Solution Algorithm

In the following the solution algorithm is represented in its easiest form. The general idea is that not all equilibria of a game are captured by the Nash equilibrium concept, but additional stable equilibria exist that fall outside the definition of a Nash equilibrium. It is assumed that an individual does not only consider those strategies currently chosen by other players, but also takes account of the subsequent and simultaneous potential reactions of other players by applying a certain form of backward induction. Though an individual does not assign specific probabilities to how likely another player chooses a certain subsequent (or simultaneous) response strategy, it is assumed that an individual refrains from playing a strategy that will trigger a response with positive probability, which will change his outcome for the worse. In addition, since the utility ranking is purely ordinal and no pay-offs are assigned to any strategy profile, equilibria are only defined by pure strategies.[3]

2.1. Stability Conditions

An n-person non-cooperative game is defined by $G = (S_1, S_2, ..., S_n; U_1, U_2, ..., U_n)$, with player set $N = (1, 2, 3, ..., n)$. S_i being individual i's strategy set and U_i being defined as i's preference function for each $i \in N$. For the given set of players and the individual strategy sets, a strategy profile is defined by $s = (s_1, s_2, s_3, ..., s_n)$, with $s_i \in S_i$ being the strategy chosen by individual i. The set of all strategy profiles is then defined by $S = S_1 \times S_2 \times S_3 \times ... \times S_n$. There exists a preference function U_i that ranks the strategy profiles according to individual i's preferences over the associated outcomes. U_i is not necessarily a utility or pay-off function, it suffices that it assigns all strategy profiles in the strategy profile set to two subsets with respect to any underlying strategy profile.[4] Hence, U_i is not required to attach a real number to all outcomes in order to present i's preferences. Assume q and p to be two strategy profiles and denote the associated outcome by $O(q)$ and $O(p)$, then

$$\begin{aligned} p \in U_i^+(q), \text{ iff } O(p) \succ_i O(q) \\ p \in U_i^-(q), \text{ iff } O(p) \preceq_i O(q) \end{aligned} \tag{1}$$

with $U_i^+(q) \cap U_i^-(q) = \emptyset$ and $U_i^+(q) \cup U_i^-(q) = S$. Hence, for each strategy profile q, set S is divided into a set of strategy profiles, whose outcomes are strictly preferred to the outcome

[3] On the issue of pure strategies, see Harsanyi, 1973; Morris, 2006. The assumption, by which a minimal loss probability, deriving from a strategy switch, is sufficient to deter a player from choosing this strategy, is similar to the maxmin criterion used in Decision Theory, see Gilboa and Schmeidler, 1989; Hey et al., 2008.

[4] In other words, U_i is a function from set S to its Boolean $B(S)$.

associated with q and a set of strategy profiles, associated with those outcomes that are not preferred to $O(q)$. Consequently, completeness is the only axiom that must hold for the underlying preferences. Owing to the pairwise comparison of outcomes, transitivity is unnecessary. This chapter will, however, concentrate on games, which exhibit transitive preferences.[5] For convenience I will henceforth use the formulation that strategy profile q is preferred to p, meaning that the outcome associated to q is preferred to the outcome associated to p.[6] The preference order thus obtained will be the basis of the analysis. In order to determine equilibrium strategies, first define the set of strategies to which a player has a possible incentive to switch, given the strategies of the other players. These strategies are defined by the set of "dominant profiles". Second, these strategies need to be analysed for their validity, i.e. if the player has still an incentive to switch after taking into account the potential simultaneous and subsequent responses of other players:

I. A given strategy profile $q = (\bar{s}_i, \bar{s}_{-i})$ is defined by the strategy $\bar{s}_i$ of player i and the strategy profile $\bar{s}_{-i}$, determined by the strategic choice of all players other than i. Denote a strategy profile, which can be obtained by a unilateral strategy switch, by $z_i(q) = (s_i, \bar{s}_{-i})$, with any $s_i \in S_i$. Given the set $Z_i(q)$ of all strategy profiles that can be obtained by a unilateral switch of i, the set of "dominant profiles" for q is then defined as

$$\textbf{DP:} u_i^+(q) = Z_i(q) \cap U_i^+(q), \forall s_i \in S_i \tag{2}$$

In other words, for each underlying strategy profile, the possible better response strategies of player i are defined by a set of strategy profiles *(first)* that can be obtained by a unilateral strategy switch, given the strategy profile of all players other than the player being analysed, and *(second)* that are strictly preferred to the current strategy profile by this player.[7] Hence, we do not only assign a single strategy profile defined by the best response strategy to a given strategy profile, but all strategy profiles $p = (s_i^*, \bar{s}_{-i})$ for any $s_i^* \in S_i$, such that $O(p) \succ_i O(q)$. In other words, all those strategy profiles defined by all possible "better" response strategies of player i define a dominant profile. - Throughout this chapter a dominant profile is denoted in short as *DP*.

II. It needs to be checked whether such a possible better response is still valid, if a player reasons about sequential or simultaneous better response strategies of other players.

[5] It must clearly hold that for any strategy profile o either $o \in U_i^+(q)$ or $o \in U_i^-(q)$, but never both since both sets are disjoint, nor neither, as $U_i^+(q) \cup U_i^-(q)$ cover the entire outcome space. The presentation in 1 allows for intransitivity. Given $O(o) \succ_i O(p)$, $O(p) \succ_i O(q)$, but $O(q) \succ_i O(o)$, the intransitive preference is represented by $o \in U_i^+(p)$, $p \in U_i^-(o)$, $p \in U_i^+(q)$, $q \in U_i^-(p)$, $q \in U_i^+(o)$, and $o \in U_i^-(q)$. Transitivity should be only required, if it strictly holds that $O(p) \succ_i O(q)$ implies $U_i^-(q) \subset U_i^-(p)$.

[6] Strictly speaking, an individual does not retain a preference over strategy profiles, but over the associated outcomes/consequences. It is assumed that in a given state ω a unique strategy profile is associated to each outcome and, hence, for state ω we can simply speak of an individual preference order over strategy profiles. In another state ω', a player might associate other outcomes to a strategy profiles. Thus the preference order is not unique, but depends on the state.

[7] Notice that for complete and transitive preferences and a strict preference order, such a set can only consist of strategy profiles that lie in the direction of preference (here: to the left in the preference order in the later representation). Also notice that in this definition, a *DP* necessitates strict preference. In later games I will relax this assumption and illustrate the effect of weak preference on the equilibrium set.

As a result, in addition to the standard Nash equilibria, Conflict Analysis defines two additional criteria for stability: sequential and simultaneous stability. In general, if a strategy is the unique *valid* best response, the strategy is considered to be *stable* for this player since he has no incentive to switch. Though a slight abuse of the standard definition, a *strategy profile* defined by such a stable strategy for player i is also defined as being *stable for player i*. An equilibrium is thus given by a strategy profile, in which each component is a stable strategy given the other strategies; or using the slightly abusive definition, by a strategy profile that is stable for all players. For any strategy profile $q = (\bar{s}_i, \bar{s}_{-i})$ the following forms of individual stability exit for any player i:

(a) Rational Stability: Like in the standard Nash approach, an individual has no incentive to change his strategy, if he is already playing the *rational* best response to the strategies chosen by all other players, implying that no other possible better response strategy exists. If strategy profile q characterises the best response strategy for player i to all other players' strategies in the strategy profile, this strategy profile is defined as *rationally stable for player i*. Hence, a strategy profile q is rationally stable for player i, if the set of dominant profiles is empty. Thus for $q = (\bar{s}_i, \bar{s}_{-i})$ to be rationally stable for player i, it must hold:

$$\textbf{Rational Stability: } u_i^+(\bar{s}_i, \bar{s}_{-i}) = \emptyset, \forall s_i \in S_i \tag{3}$$

(b) Sequential stability: A switch of player i to a better response strategy can entail a subsequent switch in strategies of another player j, since j's strategy is no longer best response. This may result in a strategy profile that is not strictly preferred to the original strategy profile by player i. Consequently, player i will refrain from choosing this possible better response strategy, since a switch will not make him better off. If all possible better response strategies will eventually lead to not strictly preferred outcomes, the current strategy defined by the underlying strategy profile is best response; thus this strategy profile is defined as *sequentially stable for player i*.

Assume that player i switches from the strategy defined by strategy profile q to a possible better response strategy, thus changing the outcome to a strategy profile defined in the set of *DP*'s for q. Let this *DP* be defined as p. Remember that all possible better response strategies for any player are defined by the individual dominant profiles for this player. The set of better response strategies of another player j to the new strategy profile p is then defined by j's *DP*s for profile p. It is thus sufficient to look at the *DP*s of player i for strategy profile q and at the *DP*s of all other players different from i for p. Consequently a *DP*, or more correctly the strategy of player i defined by the *DP*, is sanctioned, if there exists some player j, who can choose a *viable* response strategy to the possible better response strategy (defined by p) of player i in such a way that the resulting strategy profile is not strictly preferred by player i to the one from which he originally deviated (q). Viable is thereby defined as a strategy switch that immediately results in a strategy profile strictly preferred by player j (i.e. defined by a *DP* of player j to strategy profile p).[8]

[8] The viability assumption avoids that a player strategically chooses a strategy that deteriorates his utility

As a result, in order for a *DP* to be *sequentially* sanctioned, it is already sufficient that at least one possible better response strategy (i.e. *DP*) of one other player to player *i*'s strategy choice exists that induces a less preferred strategy profile for *i*. If all "better response" strategies of player *i* are sanctioned in such a way, the current strategy is best response.[9]

For any player *j* define $\hat{u}_j^+(p) = Z_j(p) \cap U_j^+(p)$ as the set of *DP*s for player *j* to player *i*'s dominant profile *p* for *q*, i.e. the set of strategy profiles obtained by player *j*'s better response strategies to strategy profile $p = (s_i^*, \bar{s}_{-i})$, with $O(p) \succ_i O(q)$. In order for $q = (\overline{s_i}, \bar{s}_{-i})$ to be sequentially stable for player *i*, it must hold:

$$\textbf{Sequential stability: } \hat{u}_j^+(p = (s_i^*, \bar{s}_{-i})) \cap U_i^-(q = (\overline{s_i}, \bar{s}_{-i})) \neq \emptyset, \quad \forall s_i^* \in S_i : p = (s_i^*, \bar{s}_{-i}) \in u_i^+(q) \textit{ and for any } j \neq i \tag{4}$$

(c) Instability: If *i*'s strategy defined by *q* is not best response to the other players' strategies defined in *q* (i.e. if the set of dominant profiles for *q* is not empty and at least one dominant profile is not sequentially sanctioned by a viable response strategy of at least one of the other players) strategy profile *q* is termed *unstable for player i*. In other words strategy profile *q* is unstable, if neither condition 3 nor 4 hold. Hence, player *i* will switch to the strategy defined by the unsanctioned *DP*, as this will lead with certainty to a strictly preferred strategy profile. As a direct result from the previous definitions, for profiles *q* and *p* defined as before, *q* is unstable, if:[10]

$$\textbf{Instability: } \exists p \in u_i^+(q) : \hat{u}_j^+(p) \cap U_i^-(q) = \emptyset, \forall j \neq i \text{ and some } s_i^* \in S_i : p = (s_i^*, \bar{s}_{-i}) \tag{5}$$

(d) Simultaneous stability: In addition to the previous types of stability, simultaneous stability can occur in games that are not sequential or if the other players' strategy choices are mutually unknown. It is generally a weaker and rarer form of stability and should be checked for plausibility.[11] The main idea is that if more than one player *simultaneously* switch strategies from a current strategy profile, where possible better responses exist for those players, the resulting strategy profile may be not strictly preferred by the player currently analysed. Hence, probable occurrence of such a simultaneous strategy change deters the player from switching his strategy. If for all strategies of player *i*, which destabilise *q* through equation 5, such a simultaneous switch of other players occurs with positive probability and,

hoping that the others response strategies will eventually make him better off. The assumption also evades cycles. This is, however, not the case if we assume that equally preferred strategy profiles can be *DP*s, i.e. if only a weak preference is necessary. Therefore I am in favour of refusing this last assumption of equally preferred strategy profiles serving as *DP*s. Examples in section 4 will elaborate this issue.

[9] One might think of sequential stability as the best response strategy derived from backward induction with high ambiguity aversion. We will, however, observe in section 4 that backward induction is not equivalent to sequential stability.

[10] Note that the following definition also includes $\exists p \in u_i^+(q) : \hat{u}_j^+(p) = \emptyset, \forall j \neq i$ and some $s_i^* : p = (s_i^*, \bar{s}_{-i})$.

[11] Section 3.3 will discuss a game, where simultaneous stability is of major importance, as it captures an effect similar to risk dominance.

in addition, can lead to a strategy profile not strictly preferred to q by player i, then strategy profile q is termed "simultaneously stable for player i". Other players are only likely to switch, if they also have a valid better response for q. The set of other players is thus defined by all those players for whom q is unstable. Consequently, simultaneous stability needs only to be checked for strategy profiles that were previously defined as unstable and only for those corresponding *DPs* that are not sequentially sanctioned.

Since a player has no information about the strategy choice of other players, a simultaneous strategy switch can be effected by the entire set of players, who possess a viable, not sequentially sanctioned, better response strategy, but also only by a subset.[12] Let o be a possible strategy profile resulting from a simultaneous switch in strategies of other players for whom there exists a non-sanctioned *DP* for some strategy profile q. Hence, o is defined both by components that are identical to those in q (the players, who did not switch) and by components consisting of strategies defined by a not sequentially sanctioned *DP* of q for each player that switch. This includes also player i's switch to some strategy s_i^o fulfilling condition 5. If o is not strictly preferred to q by player i, strategy s_i^o is "simultaneously" sanctioned and will not be chosen by i. If this is the case for all *DPs* that rendered the strategy profile unstable for i, strategy profile q is *simultaneously stable for i*. The current strategy is best response with respect to the possible simultaneously chosen strategies of the other players, who have an incentive to switch.

For any player j, define $S_j^\varsigma(q)$ as the set of strategies of player j that render a strategy profile q unstable plus the strategy originally played, i.e. all those strategies that are defined by the unsanctioned *DP*s of q according to equation 5, as well as the strategy $\bar{s}_j$ corresponding to profile q. Thus, for $q = (\overline{s_i}, \bar{s}_{-i})$ the set of simultaneously attainable strategy profiles is given by $S^\varsigma(q) = S_1^\varsigma(q) \times S_2^\varsigma(q) \times ... \times S_m^\varsigma(q) \times \bar{s}_{-M}$ for a player set $M = (1, 2, \ldots, m) \subseteq N: \; h \in M$, iff $(S_h^\varsigma(q) \backslash \{\bar{s}_h\}) \neq \emptyset$. In other word, set M is defined by those players, who posses a non-sanctioned *DP* for q, including player i.[13]

$$\textbf{Simultaneous stability: } \forall s_i^\varsigma \in S_i^\varsigma(q), \exists \hat{s}_{-i}^\varsigma \in S_{-i}^\varsigma(q) : (s_i^\varsigma, \hat{s}_{-i}^\varsigma) \in U_i^-(q) \qquad (6)$$

III. The definition of an equilibrium is identical to the standard approach. The set of equilibria of the game is specified by all strategy profiles, in which each component is defined by the best response strategy of each player given the strategies chosen by all other players, i.e. all those strategy profiles that are stable for *all* players (either rational, sequential or simultaneously). Notice, however, that only if a strategy profile is *rationally* stable for all players, it is a Nash equilibrium. All those equilibria that are not rationally stable for one or more players but either sequentially or simultaneously stable would not be defined as an equilibrium in the standard approach.

[12] e.g. This implies, that seven possible player combinations for each player i have to be analysed ($\sum_i \binom{m-1}{i}$) for a case, in which a strategy profile is unstable for four players of the entire player set.

[13] Hence, simultaneous stability adds the idea of eventual simultaneous switches to the underlying assumption of backward induction.

An example will be given in the following section. The next subsection will elaborate the form of representation used in this chapter.

2.2. Representation

Since this approach goes beyond the Nash definition of an equilibrium by adding sequential and simultaneous stability, a representation of a game in normal or extensive form is insufficient. It is therefore necessary to spend a few words on the structure of analysis. Each strategy can define a set of actions, such that an individual strategy consisting of r independent actions is defined as $s_i = (a_{1i}, a_{2i}, \dots, a_{ri})$. A player has the choice of whether or not to take a certain action. Define the set $A_{ki} = (a_{ki}, \neg a_{ki})$, so set A_{ki} consists of two elements, the first meaning that action k is chosen by player i, the second that it is not. Whence we obtain that $S_i \subseteq \times_k A_{ki}$, where the equality holds if none of the actions are mutually exclusive. Hence, each strategy of a player i can be uniquely defined by a binary vector of length equal to the number of actions that player i possesses. Similarly, also each strategy profile can be uniquely identified by a binary vector equal in length to the sum of all available individual actions. Each element in this vector defines an individual action and its value whether the action is chosen or not.

Assume a game with three players, where the strategy set S_i is defined by the number of actions $x = |\bigcup_k A_{ki}|$, strategy set S_j by the number of actions $y = |\bigcup_k A_{kj}|$, and strategy S_h by the number of actions $z = |\bigcup_k A_{kh}|$, implying that the actions are mutually non-exclusive for player i, j and h, respectively.[14] In such a three player game, each strategy profile $q = (s_i, s_j, s_h)$ can be defined by a binary vector of length $x+y+z$, given by $\hat{q} = (\overbrace{I,I,\dots,I}^{x},\overbrace{I,I,\dots,I}^{y},\overbrace{I,I,\dots,I}^{z})^T$. I denotes a binary value of either 0 or 1, where 1 implies that the action is chosen, 0 that it is not. Hence, each player individually defines the sequence of this binary vector for a length equal to the number of available actions. As an example, for a three player game, in which each player has two mutually non-exclusive actions, one strategy profile p is defined by $\hat{p} = (0,1,1,0,1,0)^T$. The length of this binary vector can be reduced in the case of mutually exclusive actions. If an action A can only be chosen, if an action B is not and the inverse, but one action has to be chosen, then both action can be described by a single digit in the binary vector. $I = 1$ could be defined as A is chosen by a player i, and thus $I = 0$ would mean that B is chosen.

Each such binary vector can be again uniquely defined by a decimal code, calculated as follows: In general the binary vector has $|\bigcup_{i\in N}(\bigcup_k a_{ki})|$ digits (less the number of those actions reduced by the aforementioned simplification in the case of mutually exclusive actions) that have either the value 1 or 0. Like the binary code of a computer this can be rewritten by taking the sum over the products of the digit times two to the power of the position in the vector. Consequently, the example $\hat{p} = (0,1,1,0,1,0)^T$ can be written as $0*2^0+1*2^1+1*2^2+0*2^3+1*2^4+0*2^5 = 22 = \dot{p}$. The value of 22 does not represent a preference, but is the short representation of a strategy profile.

A preference order can thus be defined as a vector of length equal to the sum of actions ($|\bigcup_i \bigcup_k A_{ki}|$), reduced by the actions that are mutually exclusive, that can be transformed into a natural number defining a strategy profile.

[14] $|.|$ denotes the cardinality of a set, i.e. absolute number of elements in the set.

Given the assumptions, the preference function U_i orders the strategy profiles into the preference vector according to the preferences of player i over the associated outcomes. Since preferences are strictly ordinal, it suffices to note down the natural numbers, identifying each a strategy profile, in a vector, where the position of the component indicates the preference. Starting with the most preferred, strategy profiles can be ordered from the left most position to the right. This implies that for strict and transitive preferences each strategy profile can have only one position in the preference vector and it is strictly preferred to all strategy profiles noted further to the right, i.e. for $O(q) \succ_i O(p) \rightarrow U_i = (\ldots, \dot{q}, \ldots, \dot{p}, \ldots)$.

3. Example - A Prisoner's Dilemma

This section will illustrate the approach presented above. First notice that for simplicity, whenever the game representation has been changed to the game form used in the Conflict Analysis approach, I will speak of strategy profile $\dot{x}$, where $\dot{x}$ is in fact the natural number defined by the decimal code that refers to strategy profile x.

For two reasons, the Prisoner's Dilemma (PD) is chosen as an example: First, it is a simple game known to most social scientists and second, it also shows some theoretical intricacies, unapparent in other games (for a detailed discussion of theoretical issues concerning the PD, see section 6 beginning on page 31). Suppose a game G with two players $i = A, B$, where each player possesses an action, which he is free to take or not, and thus two strategies $S_i = \{\text{not confess}, \text{confess}\}$. The pay-offs represent the players' preferences over the outcomes, each defined by a strategy profile. Hence, the pay-off of player i is given by $\pi_i(s_i, s_j)$, with $i \neq j$ under strategy profile (s_i, s_j). Furthermore, assume that pay-offs are symmetric, i.e. independent of a player's position. The general symmetric 2x2 game is represented by the following normal form game:

$$\begin{array}{c} \\ \textbf{not confess} \\ \textbf{confess} \end{array} \begin{array}{c} \begin{array}{cc} \textbf{not confess} & \textbf{confess} \end{array} \\ \left(\begin{array}{cc} a,a & b,c \\ c,b & d,d \end{array} \right) \end{array} \tag{7}$$

Assuming $c > a > d > b$ and $2a > b + c$ turns the game into a Prisoner's Dilemma. Joint non-confession is welfare maximising, but joint confession is the single Nash equilibrium, since *confess* strictly dominates *not confess*.

In this game, each player has the choice over the single action ($a_i = confess$). Each of a player's two strategies can thus be defined by a single binary value, and a strategy profile can be uniquely defined by a vector with two binary components, one for each player. A vector $(1,1)^T$ means that both players confess, whereas $(1,0)^T$ implies that player A confesses, but player B does not. Each of these strategy profiles can be converted to a decimal value based on the strategy composition. Table 1 illustrates the decimal representation.

Following the previous assumption that $c > a > d > b$, the preference order for player A is represented by a vector $(1,0,3,2)$ and for player B the preference order is defined by vector $(2,0,3,1)$. By condition 2 and since *confess* strictly dominates *not confess*, it holds that $u_A^+(1) = \{0\}$, $u_A^+(3) = \{2\}$, $u_B^+(2) = \{0\}$, and $u_B^+(3) = \{1\}$, and all other sets are empty. Player A can unilaterally improve from strategy profile 0 *(not confess, not confess)* to the

Table 1. coding the binary strategy profiles into decimal digits - strategy coding: confession=1, no confession=0

Set of Strategy profiles				
Player A	0	1	0	1
Player B	0	0	1	1
Decimal Code	**0**	**1**	**2**	**3**

dominant profile 1 by choosing strategy *confess*. This enables him to increase his pay-off from a to c. Furthermore player A can unilaterally switch from strategy profile 2 to the dominant profile 3. For player B the *DP* from strategy profile 0 is strategy profile 2, from strategy profile 1 it is strategy profile 3.

By condition 3 strategy profiles 1 and 3 are rationally stable for player A, and strategy profile 2 and 3 are rationally stable for player B. Since a switch to strategy profile 3 is unsanctioned, strategy profile 2 is unstable for player A according to condition 5. The same holds for player B with respect to strategy profile 1. A strategy switch from strategy profile 0 to his *DP* is sanctioned for both players through the subsequent switch of the other player to strategy *confess*. Since $O(0) \succ_i O(3)$ condition 4 holds for both players. By definition of stability, strategy profiles 0, 1, and 3 are stable for player A, strategy profiles 0, 2, and 3 are stable for B. Consequently, 0 and 1 define the equilibria of the game.

The game can be much easier analysed, especially in the case of more strategies and players, when put into a form similar to the classical normal form. The sequential analysis renders, however, the normal form insufficient. The following presentation is thus a mix between the normal and extensive form. It will be used throughout the remaining parts of the chapter, since it succinctly represents both the game's dynamics and equilibria.

Table 2. Solution to the Prisoner's Dilemma

Stability Analysis					
equilibrium		x	E	E	x
Player A	Stability	r	s	r	u
	Preference Order	1	0	3	2
	DP		1		3
Player B	Stability	r	s	r	u
	Preference Order	2	0	3	1
	DP		2		3

In order to derive table 2, first, note down the preference order for both players. Those have already been derived above from condition $c > a > d > b$. Second, note down the dominant profiles given by condition 2 under each strategy profile in order of preference. Notice again that, as in the Prisoner's Dilemma preferences are strict, a *DP* can only be a strategy profile that appears to the left of the strategy profile, under which the *DP* is written. Consequently, the strategy profile that is farthest to the left is always rational, since a *DP* cannot exist. Based hereupon, the stability of each strategy profile can be analysed according to the conditions described above. The abbreviation for the stability should be read as follows: u - unstable, r - rationally, s - sequentially, and $\hat{u}$ - simultaneously stable.

Starting with player A, for strategy profile 1 and 3, the boxes indicating their *DPs* are empty. Hence, both strategy profiles are rationally stable. For player *A*, the *DP* of 0 is to switch to 1. Yet, the *DP* of 1 for player B is 3. Since 3 is further right than 1 in the preference order of player A (i.e. it is strictly less preferred), this strategy switch is sequentially sanctioned. Since this is the only available *DP* for 0, this strategy profile is sequentially stable for player *A*. A switch of *A* from 2 to 3, on the contrary, does not affect player B's strategy choice, since 3 has no *DP* for that player. A switch from 2 =*(not confess, confess)* to 3 =*(not confess, not confess)*, will thus not trigger any response by player *B* and thus player *A* can impose unilaterally the preferred strategy profile. The same analysis for player *B* reveals that 1 is unstable, but a switch from 0 to 2 is sanctioned by player *A*'s shift to 3.

It remains to test for simultaneous stability of strategy profile 2. Since player B has no *DP* from 2 condition 6 does not hold. The same with respect to 1, from which player *A* has no *DP*. Notice that if strategy profile 0 were not already sequentially stable, it would be simultaneously stable.[15] Each strategy profile that is not assigned a u defines an equilibrium, illustrated by E in the top row.

Thus, for the Prisoner's Dilemma we obtain two equilibria in pure strategies. One is defined by joint non-confession, the other by joint confession. None of the two strategies is strictly dominant.[16] This contrasts with the classical analysis. Conflict Analysis keeps the game's structure, and has therefore an advantage over explanations using other regarding preferences, if the original notion of Prisoner's Dilemma and its validity for social interactions should be maintained. By transforming the pay-off matrix in such a way, the game ceases to be a Prisoner's Dilemma. Inference can only be made for this new game as the rules of the game are changed and not the structure of analysis. In the Conflict Analysis approach the class of preference ordering, on the contrary, is not enlarged beyond the original definition of a PD, since preferences stay purely "self-referential". Yet, the stability of the cooperative equilibrium requires at least a supplementary assumption. Furthermore, both equilibria will not occur with equal probability and the cooperative equilibrium will only arise, if additional conditions hold. A discussion of these issues is postponed to subsection 3.2 on page 14 and section 6, beginning on page 31.

Also notice that the analysis of simultaneous stability can be simplified by using the decimal value that is attributed to each strategy profile. Assume that a strategy profile with decimal value $\dot{q}$ should be tested for simultaneous stability and $\dot{o}_i$ is the decimal value of the corresponding *DP* for player i. The new possible strategy profile given by value $\hat{q}$ is defined by

[15] Assume that for both players' the *DP*s for strategy profile 0 were not already sequentially sanctioned and simultaneous stability needed to be tested. A simultaneous switch of both player *A* and *B* to confess (which is their preferred strategy according to the *DP*s) will result in strategy profile 3, which is strictly less preferred.

[16] Those readers familiar with "Metamagical Themas" (Hofstadter, 1985) will recognise the similarity between the Conflict Analysis approach and "superrationality".

$$\hat{q} = \sum_{i=1}^{x} \dot{o}_i - (x-1)\dot{q}, \; x = 2,3,...,m \tag{8}$$

where x is equal to the number of players under consideration from the total set of players M, who possesses a DP from q.[17]

3.1. Multi-Level Hypergames

A hypergame occurs, whenever some player j is wrong about the true nature of game G and perceives a game that either or both differs with respect to the actual preference order or to the available strategies in the sets S_{-j} of the other players. Define player i's strategy set and preference order by the vector $V_i = \{S_i, U_i\}$. A non-cooperative n-player game can be represented by $G = (V_1, V_2, .., V_n)$. If one or more players misperceive the underlying rules, game G for player j is given by $G_j = (V_{1j}, V_{2j}, .., V_{nj})$ and hence, a first level hypergame is defined as $H = (G_1, G_2, ..., G_n)$. If other players are aware of the faulty perception of player j, the game turns into a second level hypergame, where the game for player j is defined by an individual first level hypergame $H_q = (G_{1q}, G_{2q}, ..., G_{nq})$. Consequently the second level hypergame is represented by $H^2 = (H_1, H_2, ..., H_n)$. The reasoning can be continued for higher level hypergames. A third level hypergame would occur in the case, where some player erroneously perceive another players misperception, which is again recognised by other players. The third level hypergame will be represented by $H^3 = (H_1^2, H_2^2, ..., H_n^2)$. In the case of two players with $i = A, B$ a first level hypergame is characterised by $H = (G_A, G_B)$. A third level hypergame will have the form

$$H^3 = (H_A^2, H_B^2) = \begin{Bmatrix} H_{AA} & H_{BA} \\ H_{AB} & H_{BB} \end{Bmatrix} = \begin{Bmatrix} (G_{AAA} & G_{BAA}) & (G_{ABA} & G_{BBA}) \\ (G_{AAB} & G_{BAB}) & (G_{ABB} & G_{BBB}) \end{Bmatrix}.$$

The equilibria of a first level hypergame depend on the stability of each player's strategies within their individual games. The set of equilibria is defined by those strategy profiles that are stable according the the individual perception given by the individual stabilities in $H = (G_A, G_B)$, i.e. by the strategy profiles stable both in V_{AA} and V_{BB}. Suppose both players erroneously believe that the other player most prefers none of them in prison and least prefers both to be imprisoned. The game that A believes to be playing is given by $G_A = (V_{AA}, V_{BA}) = (\{1,0,3,2\}, \{0,2,1,3\})$ and B's game will be given by $G_B = (V_{AB}, V_{BB}) = (\{0,1,2,3\}, \{2,0,3,1\})$. The stabilities are derived for each game individually. They are solutions to the first level hypergame, represented in table 3. The stabilities for V_{AB} and V_{BA} are not written down specifically, since they are irrelevant for determining the set of equilibria of the first level hypergame. They are only of importance, when checking for eventual simultaneous stability. In this game, however, none of the strategy profiles is simultaneously stable.

If both players believe the other player to be more *altruist*, the only possible equilibrium will be strategy profile 3. A switch to the non-cooperative strategy is not sanctioned by a

[17] Consider the example of strategy profile 0. After a simultaneous switch of both players from 0, the new equilibrium would be given by $3 = (1+2) - (2-1)0 = 3$ (since $x = 2$). If there were three players $(A, B, C) \in M$, then x can take value 2 and 3. Simultaneous stability has to be checked for the cases, in which all players choose a DP (hence $x = 3$) and only two players react ($x = 2$).

Table 3. Solution to the Prisoner's Dilemma Hypergame

Stability Analysis					
equilibrium		x	x	E	x
V_{AA}	Stability	r	u	r	u
	Preference Order	1	0	3	2
	DP		1		3
V_{AB}	Preference Order	0	2	1	3
	DP		0		1
V_{BA}	Preference Order	0	1	2	3
	DP		0		2
V_{BB}	Stability	r	u	r	u
	Preference Order	2	0	3	1
	DP		2		3

sequential switch of the other player. In the case of higher order hypergames and a two player game, the analysis of higher level hypergames can be reduced to the examination of the two games on the top left and bottom right on the main diagonal of the hypergame matrix (for an example see, Fraser and Hipel, 1984, Ch. 3 & 4). The two games are sufficient to determine the final set of equilibria in the higher level hypergame, independent of the order of the underlying hypergame.[18] The higher level hypergame then breaks down into a first level hypergame with $H = (G_{AAA\ldots}, G_{BBB\ldots})$, as the other matrix elements are of no importance. Only strategy profiles that are stable both in $V_{AAA\ldots}$ and $V_{BBB\ldots}$ form elements of the set of equilibria. This is a direct consequence from the simple fact that $H = (G_{AAA\ldots}, G_{BBB\ldots})$ represents the game that each player believes to be playing respectively.[19]

[18] An analysis of the other games is only necessary, if it is of interest what individuals believe about the outcome of their hypergame, i.e. their analysis along the various levels of misperception in the higher level hypergame. If individual perceptions about the equilibria in their game is irrelevant, the analysis can be substantially simplified.

[19] Nevertheless, Fraser & Hipel's analysis can be even further refined, if these individual games are of interest. Consider another approach: First analyse all zero level games of the original n-th level game. Each zero level game (there will be n^r, with n being the number of players and r being the order of the hypergame; but games of higher order than three are unlikely to occur) will result in a set of possible equilibria. These equilibria will define the strategy that a player will choose and hence, will determine the set of equilibria for the next higher level hypergame. The next-level equilibria are determined in the same way that new equilibria are derived, if testing for simultaneous stability (equation 8). The equilibria obtained should be checked, if it is stable according to the underlying stabilities for both players in the corresponding hypergame (i.e. the top left and bottom right elements). An example will make the idea clear: Assume a second order hypergame of a game as in matrix 7 given by some arbitrary pay-off configuration, and for simplicity that there is only one equilibrium for each zero level hypergames. Assume them to be $G_{AA} = 2$, $G_{BA} = 1$, $G_{AB} = 1$, and $G_{BB} = 2$. Hence, A assumes that the game without erroneous play would end up in equilibrium 2, but that B will perceive the game to end up in equilibrium 1. B believes the inverse. Consequently, hypergame H_A will lead to equilibrium 0 - player A chooses *not confess* in accordance with equilibrium 2, he expects the same for B in accordance with equilibrium 1, the result being equilibrium 0 (*(not confess, not confess)*. The same reasoning will lead to equilibrium 3 (*(confess, confess)* for H_B, which player B believes to be playing. Accordingly, equilibria 0

3.2. Dynamic Analysis

The approach presented so far is purely static, yet most conflicts are dynamic processes. It is necessary to adapt the approach to explain and model the dynamics of repeated games. Fortunately the static analysis is the first of two steps to derive a transition matrix and to represent the game as a Markov chain. The dynamic analysis also has a significance that goes beyond the original purpose intended by Fraser & Hipel: It is, nonetheless, also instructive to understand the *reasoning* of players that led to the stability of the equilibria in one-shot games under the assumptions of Conflict Analysis.

There are two different assumptions that can form the basis for the application of the Conflict Analysis approach to one-shot games, first, an empathic rationality of players, and second, an evolutionary incapacity of players to rationalise the singularity of such games. The first supposes that players have the potential to comprehend, experience and predict the feelings of other players that determine their strategic choice. Empathy is thus clearly distinct from sympathy and is not identical with the concept of identification. The second relies on a possible explanation of why individuals are observed to choose non-rational actions in single shot games. Caused by the rare occurrence of non-repeated interactions, evolution did not prepare us for one-shot games, thus creating a lack of apt heuristics for interactions that occur as a single incident. The discussion of both assumptions and the theoretical implications are postponed to section 6 on page 31.

The interpretation of the transition matrix of the Markov chain is therefore twofold. The transition matrix enables us to see how a game evolves in each period, if Conflict Analysis is applied to repeated interactions. It also shows the reasoning of players in a one-shot game, if players exhibit an empathic rationality or apply a repeated game solution heuristic.

Consider a game with f possible strategy profiles, let *state* X_{t-1} be defined by the probability distribution of the strategy profiles in time $t-1$ with dimension $f \times 1$. The state transition of such a game can be characterised as the discrete time Markov process $X_t = TX_{t-1}$, where T is the transition matrix of dimension $f \times f$ that describes the transition probability of moving from strategy profile x to y. X_t defines the state in t and has the same dimension as X_{t-1}.[20] Consequently a repeated game $\Gamma = (S_{1t}, S_{2t}, ..., S_{nt}; U_{1t}, U_{2t}, ..., U_{nt})$ played in successive time periods $t = 1, 2, ..., n$ can be represented, given initial distribution (*status quo* state) X_0, as

$$X_t = T^t\, X_0, \tag{9}$$

if T is time homogeneous. As a first step, one transition matrix for each player must be derived. In a second step, the final transition matrix T of Γ will result from combining the individual transition matrices T_i.

In the following I will return to the Prisoner's Dilemma to illustrate the way, in which the transition matrix is derived, but also to show how the transition matrix illustrates the reasoning of empathic rationality in one-shot games. Given the preference order of the

and 3 will lead to $H^2 = 2$ ((*not confess, confess*). Hence, 2 will define the hypergame's equilibrium. Both approaches will probably lead to identical results (as in this example, since $G_{AA} = G_{BB} = 2$), but should they not, the intersection of the two sets of possible equilibria obtained can provide a refinement.

[20] Notice that a Markov process is generally represented in the transposed form. Evidently, this has only an effect on the way the transition occurs. Instead of row defining the current and column the subsequent state in the transition matrix, the inverse holds for the representation chosen here.

strategy profiles like in table 1 and based on the *DP*s for each strategy profile (see table 2) the individual transition matrices will look as follows:

$$T_A = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array}\begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix} \end{array} \text{ and } T_B = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array}\begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix} \end{array}.$$

For illustrative simplicity, the decimal code of each strategy profile has been written on top and on the side to represent their corresponding position in the transition matrix. Generally strategy profiles are ordered according to the relative value of their decimal code. Each column in both matrices corresponds to the strategy profile in the last period $t-1$. A row defines the strategy profile in t and its value the transition probability. Consequently, the sum of all values in one column equals to 1. It should be clear that all *stable* strategy profiles can be found with value 1 on the main diagonal and the off-diagonal position is determined by the most preferred non-sanctioned *DP* (if a profile possesses more than 1 *DP*). The final transition matrix T is defined by the strategy profiles that occur if both players have chosen their best response strategy. T can be derived from T_A and T_B via an equation similar to equation 8,

$$\bar{q} = \sum_{i=1}^{x} \dot{o}_i - (x-1)\dot{q}, \tag{10}$$

where $\bar{q}$ is the new equilibrium value, and $\dot{q}$, $\dot{o}_i$, and x are defined as above. Applying the individual transition matrices to equation 10 determines the final transition matrix for game Γ as:[21]

$$T = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{pmatrix} \tag{11}$$

The absorbing states (i.e. the set of equilibria) are defined by a 1 on the main diagonal. Given any initial condition, in which both players disbelieve in mutual non-confession, i.e. assign probability 0 to strategy profile 0, the single equilibrium of the dynamic game is strategy profile 3 with certainty for all X_t and $t > 1$.

The transition matrix shows the empathic reasoning of players participating in the one-shot Prisoner's Dilemma. We know that if a player believes the other to choose defect, he will also defect. This is defined by a transition of 1 and 2 to 3. In addition, if a player expects that the other player believes that he defects, both will defect. Since he knows that the other player's best response to the belief that he defects is to defect, his own best response is also to defect, represented by T^2. The same logic applies to reasoning of higher order in a similar way. The stabilities are thus defined by the limit distribution of the transition matrix. Since $T = \lim_{n \to +\infty} T^t$ in the Prisoner's Dilemma, the cooperative equilibrium will only occur

[21] Applying equation 10 gives for 0: $0+0-0=0$, for 1: $1+3-1=3$, for 2: $3+2-2=3$, for 3: $3+3-3=3$.

if both players initially expect the other to cooperate. If one player is expected to defect, the game will result in the defective equilibrium.

Consider that there are two types of players. One player type has the self-regarding preferences as in table 2 on page 10 for A_s and B_s. The other player type are altruist cooperators with preference order $P(A_a) = \{0,1,3,2\}$ and $P(B_a) = \{0,2,3,1\}$. Consequently the possible types of interactions lead to four different games (between two altruists, two self-regarding players, one altruist and one self-regarding and the inverse, though mixed type games are identical, since preference order is "symmetric"). In the case, where simultaneous stability is considered, all games can be represented by a transition matrix identical to T defined in matrix 11.[22] Hence, we obtain, independent of the player type configuration, that if player i and j meet

$$T = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{pmatrix}$$

for any i and j. Assume that the proportion of altruists in the population is $p_a = 0.62$ and the proportion of self-regarding individuals is $p_s = 1 - p_a = 0.38$. Hence, the probability that two altruists interact is ${p_a}^2 = 0.3844$, for two self-regarding individuals ${p_b}^2 = 0.1444$, and for two different types to meet $2p_a p_b = 2 \times 0.2356$. Consequently, without sure knowledge on the other player type, only 38% of the individuals will cooperate ($X_0 = ({p_a}^2, p_b p_a, p_a p_b, {p_b}^2)^T$). This is identical to what Kiyonari, Tanida and Yamagishi (2000) found. If the second player is told that the first will always cooperate, a positive probability can be assigned only to equilibria 0 and 2. In this case 62% will cooperate ($X_0 = (p_a, 0, p_b, 0)^T$), which again corresponds to the result found by Kiyonari et al.

3.3. A Short Excursion to the Stag Hunt Game

The dynamic analysis also shows an interesting property of the Conflict Analysis approach: Given the original pay-off matrix 7 on page 9 define $a > c > d > b$ and $a + b < d + c$. The original Prisoner's Dilemma turns into a Stag-Hunt game, i.e the special type of coordination game, in which one equilibrium defined by *(not confess, not confess)* pay-off dominates the risk dominant equilibrium determined by *(confess, confess)*. The game based on Rousseau's parable in the "Discourse on the Origin and Foundations of Inequality among Men" is another illustrative example for the underlying rationality of the Conflict Analysis approach. The quantification of a hare or stag seems difficult, yet obviously hunting a stag is more risky than a hare. Conflict Analysis can incorporate the risk issue without the need to quantify in relative terms the pay-off value of both animals.

Define the strategies as *Hunt Stag =0* and *Hunt Hare =1*, since the symmetric game consists of two mutually exclusive actions for each player. The Pareto dominant equilibrium has thus decimal code 0, the risk dominant equilibrium decimal code 3, and mixed outcomes are assigned to decimal code 1 for *(stag, hare)* and to decimal code 2 for *(hare, stag)*.

In the static analysis presented in table 4, two equilibria exist, since Nash equilibria are always rationally stable for all players. The interior mixed equilibrium is neglected since

[22] This result gives an interesting basis for discussion: According to the theory described here, it is irrelevant for the final outcome, whether an individual is self-regarding or altruist in the PD.

Table 4. Static Solution to the Stag Hunt

Stability Analysis						
equilibrium			E	x	E	x
Player A	Stability		r	$\hat{u}$	r	u
	Preference Order	0	1	3	2
	DP			0		3
Player B	Stability		r	$\hat{u}$	r	u
	Preference Order	0	2	3	1
	DP			0		3

Conflict Analysis only regards pure strategies, owing to the lack of quantification. At the first impression, the static representation has no more explanatory power than the standard approach. Yet, in the mixed strategy profile, where a player chooses hare and the opponent stag, a switch towards the Pareto optimum is simultaneously sanctioned. A look at the transition matrices will make the effect of simultaneous stability more obvious. From table 4 we obtain:

$$T_A = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}, T_B = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix} \Rightarrow T = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{pmatrix} \tag{12}$$

The final transition matrix in 12 shows that simultaneous stability captures the effect of risk dominance, though not in the strict sense of $a+b < d+c$. Since Conflict Analysis is based on an ordinal ranking of the equilibria and no pay-off values are assigned, an aggregation of these values *per se* is impossible. The final transition matrix shows that the pay-off dominant equilibrium requires each player to place a high probability on the event that the other player is hunting the stag. If he assigns equal probability to all events, he will finally hunt the hare, as the risk dominant equilibrium occurs in 75% of the time according to his priors. This effect is caused by the relation $c > b$. Any coordination game with two pure Nash equilibria and $c > b$ will show the same dynamics. Hence, game $a > c > d > b$ is equivalent to $a > d > c > b$.[23]

When comparing to the Prisoner's Dilemma, it becomes obvious that both games are dynamically equivalent. The stability of outcome 0 in the PD occurs, however, through the sequential sanction of the defective strategy. In the stag hunt, a switch towards the cooperative strategy (from 1 to 0 for player *A* and from 2 to 0 for player *B*) is sanctioned by the fear the other player might switch, i.e. by a simultaneous sanction. This captures exactly the risk argument in the standard approach. Empirical evidence (Schmidt et al., 2003; Cooper et al., 1992; Huyck et al., 1990)) shows that individuals play the Prisoner's Dilemma

[23] A similar result can be found in Nowak, 2006, Ch. 6. It turns out that, in 2 player games, evolutionary selection favours the equilibrium that offers the higher pay-off in the case of miscoordination, i.e. which is defined by the strategy granting pay-off value *c*, instead of *b*.

similar to the Stag Hunt. Notice, however, that the empirical tests observe interactions after presenting a pay-off matrix to their subjects. In real world scenarios such quantification is often infeasible. The Conflict Analysis approach can nevertheless indirectly incorporate the pay-off effect (e.g. the role of the optimisation premium in Battalio et al., 2001). Both, the degree of strong altruism and the pay-off associated to each strategy profile, affect the probabilities of a player's own choice and his expectation about the other player's choice, thereby determining the initial *status quo* distribution in X_0.

4. Dilemmas and Paradoxes

The following section analyses the capacity of the Conflict Analysis approach to solve games that pose a challenge for classical game theory.[24] The games analysed here are the Traveller's Dilemma, the surprise test, and the Newcomb's Paradox. Though the sequential stability criterion resembles classical backward induction, this section will also show that the obtained results display no similarities.

The original definition of sequential stability and thus instability implies that a player refrains from all strategy switches that never lead to a strictly preferred strategy profile with certainty. Players do not bear the costs of switching, if it does not offer a benefit. This assumption, however, seems to be too strong. Assume that a player switches to a new strategy, if he knows that the subsequent switch of other players to their viable better response strategy will not lead to a less preferred outcome, but that there exists at least one potential viable better response strategy of another player that leads to a strictly preferred outcome. This means that a player will not switch if it leads him to a not strictly preferred outcome with certainty, but he will choose to do so whenever he has a chance to improve his situation without the risk to worsen it. This requires a stronger condition for sequential stability and weaker condition for instability with respect to the original definition by Fraser & Hipel. Define $q = (\overline{s_i}, \overline{s}_{-i})$, and $o \in U_i^{--}(q)$, if $O(o) \prec_i O(q)$:

Assumption 1. *A player will never switch to a strategy that leads to an equally preferred strategy profile with certainty or a less preferred strategy profile with positive probability. He will, however, change his strategy if a change does not lead to a less preferred strategy profile with certainty and there exists at least one viable response strategy of another player that defines a strictly preferred outcome, i.e. for some* $q = (\overline{s_i}, \overline{s}_{-i})$*:*
Sequential stability: $\hat{u}_j^+(p) \cap U_i^{--}(q) \neq \emptyset$*, for any* $j \neq i$ *, or*
$\hat{u}_j^+(p) \cap \left(U_i^+(q) \cup U_i^{--}(q)\right) = \emptyset, \forall j \neq i$*; both* $\forall s_i^* : p = (s_i^*, \overline{s}_{-1}) \in u_i^+(q)$

[24] The more common games, such as the battle of sexes and the chicken / hawk-dove game, have not been analysed in detail, since differences between solutions of standard game theory and those of Conflict Analysis approach are only minor. Conflict Analysis defines an additional equilibrium (both swerve) in the chicken game, if we observe that simultaneous switch may occur. This result is reasonable, but unlikely. If the chicken game is perceived as a sequential game, simultaneous stability does not apply. This exactly illustrates why the application of simultaneous stability requires an a priori justification based the rules of the game. In "The Battle of Sexes" all four outcomes are stable. The mixed are again stable owing to a potential simultaneous switch. This is the case, in which both players wait at different locations. Both players stick to their strategy and keep on waiting, hoping their counterpart is changing place, as both players fear that changing place might occur at the same time, bearing only the costs of moving without the benefit from coordination.

Instability: *$\exists p \in u_i^+(q) : \hat{u}_j^+(p) \cap U_i^{--}(q) = \emptyset\ \forall j \neq i$ and there exists at least one $\hat{s}_k : (s_i^*, \hat{s}_k, \bar{s}_{-i-k}) \in U_i^+(q) \cap \hat{u}_k^+(p)$, for $k \neq i$ and some $s_i^* \in S_i : p = (s_i^*, \bar{s}_{-1}) \in u_i^+(q)$*

where $(s_i^*, \hat{s}_k, \bar{s}_{-i-k})$ defines the new strategy profile after a switch of a player k other than i to his better response strategy given p. Furthermore, the original definition of Fraser & Hipel assumes that an equally preferred strategy profile is not regarded as a valid *DP*, meaning that a player will not switch to a strategy not defined by a *strictly* preferred strategy profile (see also footnote 8 on page 5).

The following section thus has the aim to illustrate; first, the results that Conflict Analysis obtains for the given normal form games; second, the eventual issue that arise through the relaxation in assumption 1, and third, the difference in the predicted equilibria if either or not equally preferred strategy profiles qualify as *DP*'s. (In the following analysis of the games the results that would be obtained, if equally preferred strategy profiles qualify as such, are indicated in brackets.)

4.1. Traveller's Dilemma

The Traveller's Dilemma by (Basu, 1994) tells the story of two antiquarians, who bought the same object but did not preserve the receipts. On the flight home, the airline smashes both objects, and the antiquarians ask for a refund. Thus, they are asked independently by the airline manager to state the amount (in integral numbers) they paid, constrained by a minimum amount of 2\$. In the case, where they report different amounts, they are compensated by the lower amount stated. In addition, the one, who reported the lower amount, will receive 2\$ less (as a punishment for having lied), which will be given to the other as a bonus. If both state the same amount, none will be rewarded nor punished. Backward induction tells us that both players should state 2\$, independently of the value of the duty paid. This can be seen from matrix 13, which represents the game for 2\$ to 5\$. (2,2) is the only stable Nash equilibrium.

$$\begin{array}{c} \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{array} \begin{array}{c} \begin{array}{cccc} b_2 & b_3 & b_4 & b_5 \end{array} \\ \begin{pmatrix} 2,2 & 4,0 & 4,0 & 4,0 \\ 0,4 & 3,3 & 5,1 & 5,1 \\ 0,4 & 1,5 & 4,4 & 6,2 \\ 0,4 & 1,5 & 2,6 & 5,5 \end{pmatrix} \end{array} \tag{13}$$

In the following the game will be analysed by using the Conflict Analysis approach. Table 5 shows the encoding of the 16 possible strategy profiles into decimal code in the same way as has been done in the previous section.

Consequently, the preference order U_i for player i is gives as follows, where a bar indicates equal preference, i.e. the player assigns equal preference to all strategy profiles under the same line:

$$\begin{aligned} U_A &= \left(132, \overline{66, 130, 136}, \overline{33, 65, 68, 129}, 34, \overline{17, 72}, \overline{36, 40}, \overline{18, 20, 24}\right) \\ U_B &= \left(72, \overline{36, 40, 136}, \overline{18, 20, 26, 68}, 34, \overline{17, 132}, \overline{66, 130}, \overline{33, 65, 129}\right) \end{aligned} \tag{14}$$

The game has been analysed in table 16 on page 39. *DP* of equal preference are written in brackets. In this analysis all strategy profiles that include a pay-off at least equal to the Nash equilibrium pay-off for both players are stable. The set of equilibria is defined by $E = (17, 34, 68, 136, 72, 132)$. The last two are stable, if equally preferred strategy profiles are not considered to be valid *DP*'s.[25] To see why this is the case assume that the strategy profile is defined by $(\pi - 1; \pi)$, for $\pi > 5$, i.e. player 1 declares the value $\pi - 1$, and player 2 the value π. The second player can improve by switching to a value $\pi^* \in (\pi - 2, \pi - 4)$ (for values of $\pi \leq 5$ the lower bound of π^* is 2), granting him a pay-off of $\pi^* + 2 > \pi - 3$. This would, however, entail a switch of the first player to a value that is even lower. Hence, the second player cannot win by deviating.[26] In a situation given by strategy profile $(\pi - 2; \pi)$ and $\pi > 6$ the second player has again an incentive to underbid the first player with a strategy naming the value $\pi^{**} \in (\pi - 3, \pi - 5)$. Yet, owing to the subsequent switch of the first player, he cannot augment his pay-off and will therefore not switch. In this situation also the first player has an incentive to switch his strategy by declaring $\pi - 1$, which is deterred by the second player's potential switch. The same reasoning applies to larger differences as long as both players obtain at least a value of 2. Below this value a player has an incentive to switch to the Nash strategy that cannot be deterred. If the number of strategies is k, i.e. there are k different amounts that can be stated, the limit distribution of the transition matrix assigns probability $\frac{2(2k-3)+1}{k^2}$ to the Nash equilibrium $(2, 2)$ (see transition matrix 15 on page 38). The other equilibria are played with probability $\frac{1}{k^2}$.

Then why is it more intuitive to state a value higher than 2? A possible explanation is that the game possesses two focal points that are assigned higher probability. It is likely that individuals assume with high probability that their counterpart chooses either the correct amount paid or the maximum possible value. The pay-offs also neglect loss-aversion, which a player will experience when stating a low value. Yet, the Traveller's Dilemma illustrates a drawback of the Conflict Analysis approach. Its application to specific games may result in a large number of potential equilibria.

[25] If equally preferred strategy profiles are considered as *DP*'s, the strategy profiles inside the brackets destabilise both 72 and 132. To see this consider strategy profile 72. A change to 65 leads to 17, which is equally preferred. It may also lead to 33 and 129, both are preferred to 72 by player *A*. Following Assumption 1 implies that *A* will choose 65, since this will eventually make him better off without a chance to diminish his expected pay-off. Similarly player *B* will switch from 132 to 20. Hence, if equally preferred outcomes are considered as valid *DP*'s only strategy profiles that offer a pay-off higher than the Nash equilibrium to both players and the Nash equilibrium itself will be equilibria of the game.

[26] The question that the reader might pose at this point is: Why is the player that acts as second (player 1 in the example) supposed to switch strategy as he himself will also fear a subsequent switch of the other player (the first mover, i.e. player 2), i.e. how credible is the subsequent strategy switch? The answer is connected to the discussion of the "Metagame Fallacy" and the "Newcomb's Paradox". The correlation and the answer to this question will be examined in section 6.

4.2. The Surprise Test

The Surprise Test is a game, in which a teacher announces to his student(s) that he will write a surprise exam on one day of the following week. Since the exam cannot be written on Friday, because it should not be a surprise any more, this day can be eliminated. Backward induction will then cancel each day of the following week as the student always rejects the last possible day in the remaining list. Finally the student will be sure that no exam will be written the next week and will be surprised, when it happens on one day of the week. Backward induction is no sensible reasoning, since the student ignores that whenever he eliminated one day, the teacher has an incentive to switch to that day.[27] The Conflict Analysis approach takes account of this fact.

If we consider only a week of four days, the coding in table 5 can be used to represent each possible strategy profile. Consider *A* the teacher and *B* the student. The subscripts illustrate the days ordered calendrically. Version 1 in table 6 shows a possible pay-off structure. The teacher strictly prefers all strategy profiles, in which the exam is scheduled for a different date than expected by the student. The inverse holds true for the student.

Table 6. Strategy profile Matrix

Surprise Test: pay-offs for Version 1 and 2

	17	33	65	129	18	34	66	130	20	36	68	132	24	40	72	136
1:	0,1	1,0	1,0	1,0	1,0	0,1	1,0	1,0	1,0	1,0	0,1	1,0	1,0	1,0	1,0	0,1
2:	1,5	0,0	0,0	0,0	2,4	1,5	0,0	0,0	3,3	2,4	1,5	0,0	4,2	3,3	2,4	1,5

Conflict Analysis (see table 17 on page 40) defines every strategy profile as an equilibrium and thus potential outcome of the game. The symmetric strategy profiles will be unrealistically defined as unstable only if equally preferred strategy profiles serve as *DP*'s. Any date can be chosen by the teacher and the student will randomly choose one day to study for the exam.

Consider a variant of the preference order as given in version 2 in table 6. In this version the student least prefers all strategy profiles, in which he studied too late. He prefers most the situations, in which he correctly predicted the exact date of the surprise exam. His preference is diminishing in the number of days he studied before the actual date of the exam. Simply assume that he has to revise each evening in order not to forget what he has studied a day before. The teacher prefers the student to revise as often as possible, and has no interest in the student's failure, which occurs if he studied too late. Pay-offs are as in table 6.2 and the solution is given in the lower part of table 17 on page 40.

The set of equilibria is defined by $E = (18, 36, 68, 136)$. Since both are uncertain, to which strategy the other player adheres, the student will learn for any day of the week and the teacher will schedule the exam either on Tuesday, Wednesday or Thursday (remember that in the example the school week is only 4 days long). Strategy profiles 17 and 34 are

[27] Assume the student has eliminated Friday from his list of strategy profiles and disregards this day. When contemplating about deleting Thursday the student should realize that he has to consider Friday once more, since this day is now again an option for the teacher as a surprise date.

equilibria in the original definition of Fraser & Hipel, but are destabilised by 40 and 24, respectively, if assumption 1 applies. In this version of the Surprise Exam assumption 1 thus creates a theoretical problem, absent in the original definition of sequential stability and instability.[28]

4.3. Newcomb's Paradox

This *paradox* (Nozick, 1969) has been widely discussed in the various social sciences as well as philosophy. It will turn out in the discussion in section 6 that the Newcomb Paradox plays a crucial role in understanding the theory behind the Conflict Analysis approach. The game defines a situation, in which one player *B* has the choice between taking one or two boxes. A second, omniscient player *A* chooses the value of the first box a priori to player *B*'s choice. The first box may contain either 1.000.000$ or will be empty. The omniscient player will only put one million dollar into the first box, if the other player only chooses this box but neglects the second, which contains 1.000$. The pay-off structure is presented in matrix 15 and only indicated for the first player, since the pay-offs of the omniscient player are not required.

$$\begin{array}{r c} & \begin{array}{cc} \textit{punish} & \textit{not punish} \end{array} \\ \begin{array}{r} \textit{take both} \\ \textit{take one} \end{array} & \begin{pmatrix} 1.000 & 1.001.000 \\ 0 & 1.000.000 \end{pmatrix} \end{array} \tag{15}$$

According to Nozick, this paradox illustrates a conflict between domination of strategies and maximisation of expected pay-off. Conflict Analysis defines both strategies for the first player and the corresponding strategic choice of the omniscient player as possible equilibria. Table 7 on page 24 shows the analysis of the game.

Assume that the omniscient player strictly prefers not to punish the other player. Classical game theory will define the Nash equilibrium as *(take both, not punish)*. So does Conflict Analysis (see table8 on page 24).

On the contrary, Metagame Theory predicts strategy profile *(take one, not punish)* as the unintuitive, but weakly dominant equilibrium; see table 9 on page 25 (for a formal introduction to Metagame Theory refer to subsection A in the appendix). This shows that Conflict Analysis and Metagame Theory do not necessarily determine the same strategy profiles as equilibria, and provides an example that Conflict Analysis eliminates some of the deficiencies, for which the Metagame framework has been criticized.

5. Sequential Games

In this section, the Conflict Analysis approach is applied to sequential games respresented in extensive form. It follows the same structure as the previous section. Though, to my

[28] If the student knows that the teacher will follow a strategy defined by the equilibrium set, he anticipates that the teacher will not write on Monday and he will consequently not study for this day. The teacher will conjecture this and will not schedule for Tuesday and again the student will not study for this day. Finally, only the equilibrium set $\hat{E} = (68, 136)$ should remain.

Table 7. Solution to the Newcomb's Paradox

Strategy profiles					
Player A	take both	0	1	0	1
Player B	punish	0	0	1	1
Decimal Code		**0**	**1**	**2**	**3**

Stability Analysis					
equilibrium		x	E	E	x
Player A	Stability	r	s	r	u
	Preference Order	1	0	3	2
	DP		1		3
Player B	Stability	r	r	u	u
	Preference Order	0	3	1	2
	DP			3	0

Table 8. Alternative Version of the Newcomb's Paradox - B prefers never to punish

Stability Analysis					
equilibrium		E	x	x	x
Player A	Stability	r	u	r	u
	Preference Order	1	0	3	2
	DP		1		3
Player B	Stability	r	r	u	u
	Preference Order	0	1	3	2
	DP			1	0

knowledge, Fraser & Hipel have only applied the Conflict Analysis approach to simultaneous games, it turns out that the approach can cope quite well in finding a solution, since it is able to refine the equilibrium set in a similar way as the local best response criterion (*LBR*) presented by (Gintis, 2009), which displays advantages over traditional refinement criteria.[29] In this section I have thus chosen games from Gintis (2009, Ch. 9) to illustrate that Conflict Analysis is also a powerful tool to effectively solve sequential games. For notational simplicity and to avoid redundancies, the solution will only be given in the special game form, with whom the reader should be acquainted after having read the earlier sections. Assumption 1 still applies.

Owing to the sequentiality of all games, first, simultaneous stability is inapplicable and second, the first mover can choose an equilibrium as long as it is defined by an unambiguous

[29] such as subgame perfect, perfect Bayesian, sequential and proper equilibria

Table 9. Metagame Solution to the alternative version of the Newcomb's Paradox: B - one box; b - both boxes; p - punish; np - not punish, supposing $\pi_o(p,B) = 1, \pi_o(p,b) = 2, \pi_o(np,b) = 3, \pi_o(np,B) = 4$, where $\pi_o(s)$ defines the pay-off of the omniscient player under strategy profile s. Though there are many stable equilibria, (b,B,b,b;p,np) weakly dominates the others, implying equilibrium (np,B).

	p,p	p,np	np,p	np,np
b,b,b,b	1.000,2	1.000,2	1.00.1000,3*	1.00.1000,3*
B,b,b,b	0,1	1.000,2	1.00.1000,3*	1.00.1000,3*
b,B,b,b	1.000,2	1.000.000 , 4*	1.00.1000,3	1.00.1000,3
b,b,B,b	1.000,2	1.000,2	0,1	1.00.1000,3*
b,b,b,B	1.000,2	1.000,2	1.00.1000,3	1.000.000,4
B,B,b,b	0,1	1.000.000,4*	1.00.1000,3	1.00.1000,3
B,b,B,b	0,1	1.000,2	0,1	1.00.1000,3*
B,b,b,B	0,1	1.000,2	1.00.1000,3	1.000.000,4
b,B,B,b	1.000,2	1.000.000,4*	0,1	1.00.1000,3
b,B,b,B	1.000,2	1.000.000,4*	1.00.1000,3	1.000.000,4
b,b,B,B	1.000,2	1.000,2	0,1	1.000.000,4
B,B,B,b	0,1	1.000.000,4*	0,1	1.00.1000,3
B,B,b,B	0,1	1.000.000,4*	1.00.1000,3	1.000.000,4
B,b,B,B	0,1	1.000,2	0,1	1.000.000,4
b,B,B,B	1.000,2	1.000.000,4*	0,1	1.000.000,4
B,B,B,B	0,1	1.000.000,4*	0,1	1.000.000,4

path. Preplay conjectures are conducted by players according to the rationalities underlying the Conflict Analysis approach, and players will only choose strategies according to an equilibrium profile. This reasoning leads to the following additional assumption that is applied in this section:

Assumption 2. *In a sequential game, if the strategy profile granting highest pay-off to the first-mover, in the set of equilibria, is defined by a unique path in the* ***reduced game tree****, such that no player is ambiguous about the position of his decision node at the time of his decision, then it determines the outcome of the game. The* ***reduced game tree*** *is thereby solely defined by the paths of the strategy profiles in the equilibrium set.*

The outcome of each game will be indicated by an E^*. The solution to the first game and its extensive form is shown in table 10. Notice that $\emptyset$ means that the player has no choice at that node. Hence $\emptyset$ can take both value 1 and 0, but pay-offs are identical (in order to keep notation as easy as possible, I thus reduced the strategy profile set, where it did not effect the identification of equilibria). The set of possible equilibria for the first game, represented in table 10 and figure 1, is given by $E = \{0,3\}$. Each equilibrium defines an unambiguous paths, separated by Alice's initial strategy choice. Alice will act in accordance with the path defined by strategy profile 3, which ranks higher than 0 according to her preference order, and Bob will also choose according to 3. The result is identical to *LBR*.

Table 10. Incredible Threats

Strategy profile set:

Choice Alice	R	0	1	1
Choice Bob	r	∅	0	1
Decimal		0	1	3
pay-off		1,5	0,0	2,1

Solution:

equilibrium	E*	E	x
stabilities	r	r	u
A. preference	3	0	1
DP			0
stabilities	r	r	u
B. preference	0	3	1
DP			3

Figure 1. Game.

The second game is shown again to the right in extensive form, and its solution is given in the next table 11. The bar indicates the set of equally preferred strategy profiles. Strategy profiles that are equally preferred can be interpreted as mutual *DPs* or not. Following the discussion in the previous section (see assumption 1 in section 4), it is assumed that a strategy profile will only be a viable *DP* as long as a switch to this strategy profile results in a preferred outcome with positive probability. Furthermore, like in the previous section, the stabilities occurring if equally preferred strategy profiles qualify as *DP*'s, are indicated by the brackets. Since Bob cannot improve the outcome by switching from 9 to 1 or vice versa, a strategy switch will not happen through assumption 1. In addition, the consideration of weak or strict preference for *DP*s is of no importance for the equilibrium set of this game.

Hence $E = \{1, 10\}$. Given the set of equilibria, both 1 and 10 define unique paths,

Figure 2. Game.

Figure 3. Game.

since Bob knows for sure whether or not he is on the path defined by 1 or 10, depending on whether he can choose a strategy or not. If Alice plays according to her most preferred equilibrium 10, Bob knows he can choose and takes strategy a, leading to 10, which is identical to the result predicted by *LBR*.

The third game is again illustrated in extensive form to the right in figure 3 and in table 12. A switch of Bob from 2 to 10 will not improve his outcome with positive probability, as no player has a *DP* to a preferred strategy profile. A switch from 10 to 2 can trigger a subsequent change of Carole to 18. By switching, Bob can attain a preferred outcome with positive probability. The set of equilibria is given by $E = \{1, 18\}$, each defining a unique path. Whenever Bob and Carole are free to choose, they will take the strategy defined by 18. Hence, Alice will initially choose strategy B, as predicted by *LBR*.

The following game, named after Reinhard Selten, is shown below and the solution is given in table 13.

The game is analysed in the same way as the previous games. Again a strategy profile

Table 11. Picking the Sequential equilibrium I

Strategy profile set:							
Choice Alice	A	1	1	0	0	0	0
	B	0	0	1	1	0	0
	C	0	0	0	0	1	1
Choice Bob	a	0	1	0	1	0	1
Decimal		1	9	2	10	4	12
pay-off		0,2	0,2	-1,-1	2,1	-2,0	1,1
Solution:							
equilibrium		E*	x	E	x	x(x)	x(x)
stabilities		r	u	r	u	u	u
A. preference		10	12	1	9	2	4
DP			10		10 12	1	1 2
stabilities		r(s)	r(s)	r	r	u	u
B. preference		1	9	10	12	4	2
DP		(9)	(1)			12	10

Figure 4. Game - Selten's Horse.

serves as a *DP*, only if a switch to this strategy profile results in a preferred outcome with positive probability and never in a less preferred outcome. Notice that a switch of Bob from strategy *a* to *d*, or the inverse, is only theoretical if Alice chooses *D*, since a node, where Bob's choice matters is reached with zero probability. Nevertheless, the notion of subgame-perfection moves along the same line and shows that also nodes of non-positive probability matter. The matrix in 13 thus includes 8 strategy profiles. Following the previous assumption, 0 is rationally or sequentially stable for Bob. In the case, where Bob should choose the equally preferred strategy profile 2, his change would result in 2 with certainty, which is not strictly preferred to 0. Yet, a switch from 2 to 0 entails a switch of Alice to 1, which is strictly preferred by Bob. He will thus have an incentive to change strategies if an equally preferred strategy profile serves as *DP*. The same argumentation holds for Carole with respect to 3 and 7. A switch by Carole from 7 to 3 will eventually lead to 2 or 1, one is strictly preferred the other is equally preferred. Hence, strategy profile 7 should be unstable

Table 12. Picking the Sequential equilibrium II

Strategy profile set:								
Choice Alice	A	1	0	0	0	0	0	0
	B	0	1	1	1	1	0	0
	C	0	0	0	0	0	1	1
Choice Bob	a	∅	0	1	0	1	0	1
Choice Carole	V	∅	0	0	1	1	∅	∅
Decimal		1	2	10	18	26	4	12
pay-off		1,0,0	0,0,0	0,0,1	2,2,2	0,0,0	0,1,0	0,0,0
Solution:								
equilibrium		E*	E*	x	x	x	x	x
stabilities		r	r	u(u)	u(u)	u(u)	u	u
A. preference		18	1	2	10	26	4	12
DP				1 (4)	1 (12)	1 (12)	1	1
stabilities		r	r	r	r/(s)	r/(u)	u	u
B. preference		18	4	1	2	10	26	12
DP					(10)	(2)	18	4
stabilities		r	r	r	u	s/(s)	r	r
C. preference		18	10	1	2	26	4	12
DP					18	10		

for Carole. The equilibrium set is defined by $E = (0,2,7)$ or $E = (0)$; the latter is the case if equally preferred strategy profiles qualify as *DP*'s. In both cases, whenever Carole can choose, she will take strategy λ, and whenever Bob has an option to choose, he will select *a*. Thus, Alice will opt for *D* and the final outcome is $(4,4,4)$, which is identical to what is predicted by *LBR*, though only strategy profile 2 satisfies the criterion.

The last game, denoted as the Spence Signalling Model, is shown in extensive form in figure 5 and its solution in table 14. In this game we add Nature as a player, choosing low ability worker *(L)* with probability 1/3 and high quality worker *(H)* with probability with probability 2/3. Bob can observe Alice's strategy choice whether *(Y)* or not *(N)* she invests in education. Bob can then choose whether he pays her as a skilled *(S)* or unskilled *(U)* worker.

Yet, choices by Nature entail mixed equilibrium strategies, which the Conflict Analysis approach is unable to predict. It could be assumed that Bob and Alice play two different games and assume that they are placed either in game *(L)* or *(H)* with probability 1/3 or 2/3, respectively. This assumption is, however, unnecessary for the representation, since Nature's choice already defines two separate sets of strategy profiles.

A switch of Alice from 5 to 7 (or the inverse) does not result in a strictly preferred strategy profile, whereas a shift from 1 to 3 (or the inverse) does. Independently of the strict or weak preference assumption for *DP*s, the equilibrium set is defined by $E = (2,5,7)$. The equilibria do not define a unique path, since Bob cannot observe Nature's choice. The

Table 13. Selten's Horse

Strategy profile set:									
Choice Alice	A	0	1	0	1	0	1	0	1
Choice Bob	a	0	0	1	1	0	0	1	1
Choice Carole	ρ	0	0	0	0	1	1	1	1
Decimal		0	1	2	3	4	5	6	7
pay-off		4,4,4	5,5,0	4,4,4	3,3,0	1,1,1	2,2,2	1,1,1	3,3,0
Solution:									
equilibrium		x	E*	E*(x)	x	E(x)	x	x	x
stabilities		r	s	r	u(u)	r	r	u	u(u)
A. preference		1	0	2	3	7	5	4	6
DP			1		2			5	7
stabilities		r	r/(s)	r/(u)	s	r	u(u)	r/(u)	r/(u)
B. preference		1	0	2	3	7	5	4	6
DP			(2)	(0)	1		7	(6)	(4)
stabilities		r	r	r	s	u(u)	u	r/(s)	r/(u)
C. preference		0	2	5	4	6	1	3	7
DP					0	2	5	(7)	(3)

Figure 5. Spence Signalling Model.

equilibrium set implies that Alice will never invest in education, if she is a low quality worker. If she is a high quality worker, she will either invest or not. Similarly, Bob will always assign a skilled job to Alice, if she invested in education, but Bob might assign unskilled workers both to skilled and unskilled jobs. We obtain something similar to a semi-pooling equilibrium, where any combination of S,S (assign all workers to skilled jobs) and S,U (assign educated workers to skilled jobs and uneducated workers to unskilled jobs) is best response for Bob. Bob will play S,S/U (assign educated worker to skilled jobs and uneducated worker both to skilled and unskilled jobs). Alice's best response is Y/N,N (if high quality worker, both educate and not educate; if low quality worker, never educate),

Table 14. Spence Signalling Model

Strategy profile set:									
Choice Nature	H	0	1	0	1	0	1	0	1
Choice Alice	N	0	0	1	1	0	0	1	1
Choice Bob	S	0	0	0	0	1	1	1	1
Decimal		0	1	2	3	4	5	6	7
pay-off		2,10	14,5	12,10	14,5	8,5	20,10	18,5	20,10
Solution:									
equilibrium		E	E	x	x	x	E	x	x
stabilities		r/(s)	r/(s)	r	r/(u)	r/(u)	r	u	u
A. preference		5	7	6	1	3	2	4	0
DP		(7)	(5)		(3)	(1)		6	2
stabilities		r	r	r	r	u	u	u	u
B. preference		0	2	5	7	1	3	4	6
DP						5	7	0	2

similar to *LBR*, where the best response of Alice to S,U is Y,N (invest in education if high level worker and not if low level worker) and to S,S it is N,N (never invest). Both (N,N;S,S) and (Y,N;S,U) fulfill the *LBR* criterion. Since no player knows for sure, to which best response the other player adheres, we can also assume that (Y,N;S,S) and (N,N;S,U) occur, which is captured by the equilibrium set obtained by the Conflict Analysis approach.

6. Critique and the Metagame Fallacy

We have seen that this approach has several advantages with respect to classical game theory. It has the capacity to find equilibria not predicted as such by other approaches, but which, however, appear intuitive. Though it increases the set of potential equilibria, I have described a means to discriminate between equilibria in sequential games. The approach can handle larger strategy and player sets. Conflict Analysis has also strong advantages over other similar approaches, such as the "Theory of Moves" by Brams (Brams, 1993; Brams and Mattli, 1992).[30] It allows to model situations of misperception via Hypergames. Overall it appears more adapted to real-world scenarios.

The ability to predict additional equilibria required the introduction of two additional stability criteria. This comes at a price. In *sequential games* the stability of a strategy choice, based on the sequential stability criterion, is rational for consistent preferences as defined in the preference ordering, if it is applied backwards towards the root node of the game tree. A player conjectures, which strategy a subsequent player will choose according

[30]Theory of Moves requires a strict ordinality of preferences and the solution in normal form games depends on the initial strategy profile, at which the solution algorithm starts. The greatest drawback with respect the Conflict Analysis is that the algorithm is only applicable to 2×2 games.

to his own strategy choices. Players are aware that each player considers the sequence of strategy choices of earlier players and chooses the strategy that is best response to the strategies chosen before. Upon this anticipative conjecture each player chooses his best response. Apparently, no problem is caused by the additional stability criterion. An issue eventually arises, however, in sequential games with incomplete information and if sequential stability is applied forwards in the direction of the terminal node. A player is bound by the various strategic options of his predecessors, since he knows that a previous player has already made his decision and cannot re-evaluate at a later stage. Hence, a switch *ex post* cannot occur. A similar issue applies to *one-shot games*. Conflict Analysis seems to suffer from what has been termed "Metagame Fallacy". The section discusses this issue and illustrates why the reasoning behind the Metagame Fallacy is illegitimate.

The Prisoner's Dilemma has been chosen deliberately as an introductory example in section 3, since it is also the game which best illustrates the Metagame Fallacy. The analysis of the PD has shown that there exists a stable cooperative equilibrium. The argument of the Metagame Fallacy goes as follows: If each player is convinced that his counterpart cooperates, he can still improve by defection. Thus a game of second order reasoning is created by each player that is isomorphic to the original game. The fallacy, it seems, lies in the assumption that mutual rationality implies symmetric behaviour. Yet, this is obviously not the case and cooperation cannot be a rational strategic choice.

The argument misses an important point. Apparently, a pre-commitment of a player to a strategy will never be credible as long as other players are not entirely convinced that he will stick to his commitment and, further, that the player is also sure that the other players will do the same (see Binmore, 1994, p. 179).[31] Yet, it is not a pre-commitment that stabilises the cooperative equilibrium, but the empathic knowledge of the other player's strategy. This creates a correlation between the strategic choice of each player that the Metagame Fallacy argument neglects. Each player believes that his choice of the defective strategy will impact the probability, with which the other player chooses this strategy. Hence, a player's choice to cooperate is consistent with this belief and *rational*. If p is the probability that a player 1 and 2's strategy choice are correlated, and using the pay-offs as in pay-off matrix 7 on page 9, the following condition must hold to induce a player to choose the cooperative strategy:

$$\begin{aligned} pa + (1-p)b &> pd + (1-p)c \\ p &> \frac{c-b}{a-d+c-b} \end{aligned} \tag{16}$$

Since by definition of a PD, $a > d$ and $c > b$, the probability stays within the unit interval. If, in fact, all players have a high level of empathy, such that $p = 1$, the PD turns into a Twins game (see Binmore, 1994, 1998 for more details).

$$\begin{matrix} & C & D \\ \begin{matrix} C \\ D \end{matrix} & \multicolumn{2}{c}{\begin{pmatrix} a,a & \\ & d,d \end{pmatrix}} \end{matrix} \tag{17}$$

[31] i.e. one has to assume that both players make a kind of a priori commitment, similar to a categorical imperative that dictates each player to choose the cooperative strategy. This leads to the question of free choice of players. Any player abiding by the axioms of rationality cannot, however, be assumed to have a free will a posteriori; since his choice is predetermined by the rules of the game.

which is indeed a symmetric Newcomb's Paradox. Dominance tells both players that they should defect, but this would only entail a pay-off of d, since both perfectly predict the other player's choice. Maximisation of pay-offs tells them to cooperate.

Empathy is surely a strong assumption. Yet, even Binmore frequently emphasises the importance of empathy: "*Homo economicus* must be empathic to some degree. By this I mean that his experience of other people must be sufficiently rich that he can put himself in their shoes to see things from their point of view. Otherwise, he would not be able to predict their behavior, and hence would be unable to compute an optimal response." (Binmore, 1994, p. 28). If a player believes he is empathic, then it is rational to assume that other players are also empathic. Furthermore, notice that this approach has found the defective equilibrium also to be stable, and that the cooperative equilibrium requires both players to predict that their counterpart cooperates. A look at the joint transition matrix T shows that if individuals initially assign equal probability to all strategies, the defective strategy profile will occur with a probability of $\frac{3}{4}$. The cooperative strategy profile will only arise with probability $\frac{1}{4}$, since it requires that a player believes the other player to be choosing the cooperative strategy. Furthermore "empathic errors" can occur in this approach. A hypergame, where individuals do not trust their counterpart, will result in the defective equilibrium with certainty. The assumption of empathy alone thus does not pose an issue. Yet, this assumption is only partly sufficient to maintain the cooperative equilibrium.

Before I explain the crux it is necessary to stress again an important point. At first glance Conflict Analysis seems similar to the approaches using other regarding preferences. One might think that the Conflict Analysis approach merely substitutes sympathy by empathy to explain the stability of the cooperative strategy profile. Notice, however, that this form of empathy is immanent in the structure of the Conflict Analysis approach, whereas assuming other-regarding preferences is a change of the game itself. Conflict Analysis takes an entirely different approach by changing the way games are analysed, but keeping the way the problem was originally formulated. In the Conflict Analysis approach preferences are identical to the original specification of the PD. Players are entirely self-referential and only maximise their own pay-offs. It has to be stressed that the stability of the cooperative equilibrium occurs only through the expectations about the consequences a change of strategy will have on the *own* pay-off. Other pay-offs are only considered in order to anticipate other players' strategy choices. Other regarding preferences, on the contrary, expand the assumptions on preferences beyond the original specification of the model. Strictly speaking, the necessity to introduce other regarding preferences into the PD is a sign that the game has been misspecified and the underlying game is in fact no PD. An *ex post* change of preferences creates degrees of freedom that might render the solution of games completely arbitrary.[32] Other-regarding preferences are therefore clearly distinct from the preference

[32]The issue, it seems, results from an understanding of pay-offs as a sole welfare measure, such as money, years etc. This is caused by the sometimes improper definition of the underlying game. The pay-off type is often not specified, since "[g]ame theorists [...] understand the notion of a pay-off in a sophisticated way that makes it tautologous that players act as though maximizing their pay-offs. Such a sophisticated view makes it hard to measure pay-offs in real-life games, but its advantage in keeping the logic straight is overwhelming."(Binmore, 1994, p.98) The inadmissibility to apply a utility function *ex post* becomes especially obvious for games that are defined as $\Gamma = (S_1, S_2, ..., S_n; U_1, U_2, ..., U_n)$, where U_i defines player i's utility function, and the elements of the pay-off matrix define utility pay-offs. For games defined as such, the *ex post* transformation of pay-offs by applying utility functions, implies that the original utilities are put into utility functions to define new

orderings in Conflict Analysis.

In order to induce individuals to cooperate, it is insufficient that they anticipates correctly the choice of the other player with a probability of at least p. Mutual cooperation requires not only that a player believes that the counterpart correctly anticipates his choice to cooperate, but also that he believes the other to believe the same, and so on (common knowledge). The cooperative equilibrium in the PD thus demands a "common knowledge of *empathic* rationality".[33] The commonality of knowledge is clearly a very restrictive assumption, the issue is, however, not unique to Conflict Analysis.[34] In addition, the assumption that individuals anticipate other action's with high probability is a second disputable assumption.

As a last argument, in order to vindicate the approach and to weaken the magnitude of the problem raised in the last paragraph, I emphasize that this line of argument does not hold for repeated or pseudo one-shot games[35] (as has been illustrated at the beginning of this section), but for one-shot games. According to Larry Samuelson and Andrew Postlewaite such games are an unrealistic representation of life (see Mailath, 1998). In fact, the argument that individuals play Nash equilibria seems to hinge on the assumption of the repetitiveness of games.[36] Thus, there is doubt as to whether individuals really apply the

utilities (-the same line of argument applies to pay-offs as incremental fitness). Even for a general game of the form $\Gamma = (S_1, S_2, ..., S_n; \pi_1, \pi_2, ..., \pi_n)$, however, such a transformation by utility functions is inapplicable. A sophisticated interpretation of pay-offs implies that they already include the contextual essence, i.e. all the information necessary to solve a game. In addition, the equivalence of games that are derived from positive affine transformations of the pay-off matrices is no longer maintained if pay-offs are transformable *ex post*. A sufficiently large positive affine transformation of the pay-offs will thus not necessarily maintain the preference relations and strategic choice will be incongruent with the original game. General claims about the Prisoner's Dilemma, stag hunt, battles of sexes, or chicken game etc. are infeasible. In contrast, Conflict Analysis has only limited degrees of freedom as equilibria are strictly defined by the preference relation of the original specification and the three stability criteria. In conclusion, whenever it is observed that players do not choose the strict dominant strategy, the only valid deduction in the context of standard Game Theory is that the PD is an incorrect representation of the real interaction.

[33] In fact, the condition is slightly weaker. Mutual cooperation does not require that individuals possess the necessary level of empathy to predict other's strategy choices, but that they believe that others do and further that others believe they do.

[34] Neither common knowledge, nor mutual knowledge of conjectures cannot be supposed *per se*. This is, however, a prerequisite for a rational Nash equilibrium (see Aumann and Brandenburger, 1995).

[35] This is the case in a version of the PD by Wagner, 1983, where both prisoners have friends in the District Attorney's office, who inform them about whether or not the other has confessed. The game is sequential, but there exists no definite first and second mover. The game only ends with certainty, when both confessed (i.e. when one player confessed, since the best reply strategy for the other player is also to confess).

[36] According to Mailath, 1998, a Nash equilibrium requires mutual knowledge of what other players do, i.e. knowledge, which can only be derived either by preplay communication, self-fulfilling prophecy (what Mailath describes here as self-fulfilling prophecy might be more familiar by the term "Common Knowledge of Logic", see Gintis, 2009 for a critical discussion), focal points, or learning. Laboratory experiments have shown that preplay does not necessarily prevent coordination failures (Cooper et al., 1992), even more so in those cases, where a player benefits from a specific strategy choice of the other player irrespective of his own choice. Such a case is the Stag Hunt game with preference order $a > c > d > b$ for pay-off matrix 7 on page 9. The argument of self-fulfilling prophecy assumes that if a theory to uniquely predict outcomes is universally known, it will determine the Nash equilibria, i.e. the theory is consistent with the theory used by other individuals. First, the underlying logic is circular: The theory is correct if it is self-fulfilling, and thus if adapted by all agents, who believe it to be correct. Second, it requires a link between what an agent is expected to do and what he does, and further what he assumes others to do. Experiments indicate that individuals do not believe that others respond to pay-offs in symmetric games in the same way as they do (Huyck et al., 1990). This does not contradict mutual

rationality of one-shot games to the PD. If rational behaviour is explained as a result of an evolutionary selection process[37], it is dubious whether one-shot games regularly occurred in the historical context that shaped our behaviour. The empirical data shows that both equilibria predicted by Conflict Analysis are entirely plausible. Individuals apply heuristics (Page, 2007), such as the belief that others will able to predict one's choice, or the belief in a poetic justice of fate, which may ultimately lead to a strategy choice predicted by Conflict Analysis.[38]

7. Conclusion

This chapter has developed and illustrated a game theoretic approach that provides an interesting alternative to model interactions. This approach offers undeniable advantages over other standard approaches: It can handle larger (non-quantitative) strategy and/or player sets more easily. It incorporates the higher order reasoning of Metagame Theory and allows for modeling higher level hypergames. It can therefore explain the existence of equilibria not captured by the Nash concept, but which are frequently observed in real-world interactions. It can be efficiently applied to sequential games to discriminate between equilibria. It does not require a transition from a perceivable preference order to a cardinal description of preferences, nor the strict transitivity of preferences.

Conflict Analysis is, however, not as elegant as standard game theoretic approaches, when it comes to two player interactions with limited strategy sets. The addition of two supplementary stability criteria exhibits drawbacks in the axiomatic foundation of this approach and its theoretic basis is by far not as rigorously developed as the "pure" Nash approach. Furthermore, it is also restrictive in some assumptions. Players are required to exhibit a certain level of empathy that enables them to predict others choices, as well as the common knowledge of this capacity. Alternatively they are assumed to misinterpret one-shot games as repeated games. Though players are supposed to enjoy a higher order reasoning that allows them to anticipate the reaction of others to their strategy choice, they are not supposed to deliberately choose a non-best response that makes them immediately worse off if others do not change their strategy, but could improve the outcome to their favour after eventual strategy switches of other players. This is obviously an assumption to keep the approach tractable, but one might wish for the possibility of such "tactical" choice. Furthermore, though the approach handles most games quite well, the determination of mixed equilibria represents an issue, owing to the structure of the Conflict Analysis approach and the absence of specific pay-offs. Yet, this last issue only presents a drawback for the small minority of games that possess quantifiable pay-offs.

rationality, but shows that people do not assume that a unique way to predict behaviours exist. Further, seldom a unique "obvious" strategy exists in a game, which defines a focal point. Efficiency, egality, justice, risk are all different approaches of such "obvious" play. Finally, learning requires that (almost) identical games are played repetitively. Thus, the theoretical argument against the Conflict Analysis approach under the classical rationality paradigm is not substantive as it, in fact, applies to a game type, which cast already scepticism on the feasibility of Nash equilibria.

[37] see Choi and Bowles, 2007; Gintis, 2009; Gintis et al., 2005; Robson, a,b

[38] Consider the English and German idiom: "You always meet twice." Under the condition that a game is played repetitively the rationality of Conflict Analysis does not seem far-fetched.

Overall Conflict Analysis is an attractive approach, especially, when applied to repeated games. It should not be regarded as an alternative to standard game theory or even as its substitute. It should be more considered as an alternative in perspective. Conflict Analysis is meant as a positive approach that illustrates an argument of why certain equilibria are frequently observable that are indeed not Nash equilibria. This approach is embedded in the spirit of theoretical pluralism, and an eventual step forward in redefining the theoretical development in game theory. It should complement with other alternative attempts and its methodology should thus provide sources for new conceptions that can enrich game theory and offer new starting points for future research.

Appendix

A. The Fundamentals of Metagame Analysis

This section will illustrate the basic principles of the Metagame Theory. Most of the basic argument is equivalent to standard game theory. Yet, the higher order reasoning necessitates to depart from standard definitions at various points.

In a game G let there be a set of players N with player $i = 1,2,...,n$ and let there be also an individual set of strategies S_i for each player, so that each individual strategy is denoted by $s_i \in S_i$. The entire set of strategy profiles is represented by $S = S_1 \times S_2 \times .. \times S_n$. A strategy profile of a game G in period t is then defined by $s_t \in S_t$, where $s_t = (s_{1t}, s_{2t}, ..., s_{nt}) \in S$. For each player i there exists a preference function U_i, which orders (not necessarily completely) the set of strategy profiles S. Consequently game G can be defined as $G = (S_1, S_2, ..., S_n; U_1, U_2, ..., U_n)$.

A metagame kG is derived from the underlying game G in the following way: Replace the strategy set of player k by a new set of strategies F. Each element $f \in F$ is a function that defines a response strategy to each possible strategy profile s_{-k}.[39] The preference functions for the new game are denoted as $U_i^{'}$ of player i in kG. Consequently, the metagame is defined as $kG = (S_1, S_2, ..., S_{k-1}, F, Sk+1, ..., S_n; U_1^{'}, U_2^{'}, ..., U_n^{'})$. Since the set of strategy profiles of kG is defined by the the Cartesian product $F \times S_{-k}$, we obtain a strategy profile $q = (f, s_{-k} \in F \times S_{-k})$. An rth level metagame L is then represented by $L = k_1 k_2 ... k_r G$. Let there be an operator β that transforms any strategy profile of an rth level metagame L to a strategy profile of the lower (r-1) level metagame, such that $\beta(f, s_{N-k}) = (f(s_{N-k}), s_{N-k})$ and $\beta^r(f, s_{N-k}) = (s_k, s_{N-k}) = q$.[40]

[39] Assume a game with two players and 4 strategies each. F is then a set of $4^4 = 256$ functions, where each f is defined by four response strategies, one for each strategy of the other player.

[40] Suppose that both players have strategies *(u)p, (d)own, (l)eft* and *(r)ight*. For $1G$ let there by an $f = (u/u/d/d)$, meaning that player 1 plays strategy up as a response to both up and down, and down as a response to both left and right. A strategy profile $p = (u/u/d/d, l)$ in $1G$ can than be transformed to a strategy profile in G, since $\beta(u/u/d/d, l) = (f(l), l) = (d, l)$.

For each strategy profile q, let S be divided into two subsets, where $U_i^+(q)$ denotes the set of strategy profiles that are strictly preferred by player i to strategy profile q and where $U_i^-(q)$ includes all the strategy profiles that are not strictly preferred by player i to q. Hence, for any given strategy profile $\bar{s}_{-i}$ by all players other than i, the set of strategy profiles that can be obtained by a unilateral strategy switch of player i is given by $z_i(q) = (s_i, \bar{s}_{-i})$. An individually attainable dominant profile p with respect to q is then defined as $p \in u_i^+(q) = z_i(q) \cap U_i^+(q)$.[41] That means that a player i can improve on the current strategy profile $q = (\bar{s}_i, \bar{s}_{-i})$ given strategy profile $\bar{s}_{-i}$ by choosing a strategy s_i^* to strategy profile $p = (s_i^*, \bar{s}_{-i})$. Notice that it follows from the paragraph above that the preferences in G determine the preferences in L, i.e. in the case, where q is strictly preferred to another strategy profile p: $p \in U_i^{-'}(q) \Leftrightarrow \beta p \in U_i^-(\beta(q))$.[42]

A strategy profile $q = (\bar{s}_i, \bar{s}_{-i})$ is rational for any player i, if $\bar{s}_i$ is the best response strategy to $\bar{s}_{-1}$. Hence, the set of rational strategy profiles for player i is given by $R_i = \{q | \forall s_i, (s_i, s_{-i}) \in U_i^-(q)\}$. The metarational strategy profile for G can then be derived from the underlying metagame L by applying β r-times to the set of rational strategy profiles R. Let $\hat{R}_i(L)$ denote the set of metarational strategy profiles for G, then $\beta^r R_i(L) = \hat{R}_i(L)$. Similarly the set of equilibria $E(L)$ for a metagame L, which is the set of strategy profiles rational for all players in L, connects to the set of equilibria for game G via β and $\beta^r E_i(L) = \hat{E}_i(L)$.[43] The general set of rational strategy profiles R_i^* for player i is then given by the union of all rth level metagames and thus $R_i^* = \bigcup_j \hat{R}(L_j) \cup R_i(G)$.

On the basis of the "Characterization Theorem" by Howard (1971), the determination of the general set of rational strategy profiles does not necessitate the analysis of the infinite metagame tree, but only a three step analysis. The first step is to check for the absence of unilaterally attainable dominant profiles $p = (s_i, \bar{s}_{-i})$, such that $\nexists s_i : (s_i, \bar{s}_{-i}) \in U_i^+(q)$. In this case q is metarational. A *symmetric metarational outcome* is defined if for all such better response strategies s_i^* other players possess a strategy that garantees a new strategy profile not preferred by the particluar player under analysis. Hence, it must hold $\exists s_{-i} \forall s_i^* : (s_i^*, s_{-i}) \in U_i^-(q)$. A *general metarational outcome* occurs if the player under analysis has again a response strategy to improve to a preferred strategy profile. A cycle may evolve and the play can continue indefinitely, which is assumed to be strictly less preferred to q (similar to a deadlock point, see Binmore, 1998). A strategy profile g therefore has to be finally checked for the absence of such an infinite cycle; $\nexists s_i \forall s_{-i} : (s_i, s_{-i}) \in U_i^+(q)$. If this is the case, q is metarational. Thus, if a strategy profile q is metarational for player i, it is also stable for player i and the set of stable strategy profiles for all players defines the set of equilibria of the general (n-th level) metagame.

[41] An alternative formulation is $\exists s_i : (s_i, \bar{s}_{-i}) \in U_i^+(q)$ such that $p \in \{(s_i, \bar{s}_{-i}) : (s_i, \bar{s}_{-i}) \in U_i^+(q)\}$.

[42] $(u/u/d/d, l)$ generates the same outcome as (d, l) and thus preference orders with respect to each must be identical.

[43] Suppose that$(u/u/d/d, l)$ is rational for *both* players, then (d, l) is meta-rational for both players and the equilibrium of the underlying game G.

B. Tables

Table 15. Dynamics of the Traveller's Dilemma - Individual Transition Matrices

Transition Matrix for *A*

	17	18	20	24	33	34	36	40	65	66	68	72	129	130	132	136
17	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
33	0	0	0	0	1	0	1	1	0	0	0	0	0	0	0	0
34	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
40	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
65	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
66	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
68	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
72	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
129	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
130	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
132	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
136	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1

Transition Matrix for *B*

	17	18	20	24	33	34	36	40	65	66	68	72	129	130	132	136
17	1	0	0	0	1	0	0	0	1	0	0	0	1	0	0	0
18	0	1	0	0	0	0	0	0	0	1	0	0	0	1	0	0
20	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
24	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
33	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
34	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
40	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
65	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
66	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
68	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
72	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
129	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
130	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
132	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
136	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1

Final Transition Matrix

	17	18	20	24	33	34	36	40	65	66	68	72	129	130	132	136
17	1	1	1	1	1	0	0	0	1	0	0	0	1	0	0	0
18	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0
20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
33	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0
34	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
40	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
65	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
66	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
68	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
72	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
129	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
130	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
132	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
136	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1

Table 16. Traveller

overall stabi
stability for *A*
As preference
DPs
stability for *B*
Bs preference
DPs

Table 17. Static An

overall stability	E(E)
stability for *A*	r(s)
As preferences	33
DPs	(36) (40)
stability for *B*	r
Bs preferences	17
DPs	

overall stability	x
stability for *A*	r
As preferences	24
DPs	
stability for *B*	r
Bs preferences	17
DPs	

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