# State-Dependent Stochastic Stability and the Non-Existence of Conventions

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**Abstract**

Arising from criticism in the literature and focusing on 2x2 coordination games, the concept of stochastic stability is extended to take account of state dependent error and sample sizes. Both, error and sample size are expected to be correlated with the loss that occurred, if a player chooses a non-best response strategy. The state independent and state dependent predictions determine the same Stochastically Stable State (SSS) if the game's pay-off matrix exhibits a form of symmetry, or if only the relative potential loss from idiosyncratic play defines the state dependent variable. Predictions may differ if neither of these conditions is met. In addition, the paper raises a second crucial point. Even if these conditions are met, the minimum stochastic potential is only a necessary but not a sufficient condition for the evolution of an SSS. The SSS must further be sufficiently risk dominant, otherwise no convention will evolve.

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## 1. Introduction

An institutional framework can be considered as the outcome of an interaction of a large number of social and economic agents. It can thus be perceived as a pure Nash equilibrium of a complex game between various players, which defines the convention and thus the institutions to which these agents adhere. Yet, the problem of such games is that they oftentimes exhibit a large number of stable pure Nash equilibria. Thus it is important to look at the dynamics underlying the evolution of such conventions, to analyse how and under which conditions a structural change occurs and how this change defines the long-term institutional framework.

Along the same line, Young (1993) illustrated in "The Evolution of Convention" a well-known approach that is part of a larger literature on stochastic stability.[1] Starting with the analyse of the long-term properties of stochastic play with random matching in a larger population (Turnovsky and Weintraub, 1971; Foster and Young, 1990; Kandori et al., 1993) the approach has been extended to different matching rules, especially local interactions (Blume, 1993; Ellison, 1993; Young, 1998; Ellison, 2000; Morris, 2000) and interactions on graphs (Boncinelli and Pin, 2010). In addition, other extensions focused on the way a reply is chosen by the players assuming e.g. choice trembles

[1] See also the similar approach by Kandori et al. 1993.

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(Samuelson, 1994, 1997) or average performance of each strategy (Robson and Vega-Redondo, 1996). General criticism has been raised in Orléan (1995); Bowles (2006).

These variants of stochastic play provide a mean to discriminate between a potentially large number of pure Nash equilibria in an evolutionary game, which define plausible conventions between players on the long-run. It is assumed that players illustrate random and infrequent *idiosyncratic* play and rely on past interactions to make a choice. Thus, the approaches generally do not attempt to explain the subtle procedures leading to a transition between conventions, but why certain conventions and institutions appear to be more stable and persist longer than others.[2] Yet, especially the assumption of the initial framework by Young (1993) neglects the fact that social norms and conventions are characterised by an environment that works towards a particular behaviour. Hence, approval and disapproval are vital for the dynamics defining social conventions and norms. The disapproval of a certain strategy or action can be assumed as being captured by the pay-off loss that results from the lack of coordination in a game, i.e. the forgone pay-off a player if he does not adhere to the conventional strategy.

Restricting the analysis to $2 \times 2$ coordination games, section 2 of this article uses Young's approach as a basis, but extends it along two dimensions: Firstly, it is assumed that a player's likelihood of idiosyncratic play depends on the potential loss he faces in the case of miscoordination, i.e. if the player chooses a non-conventional strategy given his "opponent" adheres to the current convention. The higher the loss of a player is if he deviates from conventional behaviour (i.e. the higher the social pressure to comply) the less willing is this player to try alternative forms of behaviour. The same argument, however, also holds with respect to the sample size.[3] In order to minimize the risk of being exposed to social pressure, a player wishes to be better informed about what is considered conventional behaviour. Thus secondly, the original approach is also extended to incorporate a varying sample size that depends, like the error rate, on the potential loss a player endures if he does not adhere to the currently prevailing convention. Both cases are of interest, since we shall discover that assuming a state dependent error size is not symmetric to assuming a state dependent sample size. Predicted results differ in the case if more than one player type interacts (i.e. if the underlying pay-off matrix is not symmetric). This is an important point, since criticism on the approach generally seems to concentrate on the error size but neglects the sample size.

The connection of the potential loss from mis-coordination to the error and sample size creates a *weighted* stochastic potential for each equilibrium of a game. In the original approach, the minimum stochastic potential defines the long-run convention. The minimum *weighted* stochastic potential, as described in this paper, however, does not necessarily determine the same state as the minimum stochastic potential of the original approach. Yet if the pay-off matrix illustrates certain forms of symmetry, or the loss in each conventional state relates to the error and sample size relative to the total loss over all conventions, the state dependent approach defines the same long-term convention as the original approach. In this respect, the results in this paper call for prudence of applying state independent stochastic stability as a discrimination criterion for any game, but also seem to confirm the original results for a large number of games, namely those that exhibit some form of symmetry. This rather positive conclusion is, however, premature.

Section 3 points out a more fundamental issue in this context: Is it realistic to assume that the

[2]We are, in fact, looking here at the so-called organic evolution of institutions, see (Menger, 1963).
[3]This relation indeed captures the essence of risk dominance, see footnote 7 for details.




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error rate approaches zero in the long-run as required by Young's adaptive play? If the error rate is state (loss) dependent, this assumption cannot be supported. If the expected loss is close to zero, individuals can be presumed to illustrate a high frequency of idiosyncratic play.[4] Consequently, if the error rate remains close to one at some states, the Stochastically Stable State (*SSS*) is not sufficiently defined by the risk dominant equilibrium (as defined in Harsanyi and Selten, 1988). In this context, the entailed *randomness* of idiosyncratic play will require an even larger basin of attraction as a counter-force, in order for a SSS to evolve in the long-run, but, at the same time, can also act as a barrier to transition between conventions.

This result does not contradict the finding that in case of a high error rate, the risk dominant equilibrium is a more likely long-term convention than the risk inferior equilibrium (see e.g. theorem 4.2 in Young, 1998). However, section 3 shows that oftentimes a transition between pure Nash equilibria does simply not occur. If a population starts out in a state of a pure Nash equilibrium, this equilibrium will remain a convention in the long-run if its has a sufficiently large basin of attraction. This result is similar to the adequate setting studied in the biological literature, in which mutual invasion is not feasible (see Taylor et al. 2004 for an example). (Similarly Antal and Scheuring 2006; Tarnita et al. 2009 show how weak selection shifts the best response strategy.) This paper demonstrates, in addition, that if the equilibria's basins of attraction have roughly equal sizes, a population will end up fluctuating in around a completely mixed state in the long-run, and no unique conventional strategy will exist. Although this mixed state is not an equilibrium of the unperturbed game, it defines the long-run *convention*, No further transition between pure Nash equilibria occurs. Thus, this paper provides an explanation of why situations without clearly defined conventions can be frequently observed, and illustrates a crucial weakness of the stochastic stability approach.

### *1.1. A short Introduction to Stochastic Stability*

In the following, we shall focus on the simplest definition of stochastic stability, as is defined by Young (1993). Agents are randomly matched to play a game. Since players do not know which strategy will be chosen by their counterpart, they choose their best response based on their prior experiences (commonly known as fictitious play). However, players do not take into account the entirety of past play, but posses only a limited memory. In addition, players draw a sample from this memory, whereupon they choose their best response, and, at the same time, are prone to commit infrequent errors (which Young terms adaptive play). Thus with a small given probability, players do not choose their (myopic) best response that would maximise their expected pay-off, but choose a strategy at random. Consequently, by adding noise (or periodic shocks) to the game, the process does not fully settle down in a single state, but tends to occasionally switch between equilibria. In a $2 \times 2$ player game, the population visits the *risk dominant* equilibrium (i.e. the stochastically stable state) more often and eventually settles in this state once the error subsides.

To understand this better, imagine two pure Nash equilibria (all play strategy $A$ and all play strategy $B$) of a $2 \times 2$ coordination game. Assume that *all* $B$ risk dominates *all* $A$. Let us also assume that the population consists of only two players and is initially in a state, in which both players remember to have always played only $A$. We define this as a history $h_A$. It requires a

[4] A second issue regarding adaptive play is that the the most recent play makes players forget the oldest play. This would require players to associate a time frame to each interaction. This is not consistent with the idea that players sample, and thus past play should be forgotten randomly. Section 3 indicates that relaxing this assumption might not change the conclusions of this paper.

certain number of erroneous play by one player, in order for them to occur in the sample of the other player at a frequency that makes strategy $B$ his best response. Once strategy $B$ occurs sufficiently frequently in the samples of both players (either by error or by best-response), both choose $B$ as a best response. From this point on the population converges to the state, in which both remember only having played $B$, denoted by the history $h_B$. Looking at the other case, in which the population is in the state $h(B)$, we observe that a larger number of errors are required for $A$ to become a best response. Since *all B* risk dominates *all A* by assumption, the former has a larger basin of attraction, and thus more instances of idiosyncratic play need to be sampled. Consequently, the equilibrium in which both players play $B$ is more likely to occur. It is both easier accessible and more *resistant* to stochastic shocks. For the interested reader, the original concept is more extensively discussed in section Appendix A.

The assumption that errors are state and pay-off independent has been criticized (Bergin and Lipman, 1996; Bowles, 2006). Yet, the same criticism applies to the state independent sample size. It will be illustrated herein that no major changes are required in Young's approach to take this condition into account. It is only necessary to weigh the resistances accordingly. The following section will thus introduce pay-off dependent sample and error size into the calculation of the resistances. The assumption is simple: If an individual has more to loose from choosing a wrong response leading to miscoordination, he either tries to directly control his error rate (i.e. is less willing to try out alternative strategies) or increases the sample size (thus having a better view of what has been played before). Yet, both increasing the sample size or reducing the error size is costly. We can assume, e.g. that in the case in which a player controls his error rate by increasing his sample size, the player spends time to ask around causing a higher organisational and also opportunity cost. In the case, in which he tries to control the error rate directly and hence his idiosyncratic play, the player experiences costs of prior mental processing and analyses, in addition to controlling his impulses. Consequently, only a higher potential loss will induce a player to take such preventive actions.[5] It will be shown that the *minimum stochastic potential* is not a sufficient criterion for an *SSS* in $2 \times 2$ games. Additional assumptions about the pay-off matrix or the correlation between potential loss from mis-coordination and the error or sample size have to be made to guarantee that the predicted results remain identical to the original approach. It is further illustrated that a significant difference exists between assuming a state dependent sample size on the one hand and a state dependent error size on the other hand, if the interacting population is not homogeneous. Sample size affects the rate at which a player type directly *observes* mutations and is dependent on the pay-off this player type has at the current convention. In contrast, the error size affects the rate at which an error is *committed* by the other player types and therefore depends on the pay-off of those other player types.

## 2. State Dependent Sample and Error Size

Adaptive play is defined by the rate at which an individual samples past play and by the rate at which he idiosyncratically chooses a strategy (i.e. explores alternatives). We can assume that both variables, sample and error size, are controlled by a player in order to maximise his expected pay-offs. By manipulating these two variables he can minimize the loss from mis-coordination caused by

[5] We can, for example, assume a control cost function, similar to Damme and Weibull (1998), which connects potential loss to both error and sample size.

the non-observance of the conventional strategy.[6] Both increasing the sample size and stabilising his trembling hand (or reducing his willingness to explore) affect the rate of erroneous choice, but both come at a cost. Thus, if his potential loss is high, an individual is more careful in exploring alternative strategies, more inclined to stabilise his "trembling hand", as well as to increase his sample size to make sure that the latter correctly represents the prevailing convention.

Given that $\epsilon_i$ defines individual $i$'s rate of erroneous choice and that $s_i(\omega)$ determines the size of the sample a player draws from the collective memory $m$ in state $\omega$. The option of directly controlling a player's error rate by stabilising his *trembling hand* is defined by the individual's choice of $\gamma_i(\omega)$ in state $\omega$ that, given an exogenous "baseline error $\varepsilon$", defines the error rate as $\epsilon_i = \varepsilon^{\gamma_i(\omega)}$. Hence, $\partial\epsilon_i/\partial\gamma_i < 0$, given $\varepsilon \in (0,1)$ and $\gamma > 0$. For simplicity, assume that the baseline error $\varepsilon$ is exogenous and identical for all players.

Consider a $2\times 2$ coordination games with two strict Nash equilibria in pure strategies, generally of the form presented in matrix 1, with $a_i > c_i$ and $d_i > b_i$. (In the following index 1 is always assigned to row players and index 2 to column players.)

$$\begin{array}{cc} & \begin{array}{cc} A & \quad B \end{array} \\ \begin{array}{c} A \\ B \end{array} & \begin{pmatrix} a_1, a_2 & b_1, c_2 \\ c_1, b_2 & d_1, d_2 \end{pmatrix} \end{array} \tag{1}$$

The pay-off from coordination is generally defined by $g_i(\omega) = max[\pi_i(A,\omega), \pi_i(B,\omega)]$, given that $\pi_i(S,\omega)$ defines player $i$'s pay-off if he chooses strategy $S$ when $\omega \in (A,B)$ defines the conventional strategy adhered to by a second player. Furthermore, we equivalently define $w_i(\omega) = min[\pi_i(A,\omega), \pi_i(B,\omega)]$. The potential loss in the case, if player $i$ erroneously chooses the non-conventional (non-best response) strategy, is thus $l_i(\omega) = g_i(\omega) - w_i(\omega)$. For a game defined by matrix 1, we have $l_i(A) = a_i - c_i$ and $l_i(B) = d_i - b_i$. For simplicity assume an identical baseline error and cost structure for all players, and that error and sample size depend only on the potential loss, they are thus independent of the player type. We assume the following

$$\begin{aligned} l_i(\omega) < l_j(\omega') &\Leftrightarrow \epsilon_i(\omega) > \epsilon_j(\omega') \\ &\Leftrightarrow s_i(\omega) < s_j(\omega'), \quad \text{with } i,j = \{1,2\} \text{ and } \omega,\omega' = \{A,B\} \\ &\Leftrightarrow \gamma_i(\omega) < \gamma_j(\omega') \end{aligned} \tag{2}$$

These assumptions can be supported by a cost control function as in Damme and Weibull (1998) (see page 17 in the appendix for details) and empirical evidence (Elder and Allen 2003, but also Weber 2007, p. 616.) Relation 2 implies that the player type with the highest potential loss, either relies on the largest sample, or has the smallest error probability. It also means that a player samples more or has a lower error rate in the convention that he perceives as risk dominant.[7]

In $2 \times 2$ coordination games, only two equilibria in pure strategies exist and both equilibria are connected by direct paths. The *reduced resistance* is defined by the minimum share of non-best response plays in the sample, which is necessary to induce best response players to switch

[6] This can be a monetary loss, but also be experienced discomfort from social shunning and punishment.

[7] For a symmetric $2\times 2$ game, subscripts in matrix 1 are of no importance, and equilibrium $AA$ is risk dominant, if $0.5a + 0.5b > 0.5c + 0.5d$ or if $a - c > d - b$, which equals the potential loss $l(\omega)$. Thus the higher the risk of an equilibrium, the lower is the error probability or the larger is the sample size.

their strategy. If the sample and error size are state and type independent, the sample size can be normalised to 1 and the reduced resistances will equal the stochastic potential. It suffices thus to compare only the two reduced resistances along the direct paths (one for each player type) to determine the *SSS* in the state independent case. The reduced resistances are defined as follows:

$$r_{AB} = min\left(\frac{a_1 - c_1}{a_1 - b_1 - c_1 + d_1}, \frac{a_2 - c_2}{a_2 - b_2 - c_2 + d_2}\right) \text{ and} \tag{3a}$$

$$r_{BA} = min\left(\frac{d_1 - b_1}{a_1 - b_1 - c_1 + d_1}, \frac{d_2 - b_2}{a_2 - b_2 - c_2 + d_2}\right) \tag{3b}$$

$$\text{or more succinctly: } r_{AB} = \alpha \wedge \beta \text{ and } r_{BA} = (1-\alpha) \wedge (1-\beta) \tag{3c}$$

where $r_{AB}$ ($r_{BA}$) defines the reduced resistance of a transition from a pure Nash equilibrium, where all players choose strategy $A$ ($B$) to a pure Nash equilibrium, where all players choose $B$ ($A$). Consequently, $\alpha$ and $\beta$ define the minimum population frequencies in the sample, necessary to induce best-response players to switch to strategy $B$. If $r_{AB} > r_{BA}$ ($r_{AB} < r_{BA}$), the *SSS* is defined by $h_A$ ($h_B$). Obviously in the case of a symmetric pay-off matrix, the *SSS* is equivalent to the risk dominant Nash equilibrium. (for detailed proofs, refer to Young, 1993, 1998).

In the following, we will individually focus on the case of state dependent sample size and the case of state dependent error size. In a symmetric game, a player's position is irrelevant, i.e. pay-offs are independent of the indices in matrix 1. The following two propositions hold in the presence of state dependent sample size $s(\omega)$ given conventional state $\omega$ (see Appendix B for proofs) of this section:

**Proposition 1.** *For the symmetric case with state dependent sample size the resistances are determined by*

$$r^s_{AB} = \alpha s(A) \tag{4a}$$

$$r^s_{BA} = (1-\alpha)s(B) \tag{4b}$$

**Proposition 2.** *In the case of two different player types $i = 1, 2$ and state dependent sample size $s_i(\omega)$, the resistances are defined by*

$$r^s_{AB} = \alpha s_1(A) \wedge \beta s_2(A) \tag{5a}$$

$$r^s_{BA} = (1-\alpha)s_1(B) \wedge (1-\beta)s_2(B). \tag{5b}$$

Since only the relative values of the resistances are of interest in order to determine a *SSS*, assume that in the case of symmetric pay-offs it holds by normalisation that $s(A) \neq s(B) = 1$. Then the equilibrium sample size $s^*$, at which both equilibria are stochastically stable, is given by $s^* = \frac{1-\alpha}{\alpha}$. For all $s(A) > s^*$, $h_A$ is the sole *SSS*. In the case of $s(A) < s^*$ the *SSS* is defined by $h_B$.

Consider the state dependent error size, defined by $\varepsilon^{\gamma_i(\omega)} = \epsilon_i(\omega)$, and normalise the state independent sample size ($s_{1,2} = 1$). The following two propositions hold:

**Proposition 3.** *In the symmetric case with state dependent error size, resistances are defined by*

$$r^\gamma_{AB} = \alpha\gamma(A) \tag{6a}$$

$$r^\gamma_{BA} = (1-\alpha)\gamma(B). \tag{6b}$$

**Proposition 4.** *In the general case with state dependent error size, the resistances are defined by*

$$r^{\gamma}_{AB} = \alpha\gamma_2(A) \wedge \beta\gamma_1(A) \tag{7a}$$
$$r^{\gamma}_{BA} = (1-\alpha)\gamma_2(B) \wedge (1-\beta)\gamma_1(B). \tag{7b}$$

It can be directly seen from proposition 1 and proposition 3 that, in the symmetric case, a decrease (increase) in error size from $\varepsilon$ to $\hat{\varepsilon}$, with $\hat{\varepsilon} = \varepsilon^{\zeta}$ and $\zeta > 1$ ($\zeta < 1$), is equivalent to an increase (decrease) of the sample size from $s$ to $\hat{s} = s\,\zeta$. A smaller sample size implies that fewer occurrences of idiosyncratic play are required in order to approach the boundary of the basin of attraction after which best response play switches. Similarly, a smaller exponent $\gamma_i$ leads to a larger error and thus changes the frequency of occurrences of idiosyncratic play. Both sample size and error rate operate equivalently, and the relationship between proposition 1 and proposition 3 is thus reasonable.

The result of proposition 3 is furthermore equivalent to Bergin and Lipman (1996), who illustrate that the invariant distribution $h$ satisfies

$$\frac{h_A}{h_B} = \varepsilon^{m-i^*+1-\gamma i^*}\frac{k_A[1+f_A(\varepsilon)]}{k_B[1+f_B(\varepsilon)]}$$

where $i^*$ indicates the number of players who chose strategy $B$ at the interior mixed equilibrium state, at which the error rate changes from some error rate $\varepsilon^{\gamma}$ to another defined as $\varepsilon$. The invariant distribution of the Markov process $h_A$ defines a history where players end up choosing only strategy $A$. $h_B$ is defined equivalently. For $m$ sufficiently large and $\varepsilon \to 0$ this can be normalized and rewritten as:

$$\frac{h_A}{h_B} = \varepsilon^{1-\alpha-\gamma\alpha}\frac{k_A}{k_B}$$

for $\alpha$ defined as before. In the case of $\gamma > \frac{1-\alpha}{\alpha} = \gamma^*$ the exponent is negative and the ratio goes to $\infty$. Hence $h_A \to 1$, indicating that the invariant distribution is defined by all players adhering to $A$. In the case of $\gamma < \gamma^*$ the ratio goes to zero and $h_B \to 1$, and hence all players will adhere to $B$ in the long-run. For $\gamma = \gamma^*$, $\frac{h_A}{h_B} \to \frac{k_A}{k_B}$, thus leading to a mixed state.

The general pay-off matrix in 1 can be transformed into one of four different pay-off structures by the *relative loss preserving* positive affine transformation of both players' pay-off values of the form $v = ru + k_{ik}$, with $r > 0$, and $i$ defining the player type and $k$ the column of player 1 and the row of player 2. .[8]

1. A *symmetric* pay-off matrix is defined in the literature as above, i.e. it does not matter whether an individual is a column or row player. This situation occurs in a population with only one type of player, hence $a_1 = a_2$, $b_1 = b_2$, $c_1 = c_2$, and $d_1 = d_2$.
2. A *double mirror-symmetric* pay-off matrix illustrates the case, in which the interests of both players are diametrically opposed, i.e. pay-offs are defined by a matrix, in which $a_i = d_j$ and $c_i = b_j$ for $i \neq j$. In such games pay-offs for both players are identical, but mirrored on both diagonals of the pay-off matrix.
3. A *mirror-symmetric* pay-off matrix is defined by $a_i = d_j$, $b_i = b_j$ and $c_i = c_j$ for $i \neq j$, since pay-offs are only mirrored on the main diagonal.

[8]Such an affine transformation applied to any value in matrix 1, changes any pay-off $u$ into $v = ru + k_{ik}$. The relative losses are maintained, we have, e.g. $\frac{(ra_1+k_{11})-(rc_1+k_{11})}{(ra_2+k_{21})-(rc_2+k_{21})} = \frac{a_1-c_1}{a_2-c_2}$. On the invariance of the set of Nash equilibria to such positive affine transformations, see Weibull 1995.

4. An *asymmetric* pay-off matrix cannot be transformed into one of the previous structures by a relative loss preserving positive affine transformation of all pay-off values.

In any of the first three cases, the sate dependent approach predicts the same evolutionary stable states as the state independent approach. In certain case, players might, however, consider only the relative potential losses. Either loss is defined as $\hat{l}_i(A) = (a_i - c_i)/(d_i - b_i)$ and $\hat{l}_i(B) = 1/\hat{l}_i(A)$, i.e. the perceived losses and their effect on the error and sample size are scale independent. Alternatively, a player might regard a loss as $\bar{l}_i(A) = (a_i - c_i)/(a_i - c_i + d_i - b_i)$ and $\bar{l}_i(B) = 1 - \bar{l}_i(A)$. In both cases, the long-run evolutionary properties of both state dependent cases remain identical to the state independent case. Using the previous definitions, we can summarize the results by:

**Proposition 5.** *Assume the case, where condition 2 on page 5 holds and resistances are defined by equations 5 in the case of state dependent sample size and equations 7 in the case of state dependent error size.*

*In both case, the state dependent and independent approach define the same SSS if the pay-off matrix 1 is either symmetric, mirror-symmetric or double mirror-symmetric. If the pay-off matrix is asymmetric, both approaches coincide, if the sample and error size depend on the relative instead the absolute potential loss.*

In the case, in which both error and sample size are state dependent, the resistances for pay-off matrix 1, given propositions 2 and 4, are

$$r^{s\gamma}_{AB} = \alpha s_1(A)\gamma_2(A) \wedge \beta s_2(A)\gamma_1(A) \tag{8a}$$
$$r^{s}_{BA} = (1-\alpha)s_1(B)\gamma_2(B) \wedge (1-\beta)s_2(B)\gamma_1(B). \tag{8b}$$

In this context, it is interesting to study the effect of different levels of risk aversion. Assume now the case, in which two player types coordinate on splitting a given surplus, and assume there exist two ways to split. Hence, we obtain a conflict game, in which one type prefers convention $A$ (since here this type has a higher pay-off) and the other type prefers convention $B$. For simplicity, define the pay-off matrix as being double mirror-symmetric. The state independent approach defines both equilibria as long-run conventions. Relaxing the former assumptions that sample and error size are type independent, we assume that one type is more risk-averse than the other. This type fears taking chances and thus has a higher sample, or lower error size. Following the pertinent literature (Kahneman and Tversky, 1979; Tversky and Kahneman, 1991), we assume a diminishing sensitivity to losses. In this context, the relation between risk-aversion and surplus share depends on whether risk aversion affects the sample or the error size:

**Proposition 6.** *In a double mirror-symmetric coordination game with two pure Nash equilibria, the player type that is less risk-averse can appropriate the greater share of the surplus in the case, in which sample size is state dependent. In the case, however, in which error size is state dependent, and if players exhibit diminishing sensitivity to loss, the more risk-averse player appropriates the greater surplus share.*

Being more open to taking risks benefits, ceteris paribus, a player, if he his error is state independent, but sampling is state dependent. This result is coherent with findings in the economic literature (King, 1974; Rosenzweig and Binswanger, 1993; Binmore, 1998 and for a critical discussion of empirical studies, see Bellemare and Brown, 2010) on the positive correlation between wealth and risk. If it is, however, assumed that players only exhibit a state dependent error size, proposition 6 seems to contradict either the empirical results of the previous literature or the assumptions of prospect theory. Yet, if we take risk as a measure of need,[9] this analytical result is the obvious relation that the needier one group is and the less it has to lose (from punishment, social shunning, non-conformity etc.) the more likely the convention will be defined in its favour.[10] In addition, Vendrik and Woltjer (2007) shows that the assumption of diminishing sensitivity might not hold for losses (for a different hypothesis, see also Wakker et al., 2007). In the case of an increasing sensitivity to losses, the greater surplus would be attributed to the risk-prone type in proposition 6 for both cases.

In summary, linking potential losses, and sample and error size in an intuitive way thus leads to confirm the results of the state independent approach for certain types of interactions. If conditions 2 hold, the long-term conventions defined by Young's approach are unaffected in the case of state dependence for symmetric pay-off configurations or if loss is regarded in relative and not absolute terms. Yet, conditions 2 are insufficient to guarantee identical results in the case of asymmetric pay-off matrices. Although this might indicate that state dependence only plays a role for a smaller set of games, this conclusion is premature. The state dependence of error and sample size entails two fundamental issues that contradict the basic assumptions of stochastic stability. It is first not guaranteed that a repeated transition between pure equilibria generally occurs. Second, a well-defined convention (i.e. a long-run pure Nash equilibrium) may not evolve, contrary to the adaptive play assumption; instead, a completely mixed state can define the convention. The conditions will be elaborated in detail in the following section.

## 3. The Non-existence of Conventions and The One-third Rule

In the previous analysis, players have only considered pay-off losses given a distinct convention prevails. Hence, the loss $l_i(\omega)$ has been defined as the pay-off difference that occurs if a player chooses his best response strategy with respect to the absorbing state $\omega'$, though the actual current conventional strategy is defined by the pure Nash equilibrium state $\omega$. On the one hand, this is a reasonable assumption if a player only considers his maximal potential loss or exhibit very high discount rates. On the other hand, it might be more realistic to assume that a player evaluates his potential loss according to his past sample.

Suppose that a player $i$ has to decide whether to experiment or not. Whenever a number of players experimented, the player population is in a state of transition and moves against the force of the basin of attraction of the current convention towards the other equilibrium. Our player $i$ observes that other players have been experimenting before during the sampling process. Assume that he perceives previous "experimenters" at a rate of $p$ in his sample. The basin of attraction hence still defines the pure Nash equilibrium to which the conventional strategy is a player's best response. Yet, a player expects, in addition, that in the current play his counterpart will experiment with probability $p$. He thus evaluates his potential loss based on the mixed state he sampled. The expected potential loss is then defined by the following loss function:

$$l_i(\omega^p) = \max\left[\pi_i(A,\omega^p),\pi_i(B,\omega^p)\right] - \min\left[\pi_i(A,\omega^p),\pi_i(B,\omega^p)\right], \tag{9}$$

[9] "Recall that need is to be measured in terms of the risks that people are willing to take to satisfy their lack of something important to them." (Binmore, 1998, p. 463)

[10] This also conforms with Young, 1998, Theorem 9.1., which shows that conventions are close to a social contract that maximises the pay-off of the group with the least relative pay-off.

with $\omega^p$ indicating a state in which players experiment at rate $p$. Hence, for pay-offs as in matrix 1 on page 5 a player $i$ will consider the loss

$$l_i(A^p) = (1-p)a_i + pb_i - (1-p)c_i - pd_i \text{ and} \tag{10a}$$
$$l_i(B^p) = pc_i + (1-p)d_i - pa_i - (1-p)b_i. \tag{10b}$$

If player $i$ samples less (more) than a share of $(a-c)/(a-c+d-b)$ instances of strategy $B$, he believes to be in a state of convention $A$ ($B$) and the first (second) equation applies. Notice that the case in section 2 is obtained by setting $p = 0$, i.e. the player does not expect his counterpart to experiment. Yet, given a rate $p$ of experimenters, not playing the conventional strategy in $h_A$ will always incur an expected loss greater than playing the non-conventional strategy in $h_B$ as long as $a - c > d - b$.

The former propositions could be extended from the case $l_i(\omega)$ to the case $l_i(\omega^p)$, and the general results with respect to stochastic stability should persist. This is not done here, since I believe a more fundamental issue occurs that renders the extension futile: Relaxing Young's condition of a state independent error entails that its frequency cannot be assumed to be generally small. If we consider $l_i(\omega^p)$, the absolute size of the potential expected loss varies with the strategy distribution in the sample, i.e. with the number of experimenters, implying that $l_i(\omega^p) \to 0$, as the distribution of past play approaches the interior equilibrium. As the potential loss grows smaller, the error size increases. At the interior equilibrium, expected pay-off from both strategies is identical and, hence, no expected loss results from choosing any of the two strategies at random. As a consequence, error rates will be close to 1 in the vicinity of the mixed equilibrium distribution; the zero limit of the error size is inapplicable. Also in other situations error rate can be generally high and expected loss is thus low. This is the case if potential loss is generally relatively low in comparison to the pay-offs received in both equilibria, or in cases in which sampling of information is very costly and individuals are only weakly affected by expected pay-offs.

Consider the example of driving on the left or right hand-side. Since most people are right handed, keeping left was indeed risk dominant. Nowadays, both conventions can be observed and are stable. They are imposed by law and risks are high to be punished in the case of infraction. For pedestrians this is not the case. Although to keep on the same side as driving a car is the marginal risk dominant strategy, the costs of walking on the same side as the vis-à-vis are low and people pass both on the left and right. Thus, we can observe a mixed state. Similar reasoning holds for the convention to stop at a red light. A stable convention thus requires additional properties.[11] In many cases, no single strategy defines a convention. On the other hand, if the interactions between members of population are already defined by a conventional strategy, not adhering to the convention implies a large potential loss, and repeated transition between equilibria might not occur.

Young (1998) obtained results in the case in which error rates do not approach zero, showing that the risk dominant equilibrium is still chosen as the *SSS*. However, Nowak et al. (2004); Nowak (2006); Imhof and Nowak (2006) show for similar approaches that a predominance of random errors creates an additional invasion barrier.[12] In addition, it seems intuitive to assume that if a transition takes place and the distribution of past actions approaches the interior equilibrium, random choice will continuously push the population towards a completely mixed strategy profile in which both

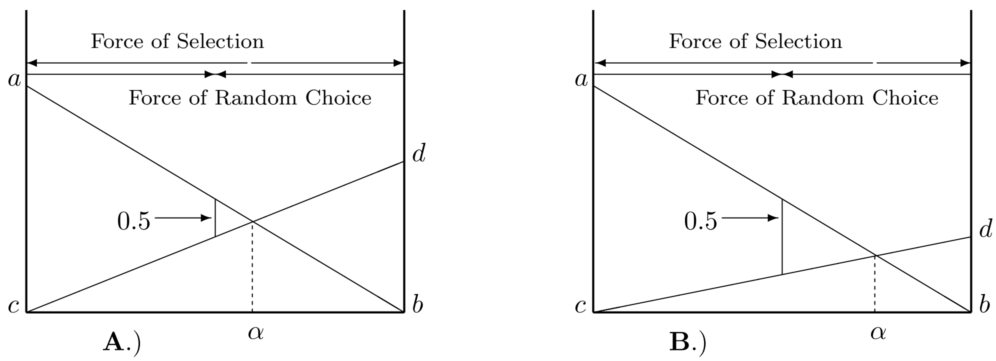
[11] Observing road users in Mediterranean countries illustrates that stopping at red lights or keeping on the right lane is only a stable convention if it is sufficiently enforced by law.

[12] The result is shown for a Moran process with constant and large error rates.

strategies are played with roughly equal probability. This counter-acts the selection process, which gravitates the population towards the Nash equilibrium inside the basin of attraction. Hence, a minimum basin of attraction is necessary for an equilibrium to exercise sufficient gravitational pull on a population at the completely mixed state to overcome the adverse effect of random choice. This should translate into a sufficiently small resistance on the path towards this equilibrium.

The graph 1 helps to illustrate both points. A.) and B.) represent the expected, normalised and

Figure 1: The one-third rule




symmetric pay-offs of two games, defined as in matrix 1 on page 5, with $a_1 = a_2$, $d_1 = d_2$, and $c_1 = c_2 = b_1 = b_2 = 0$. The frequency of strategy $B$ players defines the abscissa, the intersecting functions show the expected pay-off for each of the strategies on the ordinate. Consequently, if a player encounters strategy B players with a frequency of $f$ his expected pay-off is either $(1-f)a$ if he plays A, or $fd$ if he plays $B$. In A.) pay-off $d$ is only marginally smaller than $a$. The equilibrium frequency of strategy $B$ players given by $\alpha$ thus lies close to the 0.5 frequency. The *Force of Random Choice* pushes the distribution towards this completely mixed interior distribution.[13] The *Force of Selection*, on the other hand, pushes the distribution to one of the pure equilibria.[14] It increases with the distance from $\alpha$ and is determined by the vertical gap between the $(a-b)$ and the $(c-d)$ line. We observe that at the completely mixed distribution, the Force of Selection is very small in the direction of $h_A$ (indicated by the vertical line right of the $\alpha$-equilibrium). Random choice is frequent with respect to best response play, and thus the memory of past play mainly consists of randomly chosen actions in the long-run. In contrast, the Force of Selection is strongest at the pure equilibria, i.e. both at $h_A$ and $h_B$, minimizing non-best response play. Hence, the replacement of old memories is mainly defined by expected pay-off, impeding a transition from one convention to the other. This is not the case in B.). The completely mixed distribution lies further left to the interior equilibrium, and the Force of Selection at the mixed distribution pushes stronger in the direction of $h_A$, compared to case A.), and eventually sufficiently strong to overcome the Force of Random Choice. In addition, at $h_B$ in B.) the Force of Selection is weaker compared to graph A.). Random choice favours a transition out of the basin of attraction of $h_B$, after which selection will

[13]The Force of Random Choice determines the stochastic replacement of past memories by a randomly chosen strategy.

[14]The Force of Selection defines the replacement of old memories by a best response strategy to the individual sample.

favour $h_A$.

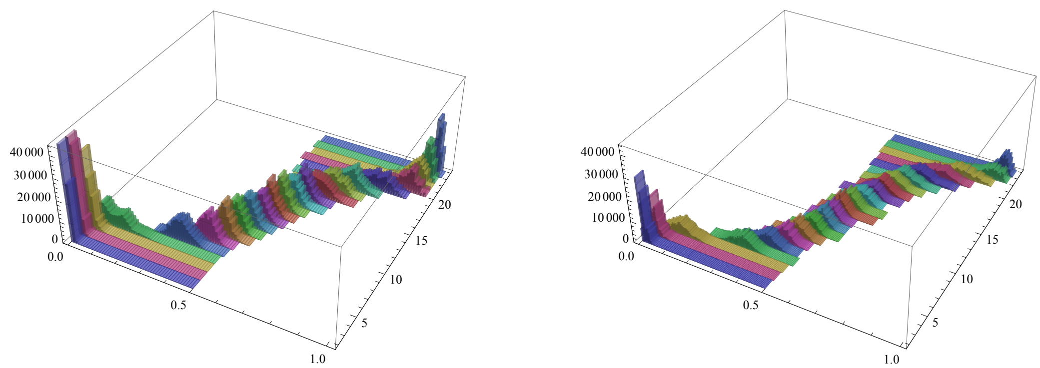
In order to test this hypothesis, a number of computer simulations have been conducted. In order to keep the setting as simple as possible, consider a symmetric $2 \times 2$ game of the shape

$$
\begin{array}{cc} & \begin{array}{cc} A & B \end{array} \\ \begin{array}{c} A \\ B \end{array} & \begin{pmatrix} a, a & 0, 0 \\ 0, 0 & b, b \end{pmatrix} \end{array} \tag{11}
$$

and

$$
\epsilon_i(\omega^p) = 1 - \left( \frac{|(1-p)a - pb|}{(1-p)a + pb} \right)^{\gamma} \tag{12}
$$

with $\epsilon_i(\omega^p) \in (0.01, 1)$ and $\gamma$ defining the sensitivity to losses. Results should depend on the size of $a$, $b$, and $\gamma$. Assuming $\gamma = 2$, for each simulation set pay-off $a$ is held at a constant value, starting at 1 and raised by 1 in each set until 60 is reached. In each simulation set consists of 21 simulations, each of a length of 100.000 periods, and in each variable $d$ is incremented by a constant $n$, thus $d \in (d_{min}, d_{min} + 21n)$. The simulation results illustrate a property complementary to the so-called one-third rule (see Nowak et al. 2004; Nowak 2006). Figure 2 shows the histograms for two simulations.




(a) $a = 10$ and $b \in (2, 23)$; $n = 1$

(b) $a = 60$ and $b \in (20, 125)$, $n = 5$

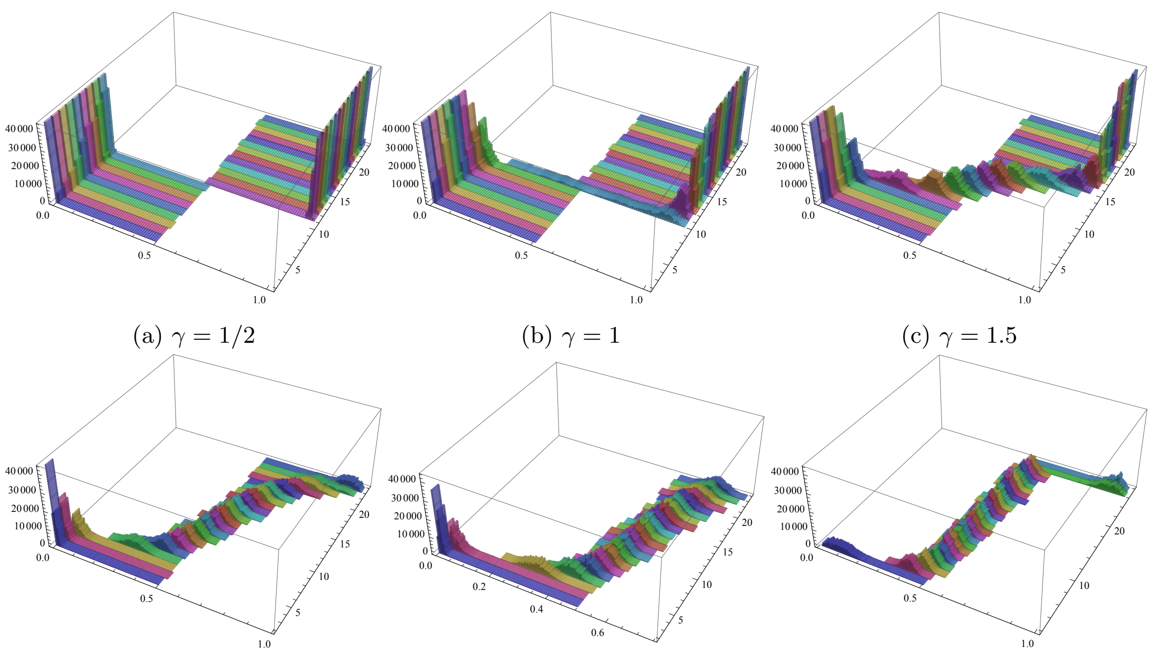
Figure 2: The histograms of strategy $B$ players for two simulation sets.

In figure 2a, the history of play is defined by a majority of strategy $A$ plays for $b < 5$, and a majority of strategy $B$ plays for $b > 20$. Similarly in figure 2b, convention $h_a$ evolves for $b < 30$ and convention $h_b$ evolves for $b > 120$. Equivalent results are obtained in all simulation sets. The one-third rule says that under certain conditions and for a $2 \times 2$ symmetric coordination game, if the basin of attraction of one equilibrium is less than $1/3$, selection favours the *fixation* at the other equilibrium, i.e. the latter will define the long-run convention.[15] The same outcome is illustrated in figure 2. In addition, we obtain a more subtle result. For a larger basin of attraction, the population moves towards an interior distribution, but remains *trapped* half way. We thus observe the following:

[15]The one-third rule has been derived for the Moran process with random replacement and in case of weak selection, i.e. a constant large error rate.

**Observation 1.** *For a symmetric $2 \times 2$ game, a state dependent error size as defined by equation 12, and $\gamma = 2$, the convention is defined by the pure Nash equilibrium with a basin of attraction larger than $2/3$. If both basins of attraction are smaller, no unique strategy will define a convention in the long-run.*

As mentioned previously, dynamics are not only defined by pay-offs, but also by the sensitivity to losses $\gamma$. Thus, simulations have been repeated for various values of $\gamma$, and the results for $a = 10$ are presented in figure 3. We observe a positive correlation between the size of the basin of attraction, which a pure Nash equilibrium requires in order to evolve as a long-run convention, and the size of $\gamma$. Hence, we have the following:




(a) $\gamma = 1/2$ (b) $\gamma = 1$ (c) $\gamma = 1.5$

(d) $\gamma = 2.3$ (e) $\gamma = 3$ (f) $\gamma = 5$

Figure 3: The histograms of strategy $B$ players for $a = 10$ and $b \in (2, 23)$; $n = 1$, with varying exponent.

**Observation 2.** *Given a symmetric $2 \times 2$ game and a state dependent error size as defined by equation 12. For $\gamma > 1$, a positive correlation exists between the size of the basin of attraction, which a pure Nash equilibrium requires to evolve as a long-run convention, and the sensitivity to losses $\gamma$. For $\gamma \leq 1$, state dependent losses have no effect in the long-run and the risk dominant equilibrium defines the SSS.*

In other words, stochastic stability is a necessary condition for a stable convention, but will not suffice in the presence of a low sensitivity to losses ($\gamma > 1$), which leads to high rate of erroneous play at the mixed state. For $\gamma = 2$, if both transitions between the equilibria face a reduced resistance larger than $1/3$, the stochastic choice of strategies does not favour any strategy. Furthermore, the lower the impact of a loss is on the error rate (i.e. the larger $\gamma$), the larger the basin of attraction of an equilibrium must be in order for it to be an *SSS*.

## 4. Conclusion

Restricting the analysis to $2 \times 2$ coordination games, this paper studied the impact of a positive correlation between potential loss from miscoordination, and the error / sample size of players on the evolutionary properties of long-term conventions. In a first step, a player's loss was defined as the immediate forgone pay-off if he does not follow the conventional strategy adhered to by his counterpart. In this case, we observe that Stochastically Stable States do not necessarily coincide with those identified by the state-independent approach. Prediction are identical in the presence of some form of symmetry of the pay-off matrix or if error and sample size are related to a *relative* measure of the potential loss. However, state dependence plays a role in the case of asymmetric pay-offs. In addition, we observe a difference in predictions between the case of a state dependent sample size and the case of a state dependent error size. Given a non-symmetric game, both variables affect the stochastic potential of an equilibrium in a different way. The paper illustrates that criticism, which predominantly focuses on the state dependence, is therefore inadequate.

In a second step, it was assumed that a player does not consider his potential immediate loss, but considers his sample to be an indication of the potentially mixed strategy of his counterpart. He thus evaluates his potential loss based on the strategies he sampled. Consequently, if a player samples a strategy distribution close to the one defined by the interior equilibrium, he expects no potential loss, since both strategies will have identical expected pay-offs. Hence, a player randomly chooses a strategy at this point, thus leading to an error rate of 1.[16] If, on the contrary, he samples only identical strategies, the error probability is arbitrarily small. Given this assumption, we observe that a pure Nash equilibrium will only define a long-run convention if its basin of attraction is sufficiently large. Its minimum size is positively correlated with the strength of a player's reaction to a loss. If the basins of attraction of both equilibria are too small, the state dependent error rate then acts as a barrier to the transition process, and no unique strategy will define a convention in the long-run. We have further seen that for a specific relation between loss and error rate, the condition is defined by the one-third rule.

It is expected that the approach and criticism of this paper can be readily extended to coordination games with a larger set of strategies. In the case of a larger number of pure Nash equilibria, the stochastic potential of an equilibrium is given by the directed path of least resistance. If error and sample rate are correlated with the immediate potential loss and if a population moves along an indirect path, the *temporal* convention will change each time the population enters a new basin of attraction. During this time, error and sample size are defined by the loss from miscoordination if this specific conventional strategy is not played. It might thus suffice to weight the resistance of each edge of a path accordingly before calculating the stochastic potential.

In the second case, in which error rate depends on the sampled strategies, a transition to a new equilibrium can only take place if it has a sufficiently large basin of attraction. Each edge of a direct or indirect path has to fulfil this requirement. It is expected that if a player population *transits* on an *illegitimate* path, it will become trapped in a mixed state, where random choice superimposes selection. At this point, both strategies will be played with roughly equal probability. The analytical elaboration of these two extensions are left to future research.

[16] $\varepsilon$ does not define the probability of choosing a wrong strategy, but choosing any strategy in the strategy set at random.

## Appendix A. A Short Illustration of the underlying framework

Assume that for a finite player population, $n$ different sub-populations exist, each indicating a player type participating in the game. Strategies and preferences are identical for all individuals in the same sub-population. The game is defined by $\Gamma = (X_1, X_2, ..., X_n; u_1, u_2, ..., u_n)$, where $X_i$ indicates the strategy set and $u_i$ the utility function of individuals of type $i$. Hence for simplicity, define each individual in such a sub-population $C_i$ as player $i$. Assume that one individual from each sub-population $C_i$ is drawn at random in each period to play the game. Each individual draws a sample of size $s < \frac{m}{2}$ from the pure-strategy profiles of the last $m$ rounds the game has been played.[17] The idea is that the player simply asks around what has be played in past periods. Hence, the last $m$ rounds of play can be considered as the collective memory of the player population. In addition to Young's assumptions, I assume explicitly that $m$ and $s$ are large. This assumption is made to guarantee that the minimum rate in the sample required to switch best-response strategy can take any value between 0 and 1. (The example in Appendix B on page 18 illustrates an instance, in which this is not the case.)

Each state is thus defined by a history $h = (x^{t-m}, x^{t-m+1}, \ldots, x^t)$ of the last $m$ plays and a successor state by $h' = (x^{t-m+1}, x^{t-m+2}, \ldots, x^t, x^{t+1})$ for some $x^{t+1} \in X$, with $X = \prod X_i$, which adds the current play to the collective memory of fixed size $m$, deleting the oldest. Each individual is unaware of what the other players will choose as a best response. He thus chooses his best reply strategy with respect to the strategy frequency distribution in his sample (fictitious play with bounded memory, which Young called *adaptive play*). He chooses, nonetheless, any strategy in his strategy profile with a positive probability. Consequently, suppose that there is a small probability that an agent inadequately maximizes his choice, and commits an error or simply experiments. The probability of this error equals the rate of mutation $\varepsilon > 0$, i.e. with probability $\varepsilon$ an individual $j$ in $C_j$ does not choose his best response $x_j^* \in X_j$ to his sample of size $s$ from a past history of interactions.[18] Instead he chooses a strategy at random from $X_j$. Since each state is reachable with positive probability from any initial state if $\varepsilon > 0$, the process is described by an irreducible Markov chain on the finite state space $\Omega \subset (X_1 \times X_2 \times ... \times X_n)^m$. Not all states are, however, equally probable. In order to shift a population from some stable equilibrium (i.e. convention), at which players only remember to have always played the same strategy, defined by strategy profile $x^{*t} = (x_1^*(t), x_2^*(t), \ldots, x_n^*(t))$ and history $h_k = (x^{*t-m}, x^{*t-m+1}, \ldots, x^{*t})$ to some other stable equilibrium defined by $x^{'t}$ and $h_l = (x^{'z-m}, x^{'z-m+1}, \ldots, x^{'z})$ in time $z$, requires that a sufficiently large number of individuals idiosyncratically chooses a non-best response strategy to move the population out of the basin of attraction of the equilibrium defined by $h_k$ into the basin of attraction of another equilibrium, so that $x_i^{'}$ is eventually a best response to any sample drawn from $m$.

For each pair of recurrent classes $E_i$, $E_j$ from the set of recurrent classes $E_1, E_2, \ldots, E_k$ in the non-perturbed Markov process, a directed $ij$-path is defined by a sequence of states $(h_1, h_2, ..., h_z) \in \Omega$ that goes from $E_i$ to $E_j$. Define the resistance $r(h, h^{'})$ as the number of mistakes (perturbations) necessary to cause a transition in each period from any current state $h$ to a successor state $h'$

[17]More precisely; the general condition is defined as $s \leq \frac{m}{L_\Gamma + 2}$, with $L_\Gamma$ being the maximum length of all shortest directed paths in the best reply graph from a strategy-tuple $x$ to a strict Nash equilibrium (see Young, 1993). Since here the analysis is restricted to $2 \times 2$ coordination games, the simplified assumption suffices.

[18]Strictly speaking the error rate is given by $\lambda_j \varepsilon$ for player $j$ and has full support, i.e. all strategies in $X_j$ are played with positive probability whenever an error occurs or the player experiments. Note, however, in the standard case the *SSS* is independent of $\lambda_j$ and the probability, with which a strategy is randomly chosen.

connected by a directed edge, implying that the transition from $h$ to the successor state $h^{'}$ in an n-person game is of order $\varepsilon^{r(h,h^{'})}(1-\varepsilon)^{n-r(h,h^{'})}$. (If $h^{'}$ is a successor of $h$ in the non-perturbed process resistance is 0. If $h^{'}$ is not a successor state both in the perturbed and unperturbed process, the resistance is equal to $\infty$.) The resistance of this path is given by the sum of the resistances of its edges, $r_\sigma = (r(h_1,h_2)+r(h_2,h_3)+\ldots+r(h_{z-1},h_z))$. Let $r_{ij}$ be the least resistance over all those $ij$-paths. Hence, there exists a tree rooted at vertex $j$ for each recurrent class $E_j$ that connects to *every* vertex different from $j$. Notice that connections can be defined by a direct or indirect path leading from any other vertex $k$ for $E_k$ to $j$ for $E_j$, with $k \neq j$, in the perturbed process. A path's resistance is thus given by the sum of the least resistances $r_{ij}$ over all the edges in the tree. The stochastic potential for any $E_j$ is defined as the least resistance among all these trees leading to vertex $j$. The recurrent class with least stochastic potential determines the Stochastically Stable State. The least resistance path can be direct or indirect, and takes further account of all strategies in the strategy set. In other words, an *SSS* is the equilibrium that is the easiest accessible from *all other states combined*.

An example will help to understand the intuition before coming to the proofs. In order to simplify as much as possible, for the length of this example, I will abstract from the loss - error rate relation 2 on page 5, and from the assumption of a relatively large sample size as well as the condition that $s \leq \frac{m}{2}$. (The example will also make it evident why this has been initially assumed.)

**Example:** Consider two players, who meet each other on a narrow road once a day, and have to decide whether to cross on the left or right. Hence, they play a $2 \times 2$ coordination game. Assume that players have a very short memory and remember only the last 2 moves ($m_{i,t} = (x_{j,t-1}, x_{j,t})$). Memory size is identical to sample size. Each state of the game can thus be represented by a vector of four components ($h_t = (m_i, m_j)$). Further assume that players are symmetric, therefore $h_t = (m_i, m_j) = (m_j, m_i)$. The 10 possible states are then defined as *(ll,ll), (ll,lr), (ll,rl), (ll,rr), (lr,lr), (lr,rl), (lr,rr), (rl,rl), (rl,rr), and (rr,rr)*. Each player chooses his best response to his memory of the opponent's last two actions. Obviously *(ll,ll)* and *(rr,rr)* are absorbing states, as the best response to $rr$ is always $r$ and to $ll$ always $l$. Assume that both equilibria provide the same strictly positive pay-off, and that mis-coordination gives zero pay-off. In the case, in which a player has a "mixed memory" of the opponent's play, i.e. $rl$ or $lr$, he chooses $l$ or $r$ both with probability $\frac{1}{2}$. In the unperturbed Markov process, states *(ll,ll)* or *(rr,rr)* will persist forever, once they are reached. State *(ll,lr)* will move to state *(ll,rl)* or *(lr,rl)*, each with probability $\frac{1}{2}$.

Now assume that a player commits an error with a low probability and does not choose his best response strategy. Let the case, in which he has memory $ll$ and chooses $r$, occur with probability $\lambda$ and the second case, in which he has memory $rr$ and chooses $l$, occur with probability $\varepsilon$. Let the states' position be as in the previous enumeration, starting with *(ll,ll)* and ending with *(rr,rr)*. The transition matrix of the perturbed Markov process is then defined as in matrix A.1

$$P^\varepsilon = \begin{pmatrix} (1-\lambda)^2 & 2(1-\lambda)\lambda & 0 & 0 & \lambda^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & (1-\lambda)/2 & \lambda/2 & 0 & (1-\lambda)/2 & \lambda/2 & 0 & 0 & 0 \\ (1-\lambda)/2 & \frac{1}{2} & 0 & 0 & \lambda/2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \varepsilon(1-\lambda) & \varepsilon\lambda & 0 & (1-\varepsilon)(1-\lambda) & (1-\varepsilon)\lambda & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \varepsilon/2 & \frac{1}{2} & (1-\varepsilon)/2 \\ \frac{1}{4} & \frac{1}{2} & 0 & 0 & \frac{1}{4} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \varepsilon/2 & \varepsilon/2 & 0 & (1-\varepsilon)/2 & (1-\varepsilon)/2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \varepsilon^2 & 2(1-\varepsilon)\varepsilon & (1-\varepsilon)^2 \end{pmatrix} \tag{A.1}$$

$P^{\sigma} = \lim\limits_{n \to +\infty; \varepsilon, \lambda \to 0} P^{\varepsilon}$ defines the limit distribution with $\varepsilon$ and $\lambda$ approaching zero at the same rate. If $\lambda = \varepsilon$ each row vector of $P^{\sigma}$ has components $(0.5 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0.5)$. Thus, both equilibrium states occur with equal probability. If $\lambda < \varepsilon$ state *(*ll,ll) is *SSS*, if $\lambda > \varepsilon$ state *(*rr,rr) is *SSS*.[19]

If we assume that equilibrium *(*l,l) generates a larger pay-off than equilibrium *(*r,r), all states, except *(*rr,rr), will converge to state *(*ll,ll) in the unperturbed Markov process. Ceteris paribus, the transition matrix looks as in matrix A.2

$$P^{\varepsilon} = \begin{pmatrix}
(1-\lambda)^2 & 2(1-\lambda)\lambda & 0 & 0 & \lambda^2 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & (1-\lambda)^2 & (1-\lambda)\lambda & 0 & (1-\lambda)\lambda & \lambda^2 & 0 & 0 & 0 \\
(1-\lambda)^2 & 2(1-\lambda)\lambda & 0 & 0 & \lambda^2 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \varepsilon(1-\lambda) & \varepsilon\lambda & 0 & (1-\varepsilon)(1-\lambda) & (1-\varepsilon)\lambda & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & (1-\lambda)^2 & 2(1-\lambda)\lambda & \lambda^2 \\
0 & 0 & (1-\lambda)^2 & (1-\lambda)\lambda & 0 & (1-\lambda)\lambda & \lambda^2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \varepsilon(1-\lambda) & (1-\varepsilon)(1-\lambda)+\varepsilon\lambda & (1-\varepsilon)\lambda \\
(1-\lambda)^2 & 2(1-\lambda)\lambda & 0 & 0 & \lambda^2 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \varepsilon(1-\lambda) & \varepsilon\lambda & 0 & (1-\varepsilon)(1-\lambda) & (1-\varepsilon)\lambda & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \varepsilon^2 & 2(1-\varepsilon)\varepsilon & (1-\varepsilon)^2
\end{pmatrix} \tag{A.2}$$

Since state *(rr,rr)* has no basin of attraction for $m = 2$, we cannot calculate the resistances for various pay-offs. Yet, a change in the relative error size can still shift the *SSS*. For $\lim\limits_{n \to +\infty} P'^{\varepsilon,\lambda}$, $\varepsilon = 0.00001$ and $\varepsilon^{\frac{1}{5}} = \lambda$ each row vector is defined by approximately $(0.03 \; 0.01 \; 0.01 \; 0.00 \; 0.00 \; 0.00 \; 0.00 \; 0.00 \; 0.00 \; 0.96)$. We observe that though state *(ll,ll)* is risk dominant, the players will spend approximately 96% of the time in state *(rr,rr)*.[20] This result is confirmed by simulations. Based on 10 runs, each for 1 million interactions, we observe that state *(rr,rr)* occurs with a probability of 96.4%, whereas state *(ll,ll)* occurs in 2.3% percent of all states. The following figure illustrates the results of one simulation.

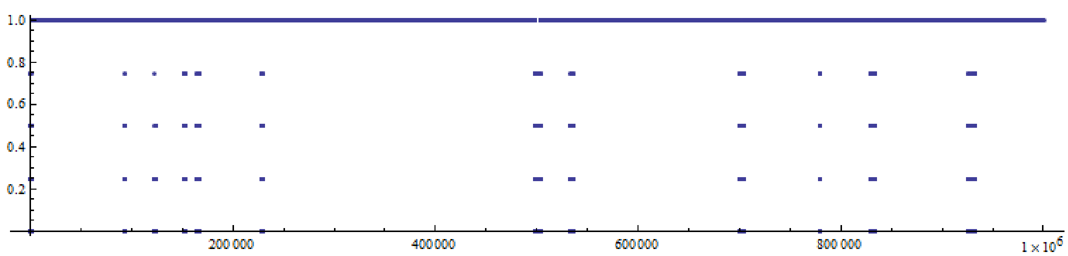



Figure A.4: Agent-based simulation of stochastic play with memory and sample size 4 among two players, and state dependent error $\varepsilon = 0.00001$, and $\lambda = 0.1$. Values illustrate the share of $r$ play in the collective memory of both players, thus 1 illustrates state *(rr,rr)*, a value of 0 represents *(ll,ll)*.

[19] e.g. if $\varepsilon = 0.0001$ and $\lambda = \varepsilon^{1.5}$, the population remains in state *(*ll,ll) almost all time (99%) and basically never in state *(*rr,rr) (< 1%).

[20] Notice that, however, in this example the *SSS* will ultimately switch to *(ll,ll)* as $\varepsilon \to 0$, since *(rr,rr)* has no basin of attraction in the unperturbed process, i.e. all states except for *(rr,rr)* converge to *(ll,ll)* with probability 1 in the case of best response play. A larger memory of 3 would require a transition matrix of size $36 \times 36$.

**Cost Control Function:** In the following we adapt the assumptions of Damme and Weibull (1998) to the current framework. Assume that these costs of sampling and steadying the trembling hand are expressed by a control cost function. The control cost function $k(\epsilon_i(s_i(\omega), \gamma_i(\omega); \varepsilon))$ is strictly convex, twice differentiable, symmetric, positive and decreasing in $\epsilon_i$. Expected profit is given by

$$\pi_i = g_i(\omega) - \epsilon_i(s_i(\omega), \gamma_i(\omega); \varepsilon) l_i(\omega) - k(\epsilon_i(s_i(\omega), \gamma_i(\omega); \varepsilon)) \tag{A.3}$$

Hence the marginal cost function $-k'(.)$ will be decreasing in $\epsilon_i$ and increasing in $s_i$ and $\gamma_i$. Maximizing the expected pay-off yields

$$l_i = -\frac{\partial k[\epsilon_i(s, \gamma; \varepsilon)]}{\partial \epsilon_i}. \tag{A.4}$$

From equation A.4, we can deduce the set of equations 2.

## Appendix B. Proofs for Stochastic Stability

Proof of Proposition (1). (This proof is with the exception of minor changes identical to the one of Young, 1998, Theorem 4.1) Let $G$ be a $2 \times 2$ coordination game with the corresponding conventions (pure Nash equilibria) $(A, A)$ and $(B, B)$, and corresponding absorbing states of history $h_A$ and $h_B$. Let $\mathfrak{B}_i$, with $i = A, B$ represent the equilibria's basins of attraction. In addition, let the pay-offs of the game be symmetric. Assume that sample size is dependent on the pay-offs at the current convention. Hence, as long as the population is inside the basin of attraction of convention $h_A$, players sample at a size $s(A)$, in the case they are in $\mathfrak{B}_B$, sample size is $s(B)$. Further, let the memory $m$ be sufficiently large ($s(\omega) \leq m/2$). Let $r_{AB}$ denote the reduced resistance for every path on the z-tree from $h_A$ to $h_B$ as a function of the sample size $s(A)$. Since after entering $\mathfrak{B}_B$ the system converges to $h_B$ without further errors, $r_{AB}$ is the same as the resistance for all paths from $h_A$ to $\mathfrak{B}_B$. Let $\alpha$ be defined as above and assume that the population has stayed in equilibrium $(A, A)$ sufficiently long, so that past play is defined only be a history $h_A$, so that all players have chosen strategy $A$ for $m$ periods in succession. Now, for a player to choose strategy $B$ and for the system to enter $\mathfrak{B}_B$, strategy $B$ must occur with a frequency of at least $\alpha s(A)$ in the player's sample. This can only happen with positive probability if $\alpha s(A)$ players successively commit the error of choosing action $B$. The probability of this to occur is at least $\varepsilon^{\alpha s(A)}$. The same logic holds for convention $h_B$, only that $(1-\alpha)s(B)$ players successively have to make a mistake. This event happens with order $\varepsilon^{(1-\alpha)s(B)}$. It follows that the resistance from $h_A$ to $h_B$ is thus $r^s_{AB} = \alpha s(A)$ and from $h_B$ to $h_A$ is $r^s_{BA} = (1-\alpha)s(B)$. $h_A$ is stochastically stable iff $r^s_{AB} \geq r^s_{BA}$.

Proof of Proposition (2). Assume the same conditions as before except that row players have sample size $s_1(A)$ near $h_A$ and $s_1(B)$ near $h_B$, and the column players have sample size $s_2(A)$ and $s_2(B)$ respectively and pay-offs are not necessarily symmetric (i.e. interaction pairs are given by one row and one column player). Keep in mind that $\alpha$ refers to the share of column players and $\beta$ the share of row players. Hence, a row player 1 currently playing strategy $x_1 = A$ will only change strategy if there is a sufficient number of column players playing $x_2 = B$ in his sample. For a positive probability of this to happen there must be at least $\alpha s_1(A)$ players committing an error in subsequent periods, occurring with probability $\varepsilon^{\alpha s_1(A)}$. For a column player 2 with $x_2 = A$ to switch there must be a sufficient number of row players playing $x_1 = B$ in his sample. Hence, there must be again at least $\beta s_2(A)$ of these players in $m$, happening with probability of at least $\varepsilon^{\beta s_2(A)}$. The same reasoning holds for the transition from $h_B$ to $h_A$. Hence, $r^s_{AB} = \alpha s_1(A) \wedge \beta s_2(A)$ and $r^s_{BA} = (1-\alpha)s_1(B) \wedge (1-\beta)s_2(B)$.

Proof of Proposition (3). Now assume that the rate of mutation is $\epsilon(A) = \varepsilon^{\gamma(A)}$ in $\mathfrak{B}_A$ and $\epsilon(B) = \varepsilon^{\gamma(B)}$ in $\mathfrak{B}_B$ and that pay-offs are symmetric. Assume that sample size is constant and, since it is equal for all player, normalised at $s(A), s(B) = 1$, thus is state and pay-off independent. Other conditions are equal to the first proof. Starting in $h_A$ for a system to enter $\mathfrak{B}_B$ with positive probability, again a share of $\alpha$ players successively has to commit the error of choosing action $B$. For a player to change strategy from $A$ to $B$ there must be thus at least $\alpha s$ players playing strategy $B$ in $m$, in order to sample a share of $\alpha$ B players with positive probability. By the same logic as above this event occurs with probability $\varepsilon^{\gamma(A)\alpha}$. Congruently, a switch from $h_B$ to $h_A$ happens with probability $\varepsilon^{\gamma(B)(1-\alpha)}$. The resistance from $h_A$ to $h_B$ is thus $r^{\gamma}_{AB} = \gamma(A)\alpha$ and from $h_B$ to $h_A$ is $r^{\gamma}_{BA} = \gamma(B)(1-\alpha)$.

Proof of Proposition (4). As in the second proof assume that pay-offs are not necessarily symmetric and that there exist two inter-acting types of players with state dependent error size $\epsilon_i(\omega)$. Row players have error size $\varepsilon^{\gamma_1(A)} = \epsilon_1(A)$ in $\mathfrak{B}_A$ and $\varepsilon^{\gamma_1(B)} = \epsilon_1(B)$ near $\mathfrak{B}_B$, and column players have error size $\varepsilon^{\gamma_2(A)} = \epsilon_2(A)$ and $\varepsilon^{\gamma_2(B)} = \epsilon_2(B)$ respectively. For convenience assume that sample size is normalised to $s(A), s(B) = 1$. A row player 1 currently playing strategy $A$ will only change his strategy if there is a sufficient number of column players playing $B$, i.e. if he encounters a proportion of at least $\alpha$ column players choosing strategy $B$ in his sampled set. For this event to happen with positive probability, there must be $s_1(A)\alpha$ of this column players in $m$. This happens with a probability of $\epsilon_2(A)^{\alpha} = \varepsilon^{\alpha\gamma_2(A)s}$. A column player has to meet a portion of $\beta$ row players erroneously playing strategy $B$. Hence, there must be at least $\beta s_2$ such players in $m$, which occurs with probability $\epsilon_1(A)^{\beta} = \varepsilon^{\beta\gamma_1(A)s}$. For $h_B$ the argument is analogous. For a normalised sample size $s_{1,2} = 1$, it thus holds $r^{\gamma}_{AB} = \gamma_2(A)\alpha \wedge \gamma_1(A)\beta$ and $r^{\gamma}_{BA} = \gamma_2(B)(1-\alpha) \wedge \gamma_1(B)(1-\beta)$.

Combining proposition 3 and 4, it follows that in the case of both state dependent error and sample size the least resistances are given by equation 8 on page 8.

Proof of Proposition (5). Assume condition 2 on page 5 holds, which was defined as:

$$\begin{aligned} l_i(\omega) < l_j(\omega') \Leftrightarrow \epsilon_i(\omega) > \epsilon_j(\omega') \Leftrightarrow s_i(\omega) < s_j(\omega') \\ \Leftrightarrow \gamma_i(\omega) < \gamma_j(\omega'), \quad \text{for } i,j = 1,2. \end{aligned} \tag{B.1}$$

Assume the general case of $2 \times 2$ conflict-coordination games, with the asymmetric pay-off structure as in matrix B.2.

$$\begin{array}{cc} & \begin{array}{cc} A & B \end{array} \\ \begin{array}{c} A \\ B \end{array} & \begin{pmatrix} a_{11}, b_{11} & a_{12}, b_{12} \\ a_{21}, b_{21} & a_{22}, b_{22} \end{pmatrix} \end{array} \Rightarrow \begin{array}{cc} & \begin{array}{cc} A & B \end{array} \\ \begin{array}{c} A \\ B \end{array} & \begin{pmatrix} a, b & 0, 0 \\ 0, 0 & c, d \end{pmatrix} \end{array} \tag{B.2}$$

The first pay-off matrix is equivalent to the second by transformation, given that $a = a_{11} - a_{21}$, $b = b_{11} - b_{12}$, $c = a_{22} - a_{12}$ and $d = b_{22} - b_{21}$. The definition in the right matrix will be used in the following, as the transformation does not affect the size of the potential loss from mis-coordination and thus results, but will simplify notation. For this pay-off matrix the frequencies are given by $\alpha = \frac{a}{a+c}$, and $\beta = \frac{b}{b+d}$. Define two positive, continuous and strictly increasing function $\mu$ and $\eta$, such that $s_i(\omega) = \mu(l_i(\omega))$, and $\gamma_i(\omega) = \eta(l_i(\omega))$. If for both player types the same equilibrium risk dominates, the solution is trivial. For $a > c$ and $b > d$, it always holds that

$min\,\{\alpha s_1(A); \beta s_2(A)\} > min\,\{(1-\alpha)s_1(B); (1-\beta)s_2(B)\}$ and also $min\,\{\alpha\gamma_2(A); \beta\gamma_1(A)\} > min\,\{(1-\alpha)\gamma_2(B); (1-\beta)\gamma_1(B)\}$. Hence, $h_A$ is *SSS*, both in the case if error and sample size are state independent and in the case, in which they are state dependent. Identically, $h_B$ is *SSS* holds for $a < c$ and $b < d$.

Assume for the following parts of the proof and without loss of generality that player type 1 prefers $h_A$ and player type 2 prefers $h_B$.
We obtain $a > c$ and $d > b$, and $\alpha > 1-\alpha$ and $1-\beta > \beta$. Hence, $\alpha > \beta$ and $1-\beta > 1-\alpha$. Consequently, there are two possibilities. Either $\beta > 1-\alpha$ ($h_A$ is *SSS* in the state independent case) or $\beta < 1-\alpha$ ($h_B$ is *SSS* in the state independent case).

*General pay-off matrix* - The case of state dependent sample size: Define as before that $s_i(\omega) = \mu(l_i(\omega)$. Then by assumption $\mu(a)\alpha > \mu(c)(1-\alpha)$ and $\mu(b)\beta < \mu(d)(1-\beta)$. Under these conditions four cases can occur:
*1. case:* If $\mu(a)\alpha < \mu(b)\beta$, then $c < a < b < d$ and thus, $\mu(c)(1-\alpha) < \mu(d)(1-\beta)$, and $h_A$ is *SSS*.
*2. case:* If $\mu(c)(1-\alpha) > \mu(d)(1-\beta)$,then $b < d < c < a$ and thus, $\mu(a)\alpha > \mu(b)\beta$, and $h_B$ is *SSS*. Hence, the results for the state dependent sample size do not necessarily coincide with the state independent case.
*3. case:* The indeterminate case occurs, if $\mu(a)\alpha > \mu(b)\beta$ and $\mu(c)(1-\alpha) < \mu(d)(1-\beta)$. Depending on the relative size of $b$ and $c$ and the order of $\mu(l_i(\omega)$ the state dependent solution will differ from the state independent approach.
*4. case:* A contradiction occurs, if $\mu(a)\alpha < \mu(b)\beta$ and $\mu(c)(1-\alpha) > \mu(d)(1-\beta)$. The case contradicts with the assumption that $a > c$ and $d > b$.

As a result only if $\mu(a)\alpha < \mu(b)\beta$ and $\beta > 1-\alpha$, and if $\mu(c)(1-\alpha) > \mu(d)(1-\beta)$ and $\beta < 1-\alpha$, the state dependent and independent results coincide.

In the case of state dependent error rate: As before define $\eta(l_i(\omega)) = \gamma_i(\omega)$. The reduced resistances are then given by $r^{\gamma}_{AB} = \eta(b)\frac{a}{a+c} \wedge \eta(a)\frac{b}{b+d}$ and $r^{\gamma}_{BA} = \eta(d)\frac{c}{a+c} \wedge \eta(c)\frac{d}{b+d}$. Without further assumptions on $\eta(l_i(\omega))$ no definite results can be obtained.

*General pay-off matrix and relative loss* - Assume that both $\hat{\mu}(l_i(\omega))$ and $\hat{\eta}(l_i(\omega))$ are defined as such that players take only account of the relative losses in both conventions. First, consider the case, in which the loss in convention $A$ is defined by $l_1(A) = a/c$ and $l_2(A) = b/d$, for player 1 and 2 respectively, and equivalently for convention $B$. Since $\alpha/(1-\alpha) = a/c$ and $\beta/(1-\beta) = b/d$, we can write $s_1(A) = \hat{\mu}(\alpha/(1-\alpha))$, $s_1(B) = \hat{\mu}((1-\alpha)/\alpha)$, $s_2(A) = \hat{\mu}(\beta/(1-\beta))$, $s_2(B) = \hat{\mu}((1-\beta)/\beta)$, and $\gamma_1(A) = \hat{\eta}(\alpha/(1-\alpha))$, $\gamma_1(B) = \hat{\eta}((1-\alpha)/\alpha)$, $\gamma_2(A) = \hat{\eta}(\beta/(1-\beta))$, $\gamma_2(B) = \hat{\eta}((1-\beta)/\beta)$.

For the state dependent sample size the resistances are $r^s_{AB} = \hat{\mu}(\alpha/(1-\alpha))\;\alpha \wedge \hat{\mu}(\beta/(1-\beta))\;\beta$ and $r^s_{BA} = \hat{\mu}((1-\alpha)/\alpha)\;(1-\alpha) \wedge \hat{\mu}((1-\beta)/\beta)\;(1-\beta)$. For $1-\alpha < \beta$, it can be directly derived that $h_A$ is *SSS*; for $1-\alpha > \beta$, it is obtained that $h_B$ is *SSS*.

For the state dependent error size the resistances are thus $r^{\gamma}_{AB} = \hat{\eta}(\beta/(1-\beta))\;\alpha \wedge \hat{\eta}(\alpha/(1-\alpha))\;\beta$ and $r^{\gamma}_{BA} = \hat{\eta}((1-\beta)/\beta)\;(1-\alpha) \wedge \hat{\eta}((1-\alpha)/\alpha)\;(1-\beta)$. Hence, for $1-\alpha < \beta$ and given the former assumptions, it must be that $\alpha > 1-\beta > \beta > 1-\alpha$ and thus $min\,\{\hat{\eta}(\beta/(1-\beta))\;\alpha; \hat{\eta}(\alpha/(1-\alpha))\;\beta\} > min\,\{\hat{\eta}((1-\beta)/\beta)\;(1-\alpha); \hat{\eta}((1-\alpha)/\alpha)\;(1-\beta)\}$. As a consequence, it follows that $h_A$ is *SSS*. In the same way, if $1-\alpha > \beta$ it must hold that $r^{\gamma}_{AB} < r^{\gamma}_{BA}$ and $h_B$ is *SSS*.

Now consider the case, in which the loss in convention $A$ is defined by $l_1(A) = a/(a+c)$ and $l_2(A) = b/(b+d)$, for player 1 and 2 respectively, and again equivalently for convention $B$. Notice that $l_1(A) = \alpha$, $l_2(A) = \beta$, $l_1(B) = 1-\alpha$, and $l_2(B) = 1-\beta$. The second part of the proof is then analogous to the first part of the proof. Thus in both variants of considering the relative loss, state dependent and independent approaches coincide.

*Symmetric pay-off matrix* - This case entails that $a = b$ and $c = d$. Without loss of generality, assume that $a > c$ and thus $b > d$. Since, $\alpha = \beta > 1 - \alpha = 1 - \beta$ is follows that $h_A = SSS$ in the state independent case. In the case of state dependent sample size we obtain $\mu(a)\ \alpha = \mu(b)\ \beta\ >\ \mu(c)\ (1-\alpha)\ = \mu(d)\ (1-\beta)$, and $h_A = SSS$. In the case of state dependent error size, it holds that $\eta(b)\ \alpha = \eta(a)\ \beta\ >\ \eta(d)\ (1-\alpha) = \eta(c)\ (1-\beta)$, and $h_A = SSS$.

*Double symmetric pay-off matrix* - In this case, $a = d$ and $b = c$. Without loss of generality, assume that $a > c$, thus $d > b$, leading to $\alpha = 1-\beta > 1-\alpha = \beta$, and hence both equilibria are *SSS*. In the case of state dependent sample size it holds that $\mu(a)\alpha = \mu(d)\ (1-\beta)\ >\ \mu(c)\ (1-\alpha) = \mu(b)\ \beta$, and both are *SSS*. In the case of state dependent error size $\eta(b)\ \alpha = \eta(c)\ (1-\beta)\ >\ \eta(d)\ (1-\alpha) = \eta(a)\ \beta$, and both are *SSS*.

*Mirror symmetric pay-off matrix* - Given a $2 \times 2$ conflict games, with pay-off matrixB.3:

$$\begin{array}{c c} & \begin{array}{cc} A & B \end{array} \\ \begin{array}{c} A \\ B \end{array} & \begin{pmatrix} a, d & b, c \\ c, b & d, a \end{pmatrix} \end{array} \tag{B.3}$$

Assume without loss of generality that $a > d > b, c$, then $r_{AB} = \beta$ and $r_{BA} = (1-\alpha)$. Equilibrium $h_A$ will be the *SSS* in the state independent case, iff $d - c > d - b$, hence iff $c < b$, and $h_B$ is *SSS* iff $c > b$. We have $l_1(A) = a - c, l_2(A) = d - c, l_1(B) = d - b, l_2(B) = a - b$.

For the state dependent sample size $s_1(A) > s_2(A) > s_1(B)$ and $s_1(A) > s_2(B)$, since B.1 holds. Hence $r_{AB} = \mu(d-c)(\frac{d-c}{(.)})$ and $r_{BA} = \mu(d-b)(\frac{d-b}{(.)})$, where $(.) = (a-b-c+d)$. Since $d-c > d-b$, equilibrium $(A, A)$ will be *SSS*. The same argument holds for $c > b$, in which case $h_B$ is *SSS*.

In the case of state dependent error size and for assumption $c < b$, we obtain $\gamma_1(A) > \gamma_2(A) > \gamma_1(B)$ and $\gamma_1(A) > \gamma_2(B)$, and thus $r^{\gamma}_{AB} = \eta(d-c)(\frac{a-c}{(.)}) \wedge \eta(a-c)(\frac{d-c}{(.)})$ and $r^{\gamma}_{BA} = \eta(a-b)(\frac{d-b}{(.)}) \wedge \eta(d-b)(\frac{a-b}{(.)})$. Hence, if $c < b$ it must hold that $\min\left\{\eta(d-c)(\frac{a-c}{(.)}); \eta(a-c)(\frac{d-c}{(.)})\right\} > \min\left\{\eta(a-b)(\frac{d-b}{(.)}) \wedge \eta(d-b)(\frac{a-b}{(.)})\right\}$ and $h_A$ is the *SSS*. By the same reasoning, for $c > b$ it holds that $r^{\gamma}_{AB} < r^{\gamma}_{BA}$ and thus $h_B$ is the *SSS*.

Consequently, in the case losses are considered relative and are independent of a positive pay-off transformation that does not change the game structure, state dependence confirms the results obtain in the standard approach. This is not necessarily the case for any function of the sample and error size, if pay-offs show no form of symmetry.

**Example:** A short example will illustrate these results. Suppose the following pay-off matrix:

$$\begin{array}{c c} & \begin{array}{cc} A & B \end{array} \\ \begin{array}{c} A \\ B \end{array} & \begin{pmatrix} 16, 6 & 0, 0 \\ 0, 0 & 10, 8 \end{pmatrix} \end{array} \tag{B.4}$$

Hence, $\alpha = \frac{8}{13}$, $(1-\alpha) = \frac{5}{13}$, $\beta = \frac{3}{7}$, and $(1-\beta) = \frac{4}{7}$. As a result it holds, that $h_A = SSS$ in the state independent case.[21] For the case of state dependent sample size we obtain $r^s_{AB} = \mu(6)\ \frac{3}{7}$ and for $h_A$ to be *SSS* under the assumption of state dependent sample size it must hold that $\mu(6)\ \frac{3}{7} > \mu(10)\ \frac{5}{13}$ (and thus also that $\mu(10)\ \frac{5}{13} < \mu(8)\frac{4}{7}$), which is not the case for all functional forms of $\mu(.)$.[22]

For the state dependent error size it holds $r^\gamma_{BA} = \eta(8)\ \frac{5}{13}$. Hence, this must be strictly smaller than $\min\left\{\eta(6)\ \frac{8}{13}\ ,\ \eta(16)\ \frac{3}{7}\right\}$, which again is not fulfilled for all functional forms of $\eta(.)$.

This result is supported by computer simulations. Each case was tested on the basis of 10 simulations, each for 500.000 periods. The initial state was a history of completely random play, and population size was 2. In the state independent case, with memory size 20, sample size 8 and $\epsilon = 0.22$, we obtain an average distribution for the share of strategy $B$ players of $(0.523, 0.049, 0, 0.018, 0.410)$, with bin size 0.2 going from 0 to 1. For $s_i(\omega) = l_i(\omega) - 4$, memory size 30 and $\epsilon = 0.2$ the average distribution is $(0.044, 0.020, 0.001, 0.029, 0.906)$. For $\gamma_i(\omega) = (l_i(\omega))^2/12$, with $\varepsilon = 0.78$, memory size 20, and sample size 8, the average distribution is $(0.006, 0.007, 0.001, 0.001, 0.985)$.

If we restrict the form of $\mu(.)$ and $\eta(.)$ to the assumptions above, we obtain: $r^{s'}_{AB} = \min\left\{\hat{\mu}(\frac{8}{5})\ \frac{8}{13}\ ,\ \hat{\mu}(\frac{3}{4})\ \frac{3}{7}\right\}$ and $r^{s'}_{BA} = \min\left\{\hat{\mu}(\frac{5}{8})\ \frac{5}{13}\ ,\ \hat{\mu}(\frac{4}{3})\ \frac{4}{7}\right\}$. Thus, $r^{s'}_{AB} > r^{s'}_{BA}$. Further $r^{\gamma'}_{AB} = \min\left\{\hat{\eta}(\frac{3}{4})\ \frac{8}{13}\ ,\ \hat{\eta}(\frac{8}{5})\ \frac{3}{7}\right\} > r^{\gamma'}_{BA} = \min\left\{\hat{\eta}(\frac{4}{3})\ \frac{5}{13}\ ,\ \hat{\eta}(\frac{5}{8})\ \frac{4}{7}\right\}$, $r^{\gamma'}_{AB} > r^{\gamma'}_{BA}$. Consequently, in the constrained case, $h_A = SSS$ both for state dependent sample and error size.

Proof of Proposition (6). Given a normalised double-mirror symmetric game with two Nash equilibria

$$\begin{array}{cc} & \begin{array}{cc} A & B \end{array} \\ \begin{array}{c} A \\ B \end{array} & \begin{pmatrix} \text{a,b} & 0{,}0 \\ 0{,}0 & \text{b,a} \end{pmatrix} \end{array} \tag{B.5}$$

In this case the frequencies are as such that $\alpha = 1 - \beta$ and $1 - \alpha = \beta$. Hence in $r_{AB} = r_{BA}$ and each equilibrium is *SSS* in the state independent case. Assume without loss of generality that player type 1 (row player) is less risk averse than player type 2 (column player) and that he has a higher surplus in $h_A$ than in $h_B$ and the inverse for type 2, i.e. $a > b$. Since player 1 is less risk averse, it is assumed that $s_1(\omega) < s_2(\omega')$, or $\gamma_1(\omega) < \gamma_2(\omega')$, where $\omega \neq \omega'$, indicating state $h_A$ or $h_B$. Further, we know that $\alpha > (1 - \alpha)$.

In the case of state dependent sample size the resistances are rewritten as: $r^s_{AB} = \alpha\ s_1(A) \wedge (1 - \alpha)\ s_2(A)$ and $r^s_{BA} = (1 - \alpha)\ s_1(B) \wedge \alpha\ s_2(B)$. It must hold that $s_1(A) > s_1(B)$ and $s_2(B) > s_2(A)$, but also that $s_1(A) < s_2(B)$ and $s_1(B) < s_2(A)$. Hence, $r^s_{BA} = (1-\alpha)\ s_1(B) < r^s_{AB}$. Consequently, $h_A$ is *SSS*. Hence, the less risk averse player type 1 can gain a higher surplus.

For the case of state dependent error size, define two positive, strictly increasing and concave functions $u$ and $v$ as such that $u(.) > v(.)$, $u(0), v(0) = 0$ (from pay-off function A.3 on page A.3), $u'(.), v'(.) > 0$ and $u''(.), v''(.) < 0$. Let $\gamma_1(\omega) = v(l_1(\omega))$ and $\gamma_2(\omega) = u(l_2(\omega))$, and hence, $\gamma_1(A) = v(a)$, $\gamma_1(B) = v(b)$, and $\gamma_2(A) = u(b)$, $\gamma_2(B) = u(a)$. The resistances are $r^\gamma_{AB} = \alpha u(b) \wedge (1 - \alpha) v(a)$ and $r^\gamma_{BA} = (1 - \alpha) u(a) \wedge \alpha v(b)$.

[21] This indicates a paradox of the standard approach, and a difference between organic (as discussed here) and pragmatic institutions. The resistance from $A$ to $B$ is defined by $r_{AB} = \beta$, i.e. the share of type 1 (row) players necessary for type 2 players to shift their best response play. However, this condition implies that the type which benefits most from equilibrium $(A, A)$ is the decisive element (see also Bowles, 2006).

[22] Notice that $b < d < c < a$ is not a sufficient condition for $SSS = h_B$, see case 2. above.

Four possible outcomes can occur -
*1. case:* $r^{\gamma}_{AB} = \alpha u(b)$ and $r^{\gamma}_{BA} = \alpha v(b)$. From the minimum conditions of the resistances it must be that $\frac{a}{b} < \frac{v(a)}{u(b)} < \frac{u(a)}{v(b)}$, leading to a contradiction of our concavity assumption.
*2. case:* $r^{\gamma}_{AB} = \alpha u(b)$ and $r^{\gamma}_{BA} = (1-\alpha)u(a)$. In this case it must hold $\frac{u(a)}{v(b)} < \frac{a}{b} < \frac{v(a)}{u(b)}$. A contradiction of the assumptions that $u(.) > v(.)$.
*3. case:* $r^{\gamma}_{AB} = (1-\alpha)v(a)$ and $r^{\gamma}_{BA} = \alpha v(b)$. Thus, $\frac{u(a)}{v(b)} > \frac{a}{b} > \frac{v(a)}{u(b)}$. Since $u(.) > v(.)$. Because of concavity, we have $\frac{a}{b} > \frac{v(a)}{v(b)}$ and $h_B = SSS$.
*4. case:* $r^{\gamma}_{AB} = (1-\alpha)v(a)$ and $r^{\gamma}_{BA} = (1-\alpha)u(a)$. For this inequality to occur, it must be that $\frac{a}{b} > \frac{u(a)}{v(b)} > \frac{v(a)}{u(b)}$, we obtain $h_B = SSS$.
Hence, if $u(.)$ and $v(.)$ are strictly concave, we have $h_B$ is $SSS$.[23]

## Appendix C. Bibliography

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[23] Clearly, we obtain $h_A$ if convexity is assumed.

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