The Dynamics of Norms and Conventions Under Local Interactions and Imitation

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Restricting the analysis to general 2×2 coordination games, this article shows how under certain conditions, it is highly likely that individuals coordinate on a (pay-off) efficient though risk inferior convention. This contrasts with other equilibrium refinement criteria, such as risk dominance or stochastic stability. Here it is assumed that players are situated on a toroidal regular lattice, interact only locally and, in each period, imitate the last period's most successful player in their neighborhood. If the set of observable players by an individual and the set that he interacts with are both identical and small, pay-off dominance plays the major role in defining the long-term convention. As the latter set of players increases, a risk dominant but pay-off inferior convention becomes more likely. The model also shows that the interaction of two player types in a non-symmetric game potentially leads to non-egalitarian conventions.

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Subject Classification: C62, C63, D73, D03, D83.

1. Introduction

The interrelationship between cultural and economic variables has already been emphasized by Max Weber in his "Wirtschaft und Gesellschaft" [2007], but also more recently, prominent scholars have stressed the necessity to give cultural fundamentals proper recognition as economic determinants [Hodgson 1996, Bollinger and Hofstede 1987, Ades and Di Tella 1996, Huntington 1998, Welzel and Inglehart 1999, Harrison and Huntington 2000]. Social and economic interactions are regulated by the prevailing conventional and normative framework forming the scaffold of institutions.[a] In turn, these interactions then define the basis for new norms, conventions and institutions, thereby altering the rules for future interactions [Bicchieri 2006]. This has spawned a growing body of research on conventions, norms and economic behavior based on evolutionary models [Boyd and Richerson 1985, Hodgson 1996; 2007, Henrich et al. 2001, Gintis et al. 2005, Huck et al. 2012].

The study of conventions has further drawn the interest of economists and game theorists for a second reason more technical in nature: Even simple games with a limited choice of strategy

[a] In this context, a differentiation between norms and conventions is non-essential. Max Weber distinguishes between convention as a mechanism that urges individuals to exhibit certain behavior by approval and disapproval, and custom, characterizing consuetude and regular behavior, which is defined by inconsiderate imitation. A norm is based on customs and conventions, whereupon conventions turn customary behavior into norms, thus creating traditional behavior. The transition between these concepts is seamless (in fact Weber speaks of conventional norm, see Weber [2007, Ch. VI]).

oftentimes illustrate a plethora of Nash equilibria, thereby requiring a refinement criterion to reduce the number of reasonable outcomes. The analysis of conventions sheds a light on which refinement criterion is adequate. Social norms and conventions serve as self-enforcing coordination mechanisms and, in their abstract form, define a stable Nash equilibrium of pure strategies in a multi-player game [Schelling 1960, Lewis 1969]. Honoring the rules (or strategy) prescribed by a convention is in each individual's interest, provided that he believes a sufficiently large number of others will do the same [Hume 2011].

In order to study how the behavior of players forms a convention in the long-term, the model of this paper generally relies on two assumptions that are deviant from *standard* game theoretical approaches: local interactions and imitation. Individuals adopt norms and conventions from *specific reference groups* that can be professional, kinsmanlike, neighborly, class-oriented, ethnic, religious, or political in nature [Weber 2007, p. 616]. Our model assumes that players only interact with their neighbors on a regular lattice. Thus, it is part of a broader body of literature which, beginning with local interaction models of Schelling [1978; 1971; 1972; 1996], focuses on peer effects, group interactions, herd behavior, panics and local technology adoption (for some recent examples, see Pesendorfer [1995], Nakajima [2007], Kremer and Levy [2008], Brock and Durlauf [2010], Shiller [2006], Conley and Udry [2010] and for an overview of local interaction methods, see Brock and Durlauf [2001], Durlauf and Young [2001], Durlauf [2004], Manski [2000]). Hence, it contrasts with the global interaction approaches often relying on random matching (for a critique of these models, refer to Potts [2000]).

Norms and conventions are considered to be subject to emulation and reproduction. Here it is assumed that players only imitate the last period's most successful neighbor which requires only very limited information and cognitive abilities with respect to other heuristics.[b] Consequently, this paper also refers to the literature relying on imitation instead of best-response play as a heuristic for strategic choices, [Bass 1969, Orléan 2002, Leskovec et al. 2008, Young 2009, Robson and Vega-Redondo 1996], and to approaches using *replicator dynamics* in a cultural context [Gintis 2000].

Given these two assumptions, the interaction model of this paper offers answers to a number of questions that lie at the heart of understanding the machinery of norms and conventions. Numerous examples of coordination failures and institutions with varying efficiencies exist [Cooper and John 1988, Murphy et al. 1989, North 1990, Hoff and Stiglitz 2001, Bowles 2006]. This paper illustrates the conditions under which long-term conventions and thus institutions vary with respect to equality and efficiency.[c] The model also provides indications as to why different levels of diffusion of norms are observed; such as the existence of *evolutionary universals* (see Parsons [1964]) in contrast to strictly local norms (see Patterson [2004]).

The model follows closely those of Nowak and May [1992], Hauert [2001], and Brandt et al. [2003]. In contrast to those papers, this analysis relies on analytical solutions and focuses on stable coordination equilibria rather than on cooperation. Additionally, it is closely related to the papers by Eshel et al. [1998] and García-Martínez [2004].[d] Nevertheless, these two models consider players

[b]The variants of best-response or fictitious play [Young 1993] require player knowledge of his own strategies and associated pay-offs; average pay-off comparison entails an interpersonal comparison of utility. In contrast, the imitation of successful members of reference groups is common, e.g., when choosing a job (such as being an actor), we do not look at the average pay-offs but orient ourselves to the richest or most popular members of our reference group. This heuristic might be explained by a number of behavioral idiosyncrasies [DellaVigna 2009].

[c]Efficiency refers to Pareto efficiency. In the symmetric setting of section 2, the efficient convention is defined by the pay-off / Pareto dominant equilibrium, i.e., the equilibrium offering the highest pay-off to all players. Institutions refer to *organic* and not to *pragmatic* institutions in this context (see Menger [1963]).

[d]García-Martínez [2004] extends Eshel et al. [1998] with the addition of group cohesion, i.e., a form of directional

situated on a circle or real line, respectively. They also assume that an individual chooses his next period's strategy based on the average pay-off of each strategy in his neighborhood rather than that of the most successful neighbor. Eshel et al. [1998] provide a result seemingly similar to the one developed in proposition 4 of this article.[e] This solution, on the other hand, is only viable for a more restricted class of games. Differing from proposition 4, it assigns positive probability to a pay-off inferior convention in the case of random seeding. In addition, both papers do not analyze the case in which the neighborhood defining the pay-off and that which is used for imitation (or learning) are unequal in size, and only consider symmetric games with a homogeneous population.

One key result of the present model is that, generally, a non-risk dominant convention (see Harsanyi [1973]) is chosen in the long-run if it is pay-off dominant and interactions are sufficiently local.[f] This contrasts with approaches closely related to stochastic stability [Young 1993, Kandori et al. 1993] which, similar to our model, assume local interactions but with a variant of best response play [Ellison 1993; 2000, Blume 1993; 1995, Young 1998, Morris 2000, Lee and Valentinyi 2000, Durlauf and Young 2001, Lee et al. 2003]. The difference in predictions is a direct consequence of assortment as an immanent evolving property in the present model, whereas other approaches explicitly assume norm adherence [Bisin and Verdier 2005, Bisin et al. 2004, Bowles 2006, Alger and Weibull 2012]. Assortment places more weight on the diagonal elements in the pay-off matrix. Additionally, the degree of interaction locality plays a crucial role: In the case where a player interacts with a large number of neighbors, the risk dominant, though pay-off inferior convention, evolves in the long-run. Thus, results differ from Robson and Vega-Redondo [1996], where players adopt the same imitation heuristic as in the present model, though players are randomly matched. Another key result is that highly unequal conventions can evolve in the case where two different player populations exist - a result deviating profoundly from other approaches (such as Young [1998]).[g]

This paper considers 2×2 coordination games given the following assumptions:

(i) All individuals interact on a toroidal, two-dimensional regular lattice, on which they are initially placed at random.
(ii) Individuals only play the game with each of their neighbors (in the Moore neighborhood) once per period.
(iii) An individual pay-offs depend only on his individual strategy and on the strategies played by his neighbors.
(iv) Each individual adopts the strategy of his neighbors with the maximum pay-off in the last period; if his strategy is ambiguous or if the individual already received no less than the maximum pay-off, he will keep his strategy.
(v) All players update synchronously and once in each period.
(vi) Updating is deterministic (no mutations) and the outcome of the game is only defined by the initial conditions, the distribution and the pay-off matrix.[h]

interaction.

[e] When adapting the result of Eshel et al. [1998] to the form and definitions used in this paper, the (pay-off) efficient equilibrium is chosen if $\mu < \min\{(c-a+\rho)/2, a-c+\rho\} = \min\{(b-2)/2, (d-\hat{c})/2\}$ for interactions on a string where strategy imitation is that of the strategy with the highest average pay-off. These results thus require that $b > a$.

[f] Given a pay-off matrix as in matrix (1), the equilibrium with the highest pay-off is pay-off dominant, e.g., equilibrium (A, A) pay-off dominates equilibrium (B, B), if $a > d$. The risk dominant equilibrium is defined as the equilibrium with the highest expected pay-off if the other player chooses both strategies with equal probability, e.g., equilibrium (A, A) risk dominates equilibrium (B, B), if $a - c > d - b$ (see also Weibull [1995]).

[g] The different types are assumed to represent different social, ethnic or religious groups.

[h] Simulations have shown that mutations at a sensibly low rate do not affect results.

The following section considers symmetric 2×2 coordination games.[i] It shows under which conditions (pay-off) efficient conventions are generally chosen and how risk dominance is a weak indicator for a long-term convention. In section 3, non-symmetric pay-off matrices, i.e., the interaction between two different populations, are analyzed, providing the argument for efficient, but non-egalitarian conventions in the long-term. Section 4 provides an answer as to why (pay-off) inefficient conventions are observed.

2. Symmetric Pay-offs

Let there be a finite but large population $\mathfrak{I}$ of N individuals in which each player is assigned to a unique individual position on a two-dimensional, torus-shaped regular lattice defined by the coordinate tuple (x, y) with $x, y \in \mathbf{N}$. Each individual interacts only with his Moore neighborhood. Define a binary relation on N indicating that "i is neighbor of j". Define this as $i \sim j$. For an individual i on a lattice point with coordinates (x_i, y_i), an individual j with $j \sim i$, is defined as $\{j : (x_j = x_i + v, y_j = y_i + w)\}$, with $v, w \in \{-1, 0, 1\}$ and $|v| + |w| \neq 0$. Consequently, it is assumed that the binary relation $\sim$ is irreflexive, symmetric and each player has eight neighbors surrounding him. Define $\mathfrak{N}(i)$ as the set of neighbors of i, i.e., $\mathfrak{N}(i) = \{j : j \sim i\}$. In each period t, a player i plays a coordination game with each neighbor once and chooses between the two pure strategies $s_t(i) = A$ and $s_t(i) = B$. In this section, the pay-offs of each interaction are given by the symmetric pay-off matrix (1). Thus, only a single player type exists and it is irrelevant whether an individual plays as a row or column.

$$\begin{array}{cc} & \begin{array}{cc} A & B \end{array} \\ \begin{array}{c} A \\ B \end{array} & \begin{pmatrix} a, a & b, c \\ c, b & d, d \end{pmatrix} \end{array} \tag{1}$$

For matrix (1) to be a coordination game, assume $a > c$ and $d > b$. Let $\mathfrak{I}_t^A$ be the set of individuals playing strategy A in period t, and $\mathfrak{I}_t^B$ the set of individuals playing B in the same period. Furthermore, let $\mathfrak{F}_t^A(i) = \#\left\{\mathfrak{I}_t^A \cap \mathfrak{N}(i)\right\}$ and $\mathfrak{F}_t^B(i) = \#\left\{\mathfrak{I}_t^B \cap \mathfrak{N}(i)\right\} = 8 - \mathfrak{F}_t^A(i, \mathfrak{N}(i))$ be the number of strategy A and B playing neighbors of i. Since each individual plays the game once with each of his neighbors at t, the pay-off of player i is thus defined as

$$\pi\left(s_t(i), \{s_t(j) : j \in \mathfrak{N}(i)\}\right) = \begin{cases} \mathfrak{F}_t^A(i)a + \mathfrak{F}_t^B(i)b, & \text{if } s_t(i) = A \\ \mathfrak{F}_t^A(i)c + \mathfrak{F}_t^B(i)d, & \text{if } s_t(i) = B \end{cases} \tag{2}$$

Define $\pi_t(i) = \pi\left(s_t(i), \{s_t(j) : j \in \mathfrak{N}(i)\}\right)$ for notational simplicity. Let $\Pi_t(i) = \{\pi_t(j) : j \in \mathfrak{N}(i) \cup \{i\}\}$ be the joint set of pay-offs for player i and his neighbors. Define $\arg(i) = \{s_t(j) | j \in \mathfrak{N}(i) \cup \{i\}, \pi_t(j) = \max \Pi_t(i)\}$. For the subsequent period, this player chooses a strategy s_{t+1} based on the imitation rule:

$$s_{t+1}(i) = \begin{cases} A, & \text{if } \arg(i) = \{A\} \\ B, & \text{if } \arg(i) = \{B\} \\ s_t(i), & \text{if } \arg(i) = \{A, B\} \end{cases} \tag{3}$$

Define the long-term convention (or equilibrium) of the game in which all players choose strategy A as h_A, whereas the convention in which all choose strategy B is defined as h_B. Though the following

[i] All results are supported by simulations. Interested readers may refer to Ille [2013b].

analysis is local, it enables us to predict the global evolution for a sufficiently large population and is based on the given pay-off configuration. The pay-off dominant strategy is defined by the pay-off dominant convention. In this and the following section, the direct implications of these assumptions are:

1. In the case where a player chooses the pay-off dominant strategy, i.e., the strategy with the largest value on the pay-off matrix's main diagonal, his pay-off increases with an increasing number of neighboring players choosing the same strategy. The maximum pay-off of this strategy is received by individuals who are only surrounded by players of the same strategy. This also holds for the pay-off inferior strategy if the matrix's main diagonal pay-off values are strictly greater than the off-diagonal values.
2. Any interior individual, who is surrounded only by players of the same strategy, never has an incentive to switch, since all players in his neighborhood play the same strategy. Transitions can only occur at the borders of clusters.
3. Under random seeding and during the initial sequence of interaction, the strategy distribution on the lattice is strongly determined by the relative average pay-off of each strategy. During this process, it is more likely a player adopts the strategy with the higher average pay-off if players initially choose their strategy at random with equal probability.

One of the principal questions is whether an efficient convention can evolve in this setting. For the moment, I suspend the element of riskiness (i.e. risk dominance) and focus solely on efficiency (i.e. pay-off dominance). In order for two equilibria to be risk equivalent, assume that $a - c = d - b$. Pay-offs can thus be written as $d = a + \rho$ and $b = c + \rho$. Define the "pay-off premium" as the pay-off difference of ρ. For $\rho > 0$, equilibrium h_B pay-off dominates h_A.

For the first set-up, consider the case in which the population originally adopted the pay-off inferior convention (i.e. every player plays strategy A). In this case, a shift to the more efficient convention can only be triggered by a number of players idiosyncratically choosing the pay-off superior strategy (frequently denoted as *mutants* in evolutionary game theory). To study the requirements for such a change in conventions, a definition is necessary for the minimum number of *mutants*.

Definition 1. *A* ***cluster of size*** r *is defined as the highest number of neighbors playing the same strategy within a set of directly connected players of an identical strategy, i.e., a cluster of size* r *is defined as a set of connected players, in which at least one player has r-1 neighbors who play the same strategy.*

For example, suppose a straight line of players where each player has a neighbor to his left and right choosing the same strategy as him, except for the corner players. Independent of its length, such a straight line always has a size of three, because each player has at least one neighbor with two players in his own neighborhood, who choose the same strategy. Hence, all players in this cluster compare any player with a different strategy to either $2a + 6b$ or $2d + 6c$.[j] Since the dynamics only depend on the player with the highest pay-off in the neighborhood, the length of a line of identical players is unimportant. This also applies to larger clusters. Notice that this definition restricts the maximum size of a cluster to nine, since a player has a maximum of eight identical neighbors. For the minimum invading cluster size of *mutants* to cause a shift in conventions, it holds (for this and the following proofs, see Appendix A):

[j]This also holds for the outer players of the line cluster, if $a, d > b, c$. If this is not the case, these outer players have highest pay-off with either $a + 7b$ or $7c + d$ given that strategy A or B is pay-off inferior.

Proposition 1. *Assume a pay-off matrix as in matrix* (1) *with two risk equivalent pure Nash equilibria, for any a, b, c, d, as long as $a-c = d-b$ holds. A population, whose convention is currently defined by the pay-off inferior strategy A, is successfully invaded by a cluster of the minimum size r who chooses the pay-off dominant strategy B, if the pay-off premium satisfies:*

$$\left.\begin{aligned} &\rho > 3(a-c) \text{ and } r \geq 4 \text{ and square} \\ &\rho > \quad a-c \quad \text{and } r \geq 5 \end{aligned}\right\} \quad \text{for } a < b$$
$$\rho > \frac{3}{5}(a-c) \text{ and } r \geq 6 \qquad\qquad \text{for } a \geq b \tag{4}$$

Hence, a minimum pay-off premium of $\rho > \frac{3}{5}(a-c)$ is sufficient for a population to abandon the inefficient convention given a minimum cluster of *mutants* occurs with positive probability. If this condition is unfulfilled, the population either remains in the inefficient convention or small clusters adopting the efficient convention endure but do not proliferate. For a sufficiently large population, we have:[k]

Proposition 2. *Clustering is an evolving property and most clusters of at least one strategy have a size equal to nine after an initial period of interaction. In addition, for $b > a$ and $\rho > 7(a-c)$, stable clusters of size $r = 1$, which play the pay-off inferior strategy A, can occur. With the case of $a > b$: Playing the pay-off dominant strategy B, clusters of size six are stable given $\frac{1}{2}(a-c) < \rho < \frac{3}{5}a - c$, of size seven given $\frac{1}{5}(a-c) < \rho < \frac{1}{3}(a-c)$, and of size eight given $0 < \rho < \frac{1}{7}(a-c)$. Clusters of size five are stable iff $a = b$.*

Until now, results have referred to the case in which a convention pre-exists. Now consider the case in which no strictly defined convention prevails *a priori*. Intuitively, the initial seeding plays a role: If a small cluster of mutants is to overtake the entire population, then stronger conditions are required when compared to the case in which the population has not yet chosen a specific convention. The initial distribution (i.e. seeding) can lie between the two extreme cases:

Definition 2. ***A balanced initial distribution*** *defines an initial distribution of a player population in which the average cluster size for all strategies is roughly identical after the first period of interaction. An* ***unbalanced initial distribution*** *defines an initial player distribution in which average cluster sizes differ strongly among strategies after the first period of interaction. However, the evolution of at least one cluster of a minimum size of six occurs with certainty for any strategy after an initial sequence of interactions.*

A balanced initial distribution has not been defined simply as the case in which players initially choose a strategy at random with equal probability due to the following: According to proposition 2, individual players agglomerate into large clusters after the initial period. The size and strategy of these clusters are defined by the average pay-off. If the average pay-off of the risk inferior strategy is substantially smaller than the average pay-off of the risk dominant strategy, most players initially choose the latter strategy. A cluster playing the former strategy, which has sufficient size for proliferation, might not occur after this initial period. A simplification to random choice is only suitable in the case of roughly equal average pay-offs.

The limits of an unbalanced initial distribution is demonstrated by the case discussed in propositions 1 and 2, in which one convention (here the inefficient convention) has been initially adopted by

[k]Proposition 2 is the first indication as to why we observe strictly local norms. The next section extends these results.

the entire population and only a minimum number of invading mutant clusters exist. Alternatively, with reference to what was explained previously, an unbalanced initial distribution also describes a situation in which players choose their strategy at random, but average pay-offs are of dissimilar size. The player population collapses into large clusters who play the risk dominant strategy and a small number of clusters, with at least one being of size six, playing the risk inferior strategy. The requirement for the evolution of an efficient convention in the balanced case is:

Proposition 3. *A population with a balanced initial distribution converges to the pay-off superior equilibrium h_B, if the pay-off premium ρ is greater than $\frac{1}{7}(a-c)$. If the pay-off premium is smaller, but positive, a player population consists of clusters playing different strategies.*

Summarizing propositions 1 and 3, we observe that the efficient equilibrium is chosen if $\rho > \frac{1}{7}(a-c)$ in the balanced case or if $\rho > \frac{3}{5}(a-c)$ in the unbalanced case. These results extend to the trade-off between risk and efficiency within this context. Let us assume as before that $d = a + \rho$ and $b = c + \rho$. Additionally, substitute c in matrix (1) by $\hat{c}$ such that $\hat{c} = c - \mu$. Consequently, h_B is pay-off dominant by a value of ρ and h_A is risk dominant by a value of μ. Define the latter value as the "risk premium". The conditions of the trade-off between riskiness and efficient are defined as follows:

Proposition 4. *Given a coordination game as in matrix* (1) *with two equilibria in which h_B pay-off dominates h_A by a pay-off premium of ρ, and h_A risk dominates h_B by a risk premium of μ, the population converges to convention h_B if:*

$$\mu < \begin{cases} c - a + 7\rho, & \text{and the initial distribution is balanced} \\ c - a + \frac{5}{3}\rho, & \text{and the initial distribution is at least unbalanced} \end{cases} \tag{5}$$

If the initial population distribution is unbalanced and $\mu > \frac{2(c-a)+4\rho}{3}$, the population chooses the risk dominant convention. However, in the case of a population that is initially sufficiently balanced, the risk dominant strategy only prevails as a convention if it is also pay-off dominant by a value greater than $\frac{a-c}{7}$. Otherwise, the population remains in a state of mixed conventions.

Thus, the model does not predict the convention to be defined solely by risk dominance. Dynamics do follow a trade-off between risk and efficiency. Furthermore, if the population is sufficiently balanced initially, then efficient conventions are observed all of the time. Section 4 elaborates on the limits of this result.

3. General 2 x 2 Coordination Game

The following section analyzes whether the dynamics lead to egalitarian long-term conventions. For this, I generalize results to non-symmetric 2×2 coordination games, in which two player types (row and column) interact with each other. Players choose their initial strategy at random. The relative average pay-off of each strategy determines whether the balanced or unbalanced case applies. The only difference with respect to the assumptions in the previous section is that on each lattice point two players coexist, one of each type. Pay-offs are defined by the pay-off matrix:

$$\begin{array}{cc} {}_{Type1}{}^{Type2} & \begin{array}{cc} A & \;\; B \end{array} \\ \begin{array}{c} A \\ B \end{array} & \begin{pmatrix} a_1, a_2 & b_1, \hat{c}_2 \\ \hat{c}_1, b_2 & d_1, d_2 \end{pmatrix} \end{array} \tag{6}$$

Similar to the former section, let $\rho_i = d_i - a_i$, $\mu_i = b - \rho_i - \hat{c}_i$, $a_i > \hat{c}_i$ and $d_i > b_i$. Correspondingly for the two player types x and y with $x, y = 1, 2$ and $x \neq y$, define for a player i of type x the set of neighbors of his own type as $\mathfrak{N}_x(i)$ and the set of neighbors of type y as $\mathfrak{N}_y(i)$. For period t, let $\mathfrak{I}^A_{t,y}$ be the set of type y players playing strategy A, and $\mathfrak{I}^B_{t,y}$ be the set of type y players playing B. Correspondingly, define $\mathfrak{F}^A_{t,y}(i) = \#\left\{\mathfrak{I}^A_{t,y} \cap \mathfrak{N}_y(i)\right\}$ and $\mathfrak{F}^B_{t,y}(i) = \#\left\{\mathfrak{I}^B_{t,y} \cap \mathfrak{N}_y(i)\right\}$ as the number of strategy A and B playing neighbors of i that are of type y. The pay-off of player i at time t: is[1]

$$\pi_{t,x}(i) = \begin{cases} \mathfrak{F}^A_{t,y}(i)a_x + \mathfrak{F}^B_{t,y}(i)b_x, & \text{if } s_t(i) = A \\ \mathfrak{F}^A_{t,y}(i)\hat{c}_x + \mathfrak{F}^B_{t,y}(i)d_x, & \text{if } s_t(i) = B \end{cases} \tag{7}$$

Analogous to the former section, let us define $\Pi_{t,x}(i) = \{\pi_{t,x}(j) : j \in \mathfrak{N}_x(i) \cup \{i\}\}$ as the joint set of agent i's pay-offs and the pay-offs of his neighbors of the same type, and also define $\arg(i) = \{s_t(j) | j \in \mathfrak{N}_x(i) \cup \{i\}, \pi_t(j) = \max \Pi_{t,x}(i)\}$. The imitation rule is then determined by condition 3 on page 4 in the former section.

Interestingly, the following proposition shows that dynamics eliminate most complexities resulting from the assumption of two player types:

Proposition 5. *Given a pay-off matrix as in matrix* (6) *and* $a_i > \hat{c}_i$, $d_i > b_i$ *for each* $i = 1, 2$, *the strategy distributions of the two player types coincide after a brief sequence of interactions. Larger clusters are unsustainable if they are defined by players on the same lattice point who play different strategies.*

Another implication of proposition 5 is that a normative or a change in convention can only be triggered by a number of individuals that involve all of the participating group (i.e. types). The finding is intuitive, since in this context I consider *organic* conventions that are based on spontaneous, uncoordinated and involuntary choice [Menger 1963]. To understand the intuition behind proposition 5, consider the case in which the same strategy for both player types has the higher average pay-off. After the first period of interaction, larger pure clusters appear where both types play this strategy. They are surrounded by smaller clusters who are either mixed, i.e., where each type on the same lattice point plays a different strategy, or who are pure but playing the strategy with the lower average pay-off. The reason why mixed clusters are unsustainable is outlined as follows: The players on the edges of the mixed cluster always choose the strategy of the pure cluster in their neighborhood. The schematic in figure 1 aids in understanding the underlying dynamics. It shows two clusters, each with two layers symbolizing the interacting player types. The pure cluster is stylized on the left side of the figure; the mixed cluster, on the right. The upper layer illustrates type 1; the lower layer, type 2. Assume, without loss of generality and with respect to the mixed cluster, that type 1 plays

[1]Notice that following the former assumptions and for that reasons of symmetry, a player does not interact with the other player type on his own lattice point. Thus, he still has eight neighbors.

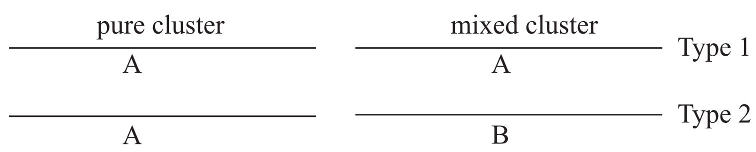

Fig. 1. Dynamics of mixed clusters.

strategy A, type 2 plays B, and the pure cluster only plays A. By assumption, type 1 players choose the same strategy in both clusters. Because imitation is horizontal, i.e., between players of the same type, type 1 players cannot imitate any other strategy. Consequently, type 2 players interact only with players choosing A. Since by definition $a_i > \hat{c}_i$ and $d_i > b_i$, a type 2 player in the pure cluster always has a higher pay-off than a player of the same type in the mixed cluster. Hence, type 2 players at the edges of the mixed cluster switch to strategy A. The dynamics are independent of what type plays which strategy and independent as to whether the strategy is risk or pay-off dominant (since the assumptions have ruled out strictly dominant strategies). Mixed clusters vanish and only small strings of mixed, unstable clusters with a maximum width of three at the borders of pure clusters remain (i.e. the *corner* elements of each cluster and the player in between).

The second case, in which the strategy with the higher average pay-off is different for both player types, is analogous. Hence, the strategy distribution on the lattice, for both player types, coincides for both player types after an initial sequence of interactions. Although, transition to this state is faster in the first case than in the second case. (Figure 2 on page 16 illustrates the behavior for two cases with identical initial distribution and the subsequent three periods of interaction.) In the following proposition, I make the simplifying assumption that the initial distribution is balanced and that $a_i > b_i$:

Proposition 6. *Given a pay-off matrix as defined in matrix* (6) *and* $a_i, d_i > b_i, \hat{c}_i$*, for each type* $i = 1, 2$*, the convergence speed of the player population towards equilibrium* h_A *is determined by the largest integer* $\lceil \eta_A \rceil_i$ *less* $\frac{-8\rho_i}{a_i - c_i - \rho_i}$. *Equivalently the largest integer* $\lceil \eta_B \rceil_i$ *less* $\frac{8\rho_i}{a_i - c_i + \rho_i + \mu_i}$ *defines the convergence speed to equilibrium* h_B.

If the population is initially sufficiently balanced, the population converges to h_B *if* $\max_i \{\lceil \eta_A \rceil_i\} < \max_i \{\lceil \eta_B \rceil_i\}$ *or to* h_A *if* $\max_i \{\lceil \eta_A \rceil_i\} > \max_i \{\lceil \eta_B \rceil_i\}$. *Otherwise, both strategies persist in the long-term.*

Simply put, subsequent to a transition period, large pure clusters with a cluster of size nine occur after an initial sequence of interaction. The cluster that expands most rapidly eventually defines the convention. However, this is only the case if the average pay-off for both strategies does not differ too greatly and the distribution is sufficiently balanced initially.[m] To get an intuition for the dynamics, consider that a cluster's edges are either horizontal, vertical or diagonal as illustrated in figure 3 in the appendix. By proposition 6, convergence speed ranges from zero to six. Hence, only six conditions for each strategy influence the dynamics of the entire population in the long-term. Notice that the solutions to $\lceil \eta_B \rceil_i$ equaling one and three return the results of proposition 4 for the single type case.

The result of proposition 6 answers the question as to whether egalitarian conventions are likely to evolve. In the case of an asymmetric coordination game of "common interest" (i.e. the same strategy is pay-off dominant for both types), the population converges to the pay-off dominant convention, if the convergence speed of one player type equals at least three or if the initial distribution is sufficiently balanced. Otherwise, the population exhibits a number of stable, rectangular shaped clusters in which some adhere to local inefficient conventions. In the matter of a "conflict game", where pay-off dominant equilibria are not identical for both player types, the strategy with the higher convergence speed defines the convention. Proposition 6 generates an interesting prediction; it illustrates a trade-off between risk and pay-off. In the case of an egalitarian and an inegalitarian pure Nash equilibrium, where the latter provides a much higher pay-off to one of the types, the ine-

[m]Remember that even if players choose a strategy at random with equal probability before the first interaction, too diverse average pay-offs can inhibit the evolution of clusters comprised of players of the risk inferior strategy which have sufficient size to overtake the player population.

galitarian convention is chosen in the long-run as long as it is insufficiently risk inferior to the former convention.[n] This implies that organic conventions and institutions may illustrate the tendency to become increasingly inequitable, benefiting one player type at the cost of the other.[o]

4. The Effect of Space

Up to this point, the model has only provided reasons for local and unequal conventions, yet it has not provided an answer as to why more and less efficient institutions are observed. The following section shows how the spatial configuration of the interactions may play a vital role. So far it has been assumed that only the eight surrounding neighbors are considered for both the calculation of pay-offs and for imitation. The group with which an individual interacts generally depends on the given social context and may often exceed the small reference group. Additionally, the observable reference group, which an individual is able to use as a benchmark for his future actions, might not be superimposable with the group actually affecting the individual's pay-off.[p] Taking this into account, this section provides general results supported by simulations without defining clear analytic conditions for each size of space.

Definition 3. *The* ***imitation radius*** *is defined as the largest Chebyshev distance between a player and a member of the set of observable neighbors whom he can imitate. The* ***pay-off radius*** *is similarly defined as the largest Chebyshev distance between a player and a member of the set of neighbors affecting his pay-off.*

The radii define the minimum number of steps a "king" requires to move from himself to his farthest neighbor (within the set of observable or pay-off affecting players) on the "chess board" lattice. In the former sections, both radii have been assumed equal to one, i.e., an individual only considers the adjacent eight players. First consider the case in which both radii are identical. Simulations show that as the radii increase, it is more likely that the population converges to the risk dominant equilibrium (see figure 4 in the appendix). This is explained by the fact that the enlarging of the pay-off radius leads to pay-offs in the first period converging to the expected pay-offs. By definition, the risk dominant strategy has a higher average pay-off than the pay-off dominant, though risk inferior strategy. Thus, individuals in areas distributed in a balanced way (i.e. where no large clusters of one player type exist) adopt the risk dominant strategy. In the aggregate we then observe a decrease in the number of pay-off dominant players in the first period. However, if this decrease occurs in a way in which a cluster of minimum size who plays the pay-off dominant strategy does not evolve, the population does not converge to the pay-off optimal equilibrium. Individuals adopt the pay-off dominant strategy only in neighborhoods where a sufficient number of individuals playing the pay-off dominant strategy agglomerate during the initial seeding process. For a random initial distribution, these agglomerations are more likely to occur the larger the population size.

Furthermore, note that the minimum sustainable cluster size depends on the imitation radius under consideration. The number of surrounding players, observed by an individual, increases with

[n]This result and proposition 6 also imply that the dynamics are not invariant to positive, affine transformations of the pay-off matrix.

[o]Under this condition, dynamics can only be counteracted by collective and revolutionary action (for an example of such a model, see Ille [2013a]). Pragmatic institutions might then serve as an antipole to organic institutions.

[p]This is, for example, the case in Palanpur where peasants used to sow their crops at a date later than one that would maximize their expected yields. A convention shift that dictates earlier planting can only be jointly implemented, since a single switching farmer would lose his yield to the birds. Farmers imitate the action that they are able to observe on the fields lying close to their own fields, yet the yield is given by the hunting ground of the birds. See Bowles [2006, Chapter 1] for a more detailed analysis.

the imitation radius. The same holds in order for the minimum cluster size of the pay-off optimal strategy to be sustainable.[q] This occurs with decreasing probability if the population size diminishes. Hence, small societies tend to move towards the risk dominant equilibrium at smaller radii in comparison to larger societies.

Oftentimes, it seems plausible that the imitation radius is much smaller than the pay-off radius.[r] Simulations show that, *ceteris paribus*, the convergence towards the risk dominant equilibrium is more likely with a higher discrepancy between the imitation radius and the pay-off radius (see figure 5 in the appendix). The effect is explained as follows: Increasing the pay-off radius benefits the risk dominant players. A large imitation radius, conversely, increases the spatial effect of a large agglomeration of pay-off dominant players on the strategy choice of neighboring players for the next period. If individual comparison of pay-offs is only highly local, the effect of such a large cluster on its surroundings is also highly local and thus negligible. To recapitulate the observations:

Observation 1. Large populations are more likely to converge to the pay-off dominant equilibrium than smaller populations.

Observation 2. With respect to larger pay-off radii, a population converges to the risk dominant equilibrium with high probability for small imitation radii.

This implies a positive relationship between the scope of individual choice effects on other players' pay-offs and individual information on the one hand, and the probability of convergence towards the risk dominant convention on the other hand. If externalities are far reaching, individuals tend to choose the risk dominant strategy (see figure 6 for the structure of the mixed stable equilibria of the former set of simulations).

5. Conclusion

Although the model is a very simplified representation of interactions determining the evolution of conventions, a straightforward interpretation of the abstract findings demonstrates an intricate and intuitive set of results. Imitation driven strategy choice and strictly local interactions foster the evolution of efficient conventions given no incumbent risk dominant convention exists. If it does, the prevailing convention is determined by a trade-off between efficiency and risk dominance. This may result in a universal convention or it may result in a situation in which both conventions are adhered to locally, depending on the specific context of interaction. If the assumptions on a strict locality of interactions are relaxed, these results no longer hold. A positive correlation exists between the scope of individual choice externalities (here defined by the pay-off radius) and the reference group used for imitation (defined by the pay-off radius) on the one hand, and the likelihood with which a risk dominant convention evolves on the other hand. Especially, if experiencing a large scope of external effects from others' choices, individuals are certain to follow the risk dominant convention. This spatial effect, where individual choice is favoring inefficient conventions, is more intense in small secluded societies.

[q]For the given pay-off, the minimum sustainable cluster size is: radius 2 = 14, radius 3=30, radius 4=48 in contrast to 6 for radius=1.

[r]See the example in footnote p: If it were not the case, peasants could easily implement the pay-off dominant equilibrium by observing all peasants in the village and collectively impose a fine on anyone sowing late. The space defining the individual's strategy for the next period is defined by those fields' last yield which the peasant can observe. It is most likely that these are the fields surrounding his own. The pay-off radius is, however, defined by the birds' hunting ground. It is highly probable that this radius is tremendously larger than the imitation radius. Consequently, the imitation radius is smaller than the pay-off radius.

In the case of more than one interacting group (i.e. non-symmetric pay-offs), a shift in conventions is only triggered by a small group of individuals, if all interacting social levels of this group (i.e. types) adhere completely to the new convention. The resulting driving force is the type of player benefiting the most from a shift in convention. Once more, this contrasts with approaches relying on stochastic stability that favor the risk dominant equilibrium. In those models, the determining force may instead be defined by the player type losing the most from a shift in convention (for the critique and an illustration, refer to Bowles 2006).

The model can be extended along several lines: In its current state, it neglects the role of path dependency. This limitation may explain persistent interaction patterns that are inferior, both in terms of risk and pay-off (for examples, see Edgerton 2004). Behavioral patterns and social customs might dictate a bearing which inhibits the evolution towards other equilibria and, thus, the adoption of certain strategies. Hence, an evolutionary process might turn out to be a blind alley. As Nelson states: "[...][B]eliefs about what is feasible, and what is appropriate, often play a major role in the evolution of institutions." [Nelson 2008, p. 7]

Since the group of possible learning algorithms is much larger than those discussed in this paper, a broader analysis might also be of interest. Furthermore, an expansion of this approach which includes more than two strategies is promising for future research, not only with respect to the *survival* of strategies given certain types of games, but also as to the spatial patterns that evolve. A comparison of proposition 4 to the result in Eshel et al. [1998] further suggests that higher dimensional interactions and thus more integrated neighborhoods increase the likelihood of an efficient convention.

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Appendix A. Proofs

Recall the aforementioned conditions $a > c$ and $d > b$, as well as that h_B pay-off dominates h_A by a positive constant ρ. We define a player entirely surrounded by neighbors playing the same strategy as him as an *internal*, otherwise we define him as an *external*. Due to the definition of cluster size r, there exists an *internal* iff $r = 9$. Denote an external and an internal in $\mathfrak{C}$ by $\mathfrak{e}_{\mathfrak{C}}$ and $\mathfrak{i}_{\mathfrak{C}}$, respectively.

Proof of proposition 1:. Assume the player population is currently in h_A. Also, assume there is an existing mutant cluster $\mathfrak{C}$ of size r playing strategy B and invading the player population. We observe for $\mathfrak{C}$ of size $r = 9$ that $\pi_t^B(\mathfrak{i}_{\mathfrak{C}}) = 8d$ and the maximum pay-off of an external in $\mathfrak{C}$ is $\pi_t^B(\mathfrak{e}_{\mathfrak{C}}) \geq 3c + 5d$. Focusing on the case in which $r < 9$, we redefine pay-offs as $\pi_t^A(i) = \mathfrak{F}_t^A(i)(a-b) + 8b$, if $i \in \mathfrak{I}_t^A$ and $\pi_t^B(i) = \mathfrak{F}_t^A(i)(c-d) + 8d$, if $i \in \mathfrak{I}_t^B$. We observe that $\frac{\partial \pi_t^B(i)}{\partial \mathfrak{F}_t^A} = c - d < 0$. Also note that no player in $\mathfrak{C}$ is *internal*. For any player $l \in \mathfrak{C}$, denote by k a player, s.t. $\pi_t^B(k) = \max\{\pi_t(h) : h \in (\mathfrak{N}(l) \cap \mathfrak{C}) \cup \{l\}\}$. The size of the invading cluster diminishes if l switches, i.e., iff

$$\max\{\pi_t(h) : h \in (\mathfrak{N}(l) \setminus \mathfrak{C})\} > \pi_t^B(k) \tag{A.1}$$

Since the relation between a cluster of size r and the pay-off of player k is positive, we have in this case:

$$\mathfrak{F}_t^A(k) = (9-r) \text{ and } \pi_t^B(k) = (9-r)c + (r-1)d \tag{A.2}$$

Two cases occur: either $a \geq b$ or $b > a$.

1. case $a \geq b$: In this case, it holds that $\frac{\partial \pi_t^A(i)}{\partial \mathfrak{F}_t^A} = a - b > 0$. Since the cluster size of the incumbent strategy is nine, all players not in cluster $\mathfrak{C}$ have at least one *internal* neighbor with the pay-off $\pi_t^A(\mathfrak{i}_{\setminus\mathfrak{C}}) = 8a$. Thus, for the proliferation of an invading cluster of strategy B players, it must hold that for some external player π_t^B(external of $\mathfrak{C}$) $> 8a$. By equation A.2, we obtain the condition for proliferation as defined by $8a < (9-r)c + (r-1)d$. Since $d = a + \rho$ and $b = c + \rho$, this condition becomes $(2r - 10)\rho > (9-r)(a-b)$. Given $a \geq b$, this condition is violated for $r \leq 5$. In addition, a cluster of size smaller than five can never be sustainable, i.e., it never resists an invasion by the incumbent strategy, since $\pi_t^B(k) \leq 5c + 3d < 5a + 3$, where the latter is the smallest pay-off of an external not in $\mathfrak{C}$. Given the results for $\mathfrak{C}$ of size $r = 9$, we observe that the condition for a cluster of size six is sufficient and necessary for larger cluster sizes to expand.

2. case $a < b$: In this case, $\frac{\partial \pi_t^\zeta(i)}{\partial \mathfrak{F}_t^A} < 0$, holds for $\zeta = A, B$. In other words, the pay-off inferior strategy benefits from the abundance of individuals playing the pay-off dominant strategy in the neighborhood. Therefore only players neighboring the invading cluster have the highest pay-off. The condition for sustainability and proliferation by the incumbent strategy are thereby identical; both are determined by equation (A.1.). By considering the geometric structure of each case, it is proven that an invading cluster of a size smaller than four cannot persist. For a cluster of size three to proliferate it must hold $6c + 2d > 5a + 3b$, which is a contradiction of $\rho > 0$. A cluster of size four can only prevail if its structure is such that all of its players have the same pay-off (a square). In this case, it must hold that $5c + 3d > 6a + 2b$ and hence $\rho > 3(a-c)$. If it is not square shaped, we require $5c + 3d > 5a + 3b$, which is a contradiction of assumption $a > c$. For a cluster of size $r = 5$, the condition is $4c + 4d > 5a + 3b$ and hence $\rho > a - c$. Any larger mutant cluster always resists invasion, since $3c + 5d > 5a + 3b$. □

Proof of proposition 2:. For the two clusters $\mathfrak{C}_A$ and $\mathfrak{C}_B$ playing strategy A and B respectively, to be neighbors there are at least two players $n \in \mathfrak{C}_A$ and $m \in \mathfrak{C}_B$ with $n \sim m$. Define a player j, such that $\pi_t(j) = \max\{\pi_t(i) : i \in \mathfrak{C}_A\}$ and a player k, s.t. $\pi_t(k) = \max\{\pi_t(i) : i \in \mathfrak{C}_B\}$. This implies that player j has the highest pay-off in cluster A and player k in cluster B.

Concentrate first on clusters of size $r < 9$. It must be that j and k are external. Either $j \sim k$, or $l \sim j, k$ for some player l with positive probability. It must then be that $\pi_t(j) = \pi_t(k)$ for none of the players to switch strategy . The pay-offs of both players can be rewritten as $a(\mathfrak{r}_A - 1) + b(9 - \mathfrak{r}_A) = c(9 - r_B) + d(r_B - 1)$. Notice that r_B defines the size of $\mathfrak{C}_B$ and $\mathfrak{r}_A = r_A$, i.e., the size of $\mathfrak{C}_A$, if $a \geq b$ or $r_A < 3$. If $a < b$ and $r_A \geq 3$, then $\mathfrak{r}_A$ does not necessarily coincide with the size of $\mathfrak{C}_A$ as the pay-off function refers to the player in $\mathfrak{C}_A$ least connected to players of the same strategy. Solving the equation shows that for some values of ρ, a set of value pairs $(\mathfrak{r}_A, r_B)$ exists which fulfills the equation.[s] Define such a set of pairs for a given ρ as $R(\rho) = \{(\mathfrak{r}_A, r_B) : \pi_t(j, \mathfrak{r}_A) = \pi_t(k, r_B)\}$. For any two stable neighboring clusters C_A and C_B and a given ρ, it must hold that their value pair $(\mathfrak{r}_A, r_B) \in R(\rho)$. This occurs with zero probability for all such neighboring clusters under

[s]Yet some of these pairs are geometrically impossible, e.g., in the case where $\mathfrak{r}_A = 1$, $r_B = 2$ and $\rho = \frac{3}{5}(a-c)$. Both clusters have an identical highest pay-off, but only a cluster of size four or larger can fully surround a cluster of size one. 11 feasible pairs remain after ruling out the geometrically impossible pairs. These are $(1,8)$ if $\rho = 7(a-c)$; $(7,4)$ if $\rho = 3(a-c)$; $(2,7), (8,3)$ if $\rho = 5(a-c)$; $(8,7)$ if $\rho = \frac{(a-c)}{5}$; $(7,6), (4,3)$ if $\rho = \frac{(a-c)}{3}$; $(2,6)$ if $\rho = 2(a-c)$; $(3,6)$ if $\rho = (a-c)$; $(8,6)$ if $\rho = \frac{(a-c)}{2}$; while $(5,5)$ is stable for all values of ρ.

the conditions of initial random distribution and a large population. At least one cluster collapses triggering the instability of others. Thus, at least one strategy develops clusters of size nine with positive probability.

From proposition 1 we know that for $a \geq b$ and a stable cluster of size r_B playing strategy B, surrounded by cluster of size $\mathfrak{r}_A = r_A = 9$ playing A, it must hold that $8a \geq d(r_B - 1) + c(9 - r_B) \geq 7a + b$. From this we obtain the second part of the proposition. For $b > a$, $\frac{\partial \pi_t^A(i)}{\partial \mathfrak{F}_t^A} < 0$ and the cluster's maximum pay-off player j is always *external* if he plays A, where it must hold that $8d \geq a(\mathfrak{r}_A - 1) + b(9 - \mathfrak{r}_A) \geq 7d + c$, which only holds for $r_A = 1$. Hence, the second result of the proposition is obtained. By proposition 1, if $b > a$ then no cluster of size $r_B < 9$, playing the pay-off dominant strategy, is stable. □

Proof of proposition 3:. The initial distribution is, by assumption, balanced. Due to proposition 2 some clusters will have a size of nine with positive probability after some initial period of interaction. Consequently, for two such neighboring clusters $\mathfrak{C}_1$ and $\mathfrak{C}_2$ of size nine, and an external player $\mathfrak{e}_{\mathfrak{C}1} \in \mathfrak{C}_1$, assign two maximum players for each cluster, i.e., player k, such that $\pi_t(k) = \max\{\pi_t(i) : i \in (\mathfrak{N}(\mathfrak{e}_{\mathfrak{C}1}) \cap \mathfrak{C}_1) \cup \{\mathfrak{e}_{\mathfrak{C}1}\}\}$ and player j, such that $\pi_t(j) = \max\{\pi_t(i) : i \in \mathfrak{N}(\mathfrak{e}_{\mathfrak{C}1}) \cap \mathfrak{C}_2\}$.

Consider the case of $a \geq b$. By definition k is internal and j is external. For cluster size nine, it must either hold that $\pi_t(k) = 8a$, if $s_t(\mathfrak{e}_{\mathfrak{C}1}) = A$ or $\pi_t(k) = 8d$, if $s_t(\mathfrak{e}_{\mathfrak{C}1}) = B$. Since $j = \textit{external}$, his maximum pay-off is either $\pi_t(j) = c + 7d$, if $s_t(\mathfrak{e}_{\mathfrak{C}1}) = A$ or $\pi_t(j) = 7a + b$, if $s_t(\mathfrak{e}_{\mathfrak{C}1}) = B$. For $\mathfrak{e}_{\mathfrak{C}1}$ to switch strategy, it must hold $\pi_t(k) < \pi_t(j)$. Since $\rho > 0$, only $c + 7d > 8a$ occurs without contradiction. In the case where $a < b$, k is external if $s_t(\mathfrak{e}_{\mathfrak{C}1}) = A$ with maximum pay-off $\pi_t(k) = 7b + a < 7d + c$. Thus h_A cannot prevail by the assumption $a < d$. □

Proof of proposition 4:. This is a direct consequence of the former proofs. In the case of an unbalanced initial distribution, given that h_B pay-off dominates h_A, the pay-off dominant strategy takes over if $3\hat{c} + 5d > 8a$, while the risk dominant strategy prevails if $7a + b > 3\hat{c} + 5d$. For a balanced initial distribution, the constraints are $\hat{c} + 7d > 8a$ and $7a + b > 8d$. Thus, the risk dominant strategy prevailing in a balanced distributed population is $\rho < |\frac{c-a}{7}|$, where the latter is the marginal perceptible unit for a pay-off dominant strategy to invade a population. □

Recall that $a_i > c_i$ and $d_i > b_i$.

Proof of proposition 5:. Consider the two clusters $\mathfrak{P}$ and $\mathfrak{M}$, where the former is pure and the latter mixed. Define the set of players of type x as X and the set of type y players as Y. Assume without a loss of generality that $s_t(j) = A, \forall j \in \mathfrak{P}$. Similarly, assume for cluster $\mathfrak{M}$ that $s_t(j) = A, \forall j \in (\mathfrak{M} \cap X)$ and $s_t(j) = B, \forall j \in (\mathfrak{M} \cap Y)$. Since strategic change occurs only at the borders of clusters, we need only consider an external player $\mathfrak{e}$ such that $(\mathfrak{N}(\mathfrak{e}) \cap \mathfrak{P}) \neq \emptyset$ and $(\mathfrak{N}(\mathfrak{e}) \cap \mathfrak{M}) \neq \emptyset$.

First, assume that $\mathfrak{e} \in (\mathfrak{M} \cap X)$. Thus not only does $s_t(\mathfrak{e}) = A$, but also $s_t(i) = A, \forall i \in \mathfrak{N}_x(\mathfrak{e})$. Consequently, $s_{t+1}(\mathfrak{e}) = A$. The same holds if $\mathfrak{e} \in (\mathfrak{P} \cap X)$. Now assume that $\mathfrak{e} \in (\mathfrak{M} \cap Y)$, thus $s_t(\mathfrak{e}) = B$. Since, $s_t(i) = A, \forall i \in (\mathfrak{P} \cup \mathfrak{M}) \cap X$, it follows that $\pi_t(f) = 8a_i, \forall f \in (\mathfrak{P} \cap \mathfrak{N}_y(\mathfrak{e}))$ and $\pi_t(g) = 8\hat{c}_i, \forall g \in (\mathfrak{M} \cap \mathfrak{N}_y(\mathfrak{e}))$. As $a_i > \hat{c}_i$, it follows that $s_{t+1}(\mathfrak{e}) = A$. The same follows for $\mathfrak{e} \in (\mathfrak{P} \cap Y)$. Equivalent results are obtained for $s_t(h) = B, \forall h \in \mathfrak{P}$, since $\pi_t(f) = 8d_i > 8b_i = \pi_t(g)$. Consequently, an external player always chooses the strategy played by the pure cluster in his neighborhood if he has not previously done so. □

Proof of proposition 6:. We make the additional assumption that $a_i > b_i$. By propositions 5 and 2, it follows for an initially balanced distribution that, subsequent to an initial sequence of interactions, large pure clusters $\mathfrak{P}_A$ and $\mathfrak{P}_B$ of size $r_A, r_B = 9$ will play strategy A and B, respectively. Furthermore, by proposition 5, player types can be neglected with respect to the dynamics. Therefore, type specific subscripts are left out when not required.

Assume an external player $\mathfrak{e}_A \in \mathfrak{P}_A$ and an external player $\mathfrak{e}_B \in \mathfrak{P}_B$ with $\mathfrak{e}_A \sim \mathfrak{e}_B$. In order to cause $\mathfrak{e}_B$ to change strategy, it must be that $\pi_t(\mathfrak{e}_A) > \pi_t(\mathfrak{i}_B)$, given internal player $\mathfrak{i}_B \in \mathfrak{P}_B$ with $\mathfrak{i}_B \sim \mathfrak{e}_B$. Since $\mathfrak{i}_B$ is internal, it must follow that $\pi_t(\mathfrak{i}_B) = 8d_i$. Define $\eta_A = \#\left(\mathfrak{I}_t^B \cap \mathfrak{N}(\mathfrak{e}_A)\right)$. Generally, the pay-off of $\mathfrak{e}_A$ is then given by $\pi_t(\mathfrak{e}_A) = (8 - \eta_A)a_i + \eta_A b_i$, leading to condition $(8 - \eta_A)a_i + \eta_A b_i > 8d_i$. Similarly, define $\eta_B = \#\left(\mathfrak{I}_t^A \cap \mathfrak{N}(\mathfrak{e}_B)\right)$. There exists an internal player $\mathfrak{i}_A \in \mathfrak{P}_A$ with $\mathfrak{i}_A \sim \mathfrak{e}_A$. Consequently, $\pi_t(\mathfrak{i}_A) = 8a_i$ and to trigger a strategy switch of player $\mathfrak{e}_A$ it must hold that $(8 - \eta_B)d_i + \eta_B \hat{c}_i > 8a_i$.

For $i = 1, 2$, define $\max_i \{\lceil \eta_A \rceil_i\}$ and $\max_i \{\lceil \eta_B \rceil_i\}$ as the largest integer fulfilling each condition, respectively, given type specific parameter values. Since the probability that $\pi_t(\mathfrak{e}_A) > \pi_t(\mathfrak{i}_B)$ is proportional to $\max_i \{\lceil \eta_A \rceil_i\}$ and the probability that $\pi_t(\mathfrak{e}_B) > \pi_t(\mathfrak{i}_A)$ is proportional to $\max_i \{\lceil \eta_B \rceil_i\}$, both values determine the likelihood by which a cluster expands and thus the speed at which each type pushes the population towards the corresponding equilibrium. $\square$

Appendix B. Simulations

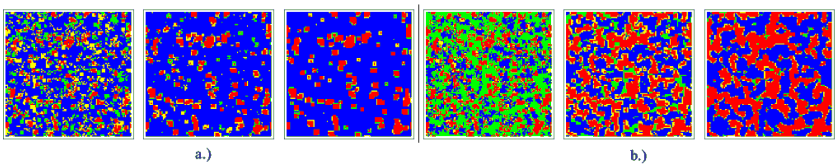

Fig. 2. The strategic distribution for three periods is: a.) $a_1 = a_2 = 6$, $b_1 = b_2 = 6$, $\hat{c}_1 = \hat{c}_2 = 0$, $d_1 = d_2 = 8$; b.) $a_1 = d_2 = 6$, $b_1 = \hat{c}_2 = 6$, $\hat{c}_1 = b_2 = 0$, $d_1 = a_2 = 8$. Using colour coding: blue: $s_i = A$, red: $s_i = B$, green: $s_1 = A, s_2 = B$, yellow: $s_1 = B, s_2 = A$.

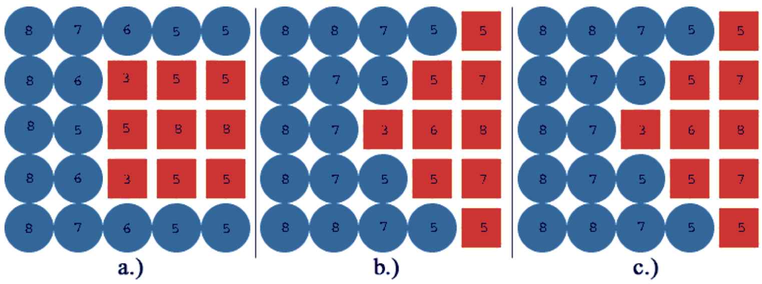

Fig. 3. The three variants of cluster edges – numbers indicate the number of players with the same strategy in the individuals neighborhood. Clusters are supposed to continue beyond the figure's frame.

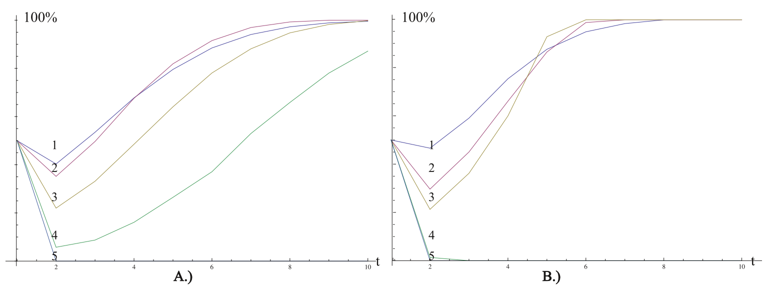

Fig. 4. Convergence for two different societies. Both figures show the number of *early seeders*: A.) 10.000 B.) 441. Radius is from one to five. The higher the radius, the higher the initial decrease in individuals playing the pay-off dominant strategy. Pay-offs: $a = 4$, $b = 0$, $c = 3$ and $d = 2$.

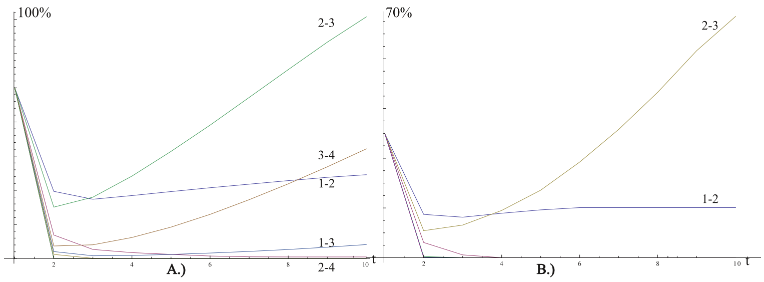

Fig. 5. Convergence for two different population sizes: A.) 10.000 B.) 441. The first value refers to the imitation radius and the second value, to the pay-off radius. Pay-offs: $a = 4$, $b = 0$, $c = 3$ and $d = 2$

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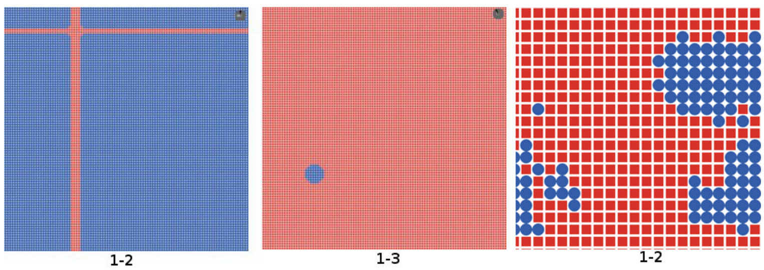

Fig. 6. Stable Radius Ratios: The first number indicates the radius of imitation and the second number the pay-off radius. First two figures refer to a population of size 10.000 and the third to a population of size 441.

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