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Contributions

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Macrodynamic Modeling of Innovation Equilibria and Traps

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Abstract: We study the interplay between the decision of firms to innovate and human capital. Based on a dynamic evolutionary model, we show that in the presence of a high stock of human capital, an advanced economy can remain caught in an "innovation trap". Following the literature on endogenous growth, R & D investments and human capital are modeled as strategic complements. Skilled workers increase productivity and enjoy a wage premium if they are employed in the R & D sector, while they receive the same wage as unskilled workers if they are employed in the production sector. We model the evolutionary dynamics of the share of innovative firms and human capital to determine the conditions under which an economy converges to a high, low or mixed state of innovation.

Keywords: behavioral macroeconomics, evolutionary dynamics, innovative firms, institutions and growth

JEL Codes: C73, E20, I25, O12, O30, O43

1 Introduction

The lack of growth in advanced economies, despite sustained human capital growth, such as in Southern European countries, is driven by a lack of research and development (R & D) and a decreasing marginal product of human capital. This

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can explain a puzzling development in some economically advanced countries, such as the Eurozone, during the last two decades. GDP per capita of the Southern EU countries has stopped converging towards the average level of Northern EU countries, despite illustrating a similar performance in terms of educational knowledge and human capital. Often, this lack of convergence has been attributed to misguided public finance and a high debt-to-GDP ratio. Other explanations refer to an excessively rigid labour market that obstructs the optimal allocation of inputs, or to an inefficient public administration and a poor financial system. Yet, Archibugi and Filippetti (2018) have demonstrated that a strong fall in public R & D shares may be a major cause behind stagnating growth and the lack of demand for human capital. The literature (Soete et al. 2020a, 2020b; Ziesemer 2020) shows that public and mission-oriented R & D can enhance growth in many countries (see Ziesemer 2019a, 2019b) in the presence of human capital growth. However less attention has been devoted to the relationship between technological innovation, human capital, and investment in decentralized economies, and to an even lesser extend has the literature focused on the externalities and complementarities of these processes. In the presence of the latter, complex co-evolutionary forces can lead to "innovation traps", even in economically advanced countries, that harm economic growth over time. This paper fills the gap by studying the factors that trap an economy in an equilibrium with a low degree of innovation as a result of a coordination failure between R & D activities and an existing large stock of human capital.[1] We illustrate that the lack of innovation can position an economy on a different growth path. Accordingly, countries in Northern Europe outperform those of the South in terms of R & D spending and technological investment.

On the basis of an evolutionary model in which firms choose whether or not to innovate, we show that long-term dynamics are determined by the complementarity between human capital and innovation. As a result, higher human capital increases the productivity of innovative firms, and in return, innovation reinforces the human capital stock via positive economic externalities. In this evolutionary scenario, the initial fraction of innovators can determine the degree of productivity and aggregate income in the long run. On the other hand, self-reinforcing dynamics limit the ability of economies to converge towards a state defined by a high level of economic growth.

1 Low-level equilibria or "traps" driven by low innovation in economically advanced countries can therefore be interpreted as self-reinforcing mechanisms that cause (relatively) low economic performance and are similar to the notion of poverty traps (see Azariadis and Starchuski 2005; Fukuda 2008; Grassetti et al. 2018; Guerrini 2006; Kikuchi and Vachadze 2015; Ponzano and Ricciuti 2018).

More precisely, we show that human capital is necessary but not sufficient to guarantee stable economic growth. In addition, the profits from innovation (R & D activities) and human capital need to be sufficiently high, i.e. above some endogenously defined threshold, to induce firms to innovate. This threshold value crucially depends on the level of the aggregate demand for skilled labor. The threshold value constitutes an inflection point that determines the critical capital stock required for economies to escape a poverty trap (see Azariadis and Drazen 1990; Askenazy and Le Van 1999; Akao et al. 2011). Conversely, if profits are either too small or entirely absent due to a low aggregate demand for skilled labor, firms do not innovate. As a consequence, an economy shifts towards a steady state that is characterized by a low degree of innovation, even in the presence of a large stock of human capital (see also the literature on "innovation equilibria": Danilov et al. 1997; Henkin and Polterovich 1991; Hritonenko and Yatsenko 2010).

In our model, a given number of firms individually decide whether to innovate and to hire skilled workers. Aggregate demand of skilled workers acts as a stimulus for innovation.[2] We show that the short-run equilibria are affected by the share of innovative firms, the stock of human capital and the level of aggregate demand determined by the degree of the output-capital ratio. The relationship between innovation and aggregate demand follows an accelerator principle which affects the co-evolution between innovation and human capital. We use this framework to determine the characteristics of the high and low innovation steady states.

The dynamics in the evolutionary model are modelled on the basis of replicator dynamics. The latter are frequently used to study dynamic social systems. While they have been initially used by biologists (Maynard Smith 1982; Nowak 2006; Page and Nowak 2002), replicator dynamics are increasingly employed by economists to understand the evolution of institutions and social norms (Boyd and Richerson 2005; Durlauf and Young 2001; Ille 2021; Lesourne et al. 2002), and are especially applied in the context of development economics (see Bischi et al. 2018; Gintis 2009; Gintis et al. 2005; Kopel et al. 2014). We further rely on earlier work by Accinelli and Carrera (2011, 2012) and

2 One may argue that technical progress creates or destroys jobs. Autor and Salomons (2017) cover 19 advanced economies between 1970 and 2007 and find that productivity growth has been mildly positive for aggregate employment. Their results suggest that – over long periods of time – the negative effect of productivity growth on employment in the same industry is more than compensated by positive "spillovers" in terms of expansions in other industries. Thus, structural change triggered by technical progress leads to a net gain in employment. Van Roy et al. (2018) investigate the impact of patents, instead of automation, on employment in Europe, and find an overall positive impact on labor. However the sectoral estimates show that the positive effects are concentrated in new and emerging sectors that are characterized by technological opportunities.

Sanchez Carrera (2019),[3] but instead of showing that low human capital accumulation and R & D can lead to low long-term economic growth, we demonstrate that despite a high level of human capital, the same may occur due to a suboptimal employment of human capital and a poor capacity for cutting-edge technological innovation.

In the following section we briefly discuss the main literature and stilized facts on which we base our model. In Section 3, we introduce the static model and derive the short-run characteristics of the economy. Section 4 extends the model to determine the evolutionary dynamics and the criteria for the longterm equilibria. Section 5 concludes.

2 Main Literature, Empirical Evidence and Stilized Facts

There is an extensive literature on the relationship between innovation, human capital and poverty traps. Recent research papers have analyzed the impact of monetary policies on human capital and innovation activities showing the existence of multiple equilibria and diverse states of economic growth (see Bondarev and Greiner 2019; Chu et al. 2019; Zheng et al. 2019). Traditionally, however, the literature mainly focuses on developing countries with a low stock of human capital and physical capital. In these countries, Tsiddon (1992) and Redding (1996) identify complementarities between human capital and R & D activities as a remedy for poverty traps. A low-growth trap can emerge endogenously in an overlapping generations model, in which asymmetric information and moral hazard play a central role (Tsiddon 1992). In Redding (1996), the interplay between firms' R & D investment and workers' human capital investment turn out to be the primary driver. Hence, the literature identifies R & D expenditure as one of the main drivers of productivity growth (Ganotakis 2012; Kuhlmann and Edler 2003; Romer 1990; Slaper et al. 2011; Storper and Scott 2009).[4] Secondly, endogenous

3 Sanchez-Carrera (2012, 2019) enriched the modeling of poverty traps by adding a spatial dimension and a strategic component, i.e. an agent's decision to acquire skills and/or to innovate depends on an evolutionary path behavior determined by strategic interactions.

4 The innovation literature stresses the role of individuals for innovation and the importance of recognizing innovation activities, economic growth, and monetary policies (see Annicchiarico and Pelloni 2019; Lundvall 2009). The theory suggests that innovation is a process of learning, both by individual agents and by organizations as a whole - through face-to-face communication (Asheim et al. 2007), team-work, absorptive capacity as well as education, occupation and work experience (Schneider et al. 2010).

growth models often relate the increase in human capital to firm innovations (Lucas 1993; Ponzano and Ricciuti 2018; Romer 1986, 1990). Barro (2001) shows that a high human capital stock tends to affect economic growth through two channels: human capital facilitates the absorption of superior technologies while it is often less able to adjust to shocks than real capital. Acemoglu (1997, 1998) proves that high-skilled labor and high-technology firms are complements. Moreover, firm performance is positively related to the quality of human capital and to the adaptation of technical and organizational change (Santos-Rodrigues et al. 2010; Storper and Scott 2009). In addition, Acemoglu (1998, 2002) shows that skill-biased technological progress and inequality between (and within) groups of skilled and unskilled workers can explain the obsolescence of their technology-specific or firm-specific skills due to technological progress. In short, this literature points to the strategic complementarities of R & D, human capital, and productivity, while a lack of either adequate human capital or innovation can be the cause of low economic growth.[5] Differences between the stock of human capital and the individual capacity of agents can then lead to asymmetric patterns of technology, productivity, and economic growth. This idea is formalized by Blackburn et al. (2000) based on an endogenous growth model which again demonstrates the existence of strategic complementarities between workers' investment in education and firms' investment in R & D. The authors argue that an economy can be caught in a "development trap" if both investments in technology and human capital are inactive.

It remains unclear whether slowing economic growth in advanced economies is caused by slowing technological progress. Similarly, the impact of reduced technological innovation on economic growth in advanced economies which already possess a high level of human capital accumulation requires further examination. The study of these questions is the principal aim of this paper. Empirical evidence shows (i) a decline in R & D activities, and (ii) a rise in human capital accumulation in advanced economies. In terms of the latter, Table 1 shows European trends of increased levels of educational participation and attainment.

5 In this vein, the World Bank and the OECD have recently pointed out that both human capital and innovation are important for sustained growth in advanced economies, and help emerging economies to catch up, see the Human Capital Project from the World Bank http://www.worldbank.org/en/publication/human-capital, Perspectives on Global Development OECD report, see http://www.oecd.org/publications/perspectives-on-global-development-22224475.htm.

Table 1: Highest Education Level Attained by Persons Aged 25–64 years, 2006–2018.

	Low (levels 0–2)				Medium (levels 3–4)				High (levels 5–8)			
	2006	**2011**	**2016**	**2018**	**2006**	**2011**	**2016**	**2018**	**2006**	**2011**	**2016**	**2018**
25–34 Years	17.5	14.9	8.1	7.6	40	36.5	37.7	36.3	42.5	48.6	54.3	56.2
35–44 Years	28.4	19.7	12.5	11.2	38.4	35	34.4	35	33.2	45.3	53.2	53.8
45–54 Years	41.5	31.1	22.9	19.5	34.2	36.4	38.1	37.6	24.3	32.5	39	42.9
55–64 Years	59.2	47.3	35.6	32.5	23.6	29	35.3	36.3	17.1	23.7	29.2	31.2
Total (25–64 Years)	**33.7**	**26.1**	**18.6**	**16.8**	**35.2**	**34.7**	**36.3**	**36.2**	**33.1**	**39.2**	**45.1**	**46.9**

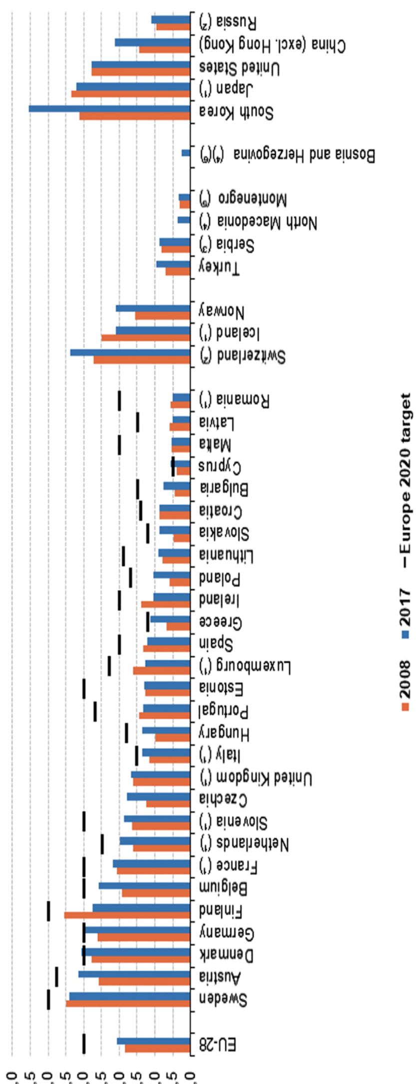
Source: Eurostat (2019).

The proportion of individuals attaining a higher degree qualification has increased significantly across all age groups. Higher level education has seen a 14 percentage point increase during a period of only 12 years.

Although the average level of human capital in the EU is high, more than half of the EU countries record a low level of innovation. Some countries are characterized by a long-lasting productivity slowdown and low aggregate demand.[6] Taking R & D intensity as a proxy for innovation, growth in the EU is lower than the Europe 2020 target (European Commission 2018, p. 81). Figure 1 shows that the EU's R & D expenditure by country in 2008 and 2017 is far from the R & D intensity target envisaged for 2020, and it is likely that the target will not be met.[7] OECD (2020) reports that Growth in R & D intensity was widespread across the majority of OECD countries in 2018, with the United States, Japan, Germany and Korea accounting for much of the increase. Real expenditure on R & D in the OECD area grew by 3.8% in 2018, mostly driven by the R & D performance behaviour of businesses, which accounted for more than 75% of this growth. The business

6 Recent empirical evidence provided by the European Commission, seems to confirm this implication. For innovation statistics on the EU, refer to https://ec.europa.eu/eurostat/statistics-explained/index.php/Innovation_statistics. The Euro zone grew at 1.88 percent in 2018, lower than the 2.3 percent gross domestic product (GDP) increase it had forecast in a previous estimate released in May 2018, and increased by 1.29 percent in 2019, see: https://data.worldbank.org/indicator/NY.GDP.MKTP.KD.ZG?end=2019&locations=XC&start=1961.

7 R & D intensity, i.e. the gross expenditure on R & D (GERD) as a percentage of gross domestic product (GDP), is one of several indicators used as targets to measure progress toward achieving the UN Sustainable Development Goal (SDG) 9 on innovation, https://sdgs.un.org/goals/goal9.



(¹) Break(s) in time series between 2008 and 2017.
(²) 2015 data (instead of 2017).
(³) 2009 data (instead of 2008).
(⁴) No data for 2008.
(⁵) 2011 and 2016 data (instead of 2008 and 2017).
(⁶) 2014 data (instead of 2017).

Source: Eurostat (online data code: t2020_20)

Figure 1: Gross domestic expenditure on R & D, by country, 2008 and 2017 (% of GDP).

enterprise sector, which generates 71% of all R & D performance in the OECD area, saw an R & D expenditure increase of 4.2% in 2018.[8]

Despite the relative lack of R & D intensity growth, educational attainment has risen steadily in advanced economies. Approximately one-third of the 25–34 year-olds have a tertiary education, while graduates with a doctoral degrees see positive growth rates.[9] Nevertheless, Pritchett (2001) pointed out that higher education often does not lead to higher wages indicating a waste of higher productivity due to higher education. Instead, the latter is, in part, used for 'directly unproductive activities'.[10] The data raise again the question of why the substantial increase in education levels in some of the EU Member States during the past two decades has not led to an acceleration of growth (Bosco and Valeriani 2018; Ermini et al. 2017).

Similarly, it is puzzling why neither the general rise of education attainment nor of skilled occupations have led to an overall employment growth.[11] Even where investment in human capital tends to produce more growth, it does not necessarily mean that jobs are available for new graduates. Figure 2 illustrates the percentage of total employment in high- and medium-high technology manufacturing and knowledge-intensive services in 2013 and 2018. European countries such as Greece, Italy, Spain, Portugal are below the European average at 44%.

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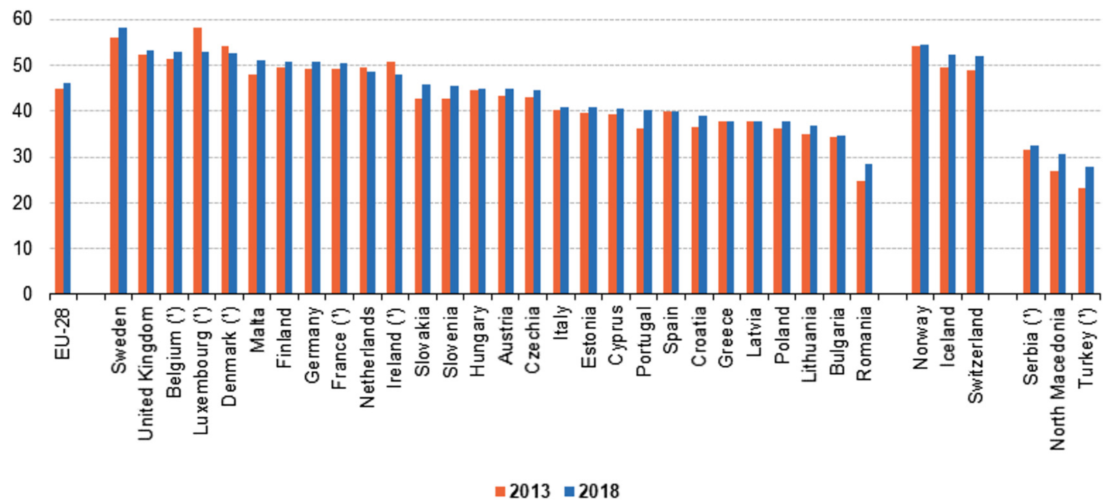
8 Other sources were R & D in the Higher Education (HE) which sector grew by 2.3%, and R & D expenditures in the Government sector which increased by 4.0% – the highest rate since 2009. Higher Education and Business Enterprise R & D have increased by 27 and 34%, respectively, compared to 2007 levels. Government R & D budget indicators show that R & D budgets rose by 5.6%, in real terms, in 2018, marking the highest increase since 2009 and pushing budget allocations for R & D above their 2009 peak. This is primarily due to growth in budgeted R & D support in Germany, Japan, the United Kingdom and the United States. Preliminary estimates also suggest a significant but more moderate increase in R & D budgets for 2019 (+2.47%). Growth in US R & D budgets came to a halt in 2019, while R & D budgets in France, Italy and Spain were still below 2007 pre-crisis levels. The United Kingdom and the United States only crossed the 2007 threshold in 2018.

9 This points to the PhD Factory problem defined by a lack of sufficient research positions and appreciation of PhD skills (high qualified workers) in the labor market (Cyranoski et al. 2011) during a time of positive growth of human capital accumulation.

10 The evidence used by Pritchett (2001) predates the strong acceleration of growth in emerging economies (EMEs).

11 Turner (2018) provides a provocative response to this question. He argues that some unproductive activities have become increasingly important in modern developed economies, especially in the service sector. Legal and accountancy services, some financial trading, regulators, etc. constitute examples in which a better educated workforce might not lead to higher welfare. Better defense lawyers on both sides of a case might lead to the same outcome. The same might happen between regulators and the representatives of the regulated industry (banking, insurance, telecommunication, utilities, etc.). The underlying thesis is that modern economies devote a larger share of labor resources to the distribution of excess profit.



(') Break(s) in time series between the two years shown.

Source: Eurostat (online data code: sdg_09_20)

Figure 2: Employment in high- and medium-high technology manufacturing sectors and in knowledge-intensive service sectors as a share of total employment.
Source: Eurostat, https://ec.europa.eu/eurostat/web/products-datasets/-/sdg_09_20.

Figure 3 compares a selection of northern EU and southern EU countries in terms of the Human Capital Index (Panel 3(a)) or the GDP per capita levels (Panel 3(b)) with respect to the number of Researchers engaged in innovation activities. Southern European countries as well as central European countries have a level of human capital that is comparable to Nordic countries, but a lower share of researchers employed in R & D. At the same time, northern EU countries tend to perform better than other european economies. Panel 3(a) and panel 3(b) suggest that not the accumulation of human capital, but rather the engagement in innovative activities generates growth, which supports the results of our dynamic model. A country's diminishing returns to human capital and the latter's specific depreciation if not utilized can cause a lack of engagement of highly skilled workers in innovative activities which in turn causes an economy to get stuck in a low-innovation trap (on human capital depreciation, see Dinerstein et al. 2020).

This paper adds to the literature in various respects. Our results complement Colonna (2014) by obtaining similar results but on the basis of a dynamic model in

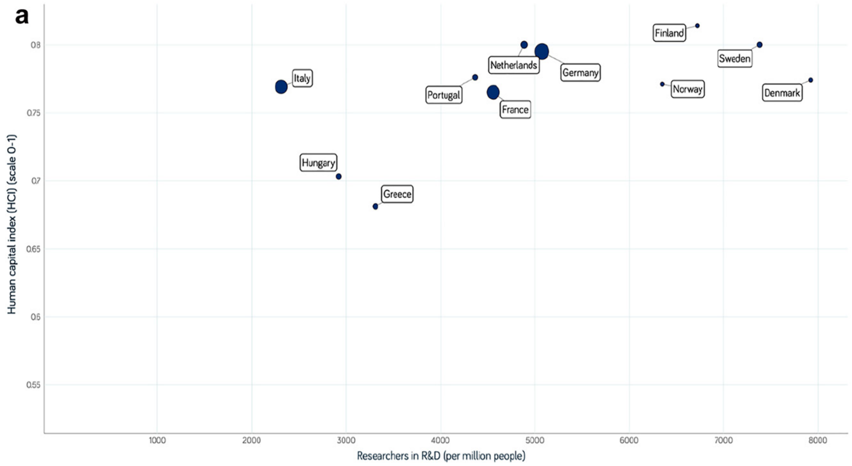


Figure 3: Researchers in R & D (per million people) are defined as the professionals who conduct research and improve or develop concepts, theories, models techniques instrumentation, software of operational methods. Source: UNESCO Institute for Statistics (http://uis.unesco.org/). (a) HCI calculates the contributions of health and education to worker productivity. The final index score ranges from zero to one and measures the productivity as a future worker of child born today relative to the benchmark of full health and complete education. (b) GDP per capita data are in constant 2010 U.S. dollars (inflation-adjusted). Source: World Bank, Human Capital Index (HCI) database https://databank.worldbank.org/data/source/human-capital-index.
Source: World Bank national accounts data, and OECD National Accounts data files.

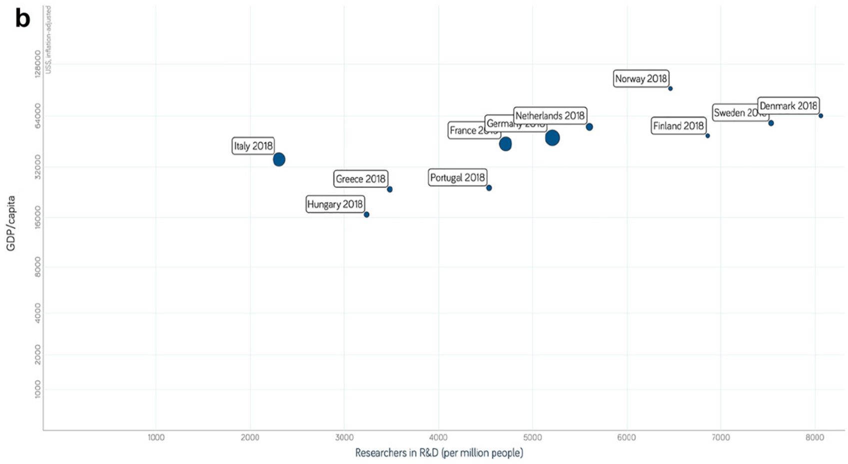


Figure 3: (continued)

which firms take strategic decisions based on the actions taken by other firms. We also provide additional reasons for a low-innovation equilibrium which occurs if the potential of innovation in an economy is low or if education exhibits strongly diminishing returns, as is supported by the empirical evidence above. Our model shares several attributes with the model developed in Acemoglu et al. (2018). We model endogenous technological change based on innovative and non-innovative firms that are linked through spill-over effects. However, in our model, the selection process between innovative and non-innovative firms is endogenous. We further add to the results in Aghion et al. (2017) by showing that innovative firms have a higher profit share. We also show that the wage of skilled workers is positively linked to the existing human capital stock, but negatively correlates with the potential an economy can achieve if all firms innovate.[12] Furthermore, and in contrast to García-Rodríguez and Sánchez-Losada (2014), we are able to show that imperfect signaling does not increase the likelihood of the low-innovation

12 Autor et al. (2020) document empirical patterns of US firms' data to assess whether the fall in the labour share can be explained by the rise of 'superstar firms'. Their hypothesis is that if globalisation or technological changes advantage the most productive firms in each industry, product market concentration will rise as industries become increasingly dominated by superstar firms with high profits and a low contribution of labour to the firm's value added and sales. As the significance of superstar firms increases, the aggregate labour-share of income will tend to fall. Such a hypothesis leads to empirically validated predictions in industry concentration and between-firm factor allocation.

trap, but will lead to a mixed equilibrium in which only some firms choose not to innovate. In addition, we show that an increase in the skilled labor force needs to exhibit sufficiently high marginal returns to encourage economic growth.

3 The Model

To analyze the impact of R & D and human capital on economic growth, we start our analyses using a static model. We assume that:

1. Two types of firms exist: innovative and non-innovative firms. Formally, we call the innovative firm an I−firm, otherwise it is an N−firm. Normalizing the total number of firms, we define $y \in [0, 1] \subset \mathbb{R}$ as the fraction of I−firms. The fraction of N−firms is then $1 - y$. Both cost of labor and productivity are higher for innovative firms.
2. Workers are employed for production, but skilled workers are hired only by innovative firms for R & D activities. Firms are price takers, and the wage rate is exogenous.
3. If both skilled and unskilled workers are employed in the production of goods their productivity is identical. Conversely, the wage of workers employed in R & D is higher than the wage earned by workers employed in direct production.
4. A firm's level of R & D is subject to cost optimization. It takes into account the average degree of human capital. We assume that R & D spending is positively correlated with the average skill of workers in the population.
5. If innovation occurs, human capital rises. It increases the wage share in the short-run, and the degree of the output-capital ratio. However, human capital does not change if innovation is absent.[13]

The production functions of the I−firm and N−firm are respectively given by:

$$\begin{aligned} q_{\mathrm{I}} &= f_{\mathrm{I}}(bK_{\mathrm{I}}, a_{\mathrm{I}}(\rho)L_{\mathrm{I}}), \\ q_{\mathrm{N}} &= f_{\mathrm{N}}(bK_{\mathrm{N}}, a_{\mathrm{N}}L_{\mathrm{N}}) \end{aligned} \tag{1}$$

13 Diebolt and Hippe (2018) empirically show that the long-run impact of human capital on innovation and economic development is a black box. Dima et al. (2018) empirically measured the impact of the knowledge economy on country competitiveness in the European Union by using Pearson coefficient and panel-data regression models. Their findings demonstrate the key role of innovation and education as the main determinants of European Union economic convergence. The authors conclude that this is particularly so in cases pertaining to the effect of technological investments on a company's performance or the relationship between innovation, human capital, and economic development, since human capital is an important driver of innovation and economic development.

where K_{I} and K_{N} denote the aggregate capital stock of I− firms and N−firms, respectively, b is the effectiveness with which K_i is used, a_i measures labor productivity. L_{I} and L_{N} define the total number of workers employed by I−firms and N−firms, respectively. The function $f_i : \mathbb{R}_+^2 \to \mathbb{R}$, $\forall i = \mathrm{I}, \mathrm{N}$ is strictly concave, differentiable and non-decreasing in each of its arguments.

The term ρ denotes the skilled labor to unskilled labor ratio ($\rho = X_{\mathrm{S}}/X_{\mathrm{I}}$). In other words, all I−firms demand X_{S} high-skilled workers for R & D activities as a fraction $\rho \in (0, 1)$ of their total workers hired for production X_{I}. This fraction ρ is determined by the firm's cost minimizing process which will be outlined below. Non-innovative N−firms demand X_{N} workers. Total demand for labor is then given by:

$$L_{\mathrm{I}} = X_{\mathrm{I}} + X_{\mathrm{S}} = (1 + \rho) X_{\mathrm{I}} \tag{2}$$

$$L_{\mathrm{N}} = X_{\mathrm{N}} \tag{3}$$

Hiring of $L_i, \forall i = \{\mathrm{I}, \mathrm{N}\}$ is indispensable for the production of each I− and N−firm, so that $q_i = \min[bK, 0] = 0$ for any K. On the other hand, R & D is not required for production, and thus $q_{\mathrm{I}} = \min\left[bK, aX_{\mathrm{I}}\right] > 0$ for $X_{\mathrm{S}} = 0$ and any positive X_{I} and K. Then, the optimum conditions of the innovative and non-innovative firms are,

$$\frac{K_{\mathrm{I}}}{L_{\mathrm{I}}} = \frac{a_{\mathrm{I}}(\rho)}{b},$$
$$\frac{K_{\mathrm{N}}}{L_{\mathrm{N}}} = \frac{a_{\mathrm{N}}}{b}.$$

For simplicity, we consider that the average quality of the human capital of skilled workers, $h > 0$, is known to all firms. Hiring skilled workers for R & D activities increases productivity, $a_{\mathrm{I}} = a_{\mathrm{I}}(\rho)$ with $a_{\mathrm{I}}'(\rho) > 0$ and $a_{\mathrm{I}}''(\rho) < 0$, while $a_{\mathrm{I}}(0) > 0$. The elasticity variation in terms of ρ on the labor productivity of I−firms is then proportional to the average quality of human capital embedded in skilled workers h and given by:

$$\varepsilon_\rho = \frac{a_{\mathrm{I}}' \rho}{a_{\mathrm{I}}} = \beta h \tag{4}$$

where $0 \le \varepsilon_\rho < 1$, and $0 \le \beta < 1$.

Firms are price takers, the wage for workers employed for production is exogenously determined at a level $W_X > 0$, and I−firms pay skilled workers hired for R & D activities a wage level $W_{X_{\mathrm{S}}} > W_X$. In other words, I−firms pay a skill premium, s, to skilled workers to avoid shirking behavior, as described by Acemoglu (2003) which we define as

$$s = \frac{W_{X_{\mathrm{S}}}}{W_X} > 1 \tag{5}$$

W_{X_S} and W_X are assumed to be constants over time. Since innovative firms hire X_S skilled workers, while N-firms hire none, the total share of skilled workers hired for R & D activities in the normalized population is given by $yX_S \geq 0$. We define the following:

Definition 1. The forgone potential of an economy is the difference between the degree of employment of skilled workers an economy can achieve if all firm innovate, and the actual rate of employment of skilled workers. It is given by:

$$f\left((1-y)\left(\rho X_I\right)\right), \tag{6}$$

where $f'(\cdot) > 0$, i.e. $\frac{\partial f}{\partial y} < 0$, $\frac{\partial f}{\partial \rho} > 0$.

The forgone potential tends to decrease as firms demand more skilled workers as illustrated in Figure 4. Thus, an "advanced" economy will be characterized by a large number of innovative firms, and profit of innovative firms increases with the proportion of the capital stock used to hire skilled workers for R & D activities. Since the forgone potential is proportional to the difference between the number of skilled labor employed by an I—firm and an average firm, it is also a relative measure of labor productivity of innovative compared to non-innovative firms, that is

$$f\left((1-y)\left(\rho X_I\right)\right) = a_I/a_N > 1. \tag{7}$$

Equation (7) indicates a complementarity between human capital and R & D activities. Labor productivity depends on the number of workers used for R & D and on their human capital. Returns to R & D increase with the level of human capital. For simplicity, assume that labor productivity in N—firms is normalized to 1.

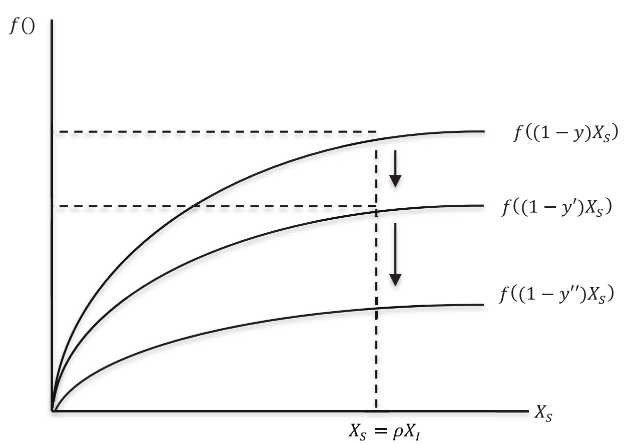


Figure 4: The forgone potential is negatively correlated with the share of innovative firms: $y < y' < y''$.

Therefore, $f(\cdot) = a_I$. Figure 4 draws the forgone potential for different shares of innovative firms, $y < y' < y''$ and a given demand of skilled workers, $X_S = \rho X_I$.

From Eqs. (2) and (5), we obtain that the relative cost of hiring skilled workers for R & D activities by I−firms is $(1 + s\rho) > 0$. Let Y_I and Y_N denote the respective total income share distribution. Given price P, $Q = yq_I + (1 - y)\, q_N$ and total revenue PQ, the income share is distributed according to

$$Y_I = \left(\frac{W_X}{P}\right) X_I + \left(\frac{W_{X_S}}{P}\right) X_S + r_I K_I, \tag{8}$$

$$Y_N = \left(\frac{W_X}{P}\right) X_N + r_N K_N, \tag{9}$$

where r_I and r_N define the profit rates of I− and N−firms, respectively.

Substituting Eqs. (2) and (5) into (8) and (9), we redefine the income share distribution as

$$Y_I = \upsilon X_P\,(1 + s\rho) + r_I K_I \tag{10}$$

$$Y_N = \upsilon X_N + r_N K_N \tag{11}$$

Remark 1. Based on the optimality conditions for innovative and non-innovative firms, the optimal output level of an innovative firm is

$$q_I = a_I(\rho) L_I = a_I(\rho)(X_I + X_S).$$

Let $\upsilon = (W_X/P) \in (0, 1)$ define the real wage of workers for production. Hence using (5), the profit of an innovative firm (where profit itself is defined as the difference between the quantity produced through skilled and unskilled labour minus the wages paid to skilled and unskilled worker) is given by:

$$\Pi_I = a_I(\rho)(X_I + X_S) - \frac{W_X}{P} X_I - \frac{W_{X_S}}{P} X_S = a_I(\rho)(1 + \rho) X_I - (1 + s\rho)\upsilon X_I.$$

The wage or unit labor cost of output is a variable unit cost of production for I−firms and is thus given by[14]

$$\sigma_I = \upsilon \frac{(1 + s\rho)}{a_I(\rho)(1 + \rho)}. \tag{12}$$

14 This measures the total compensation of employees (wages and premia). The wage share of income declines when wages grow at a lower rate than productivity, i.e. the amount of output per hour of work.

An I—firm chooses $\rho \in (0, 1)$ by minimizing the wage share (equivalently, the firm chooses ρ to maximize its profit). From $\partial \sigma_{\mathrm{I}} / \partial \rho = 0$ and Eq. (4), solving for ρ, we obtain

$$\rho = \frac{\beta h}{s(1 - \beta h)}, \tag{13}$$

Given that s is constant, the hiring of skilled workers for R & D activities will be determined by the average quality of human capital, thus $\partial \rho / \partial h > 0$. Substituting (13) into (12) yields the labor share for I—firms as a function of the quality of human capital

$$\sigma_{\mathrm{I}}(h) = \frac{\upsilon}{(1 - \beta h)\, a_{\mathrm{I}}}. \tag{14}$$

Using function (6) and (7), we rewrite Eq. (14) as a function of y and h,

$$\sigma_{\mathrm{I}}^{*}(y, h) = \frac{\upsilon}{(1 - \beta h)\, f((1 - y)(\rho X_{\mathrm{I}}))}, \tag{15}$$

Recall that $0 < \upsilon < 1$. The labor share for N—firms is given by

$$\sigma_{\mathrm{N}}^{*} = \upsilon \tag{16}$$

The profit share of firms is then[15]

$$\pi_{\mathrm{I}}^{*}(y, h) = 1 - \left[\frac{\upsilon}{(1 - \beta h)\, f((1 - y)(\rho X_{\mathrm{I}}))}\right] \tag{17}$$

$$\pi_{\mathrm{N}}^{*} = (1 - \upsilon) \tag{18}$$

For $\pi_{\mathrm{I}}^{*} > \pi_{\mathrm{N}}^{*}$, we require that $(1 - \beta h) > 0$. In the short-run, the labor share and profits are constant for N—firms, while they vary for I—firms according to the proportion of I—firms, i.e.

$$\frac{\partial \pi_{\mathrm{I}}^{*}(y, h)}{\partial y} = -\frac{\upsilon \rho X_{\mathrm{I}} f'(\cdot)}{(1 - \beta h) f(\cdot)^{2}} = -\frac{\upsilon}{f(\cdot)} \frac{\epsilon_{y}}{(1 - y)} < 0, \tag{19}$$

for all $y \in [0, 1)$, where $\epsilon_{y} = f'(\cdot) \frac{(1-y)(\rho X_{\mathrm{I}})}{f(\cdot)} = f'(\cdot) \frac{X_{\mathrm{S}} - y X_{\mathrm{S}}}{f(\cdot)}$ denotes the elasticity of the direct labor productivity differential for the production of goods in relation to the proportion of innovation spending given by $\epsilon_{y} > 0$.

The rate of profit for each firm type is given by:

$$r_{\mathrm{I}}^{*}(y, h) = \pi_{\mathrm{I}}^{*}(y, h)\, u, \tag{20}$$

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15 This measures direct and/or indirect payments to employees which depend on a firm's profitability in addition to employees' regular wages.

$$r_{\mathrm{N}}^{*}=\pi_{\mathrm{N}}^{*}u, \tag{21}$$

where $u=Q/K$ denotes the degree of the output-capital ratio. The latter indicates the additional unit of capital or investment needed to produce one unit of output. A lower capital output ratio is desirable since only a low level of investment is needed to generate economic growth. In this case, capital is very productive. Given that the total quantity produced, Q, is a weighted average of actions I and N, i.e. $Q=yq_{\mathrm{I}}+(1-y)\,q_{\mathrm{N}}$, the degree of the output-capital ratio is also uniformly distributed, $u_{\mathrm{I}}=u_{\mathrm{N}}=u=Q/K$ (as has also been shown in Madsen et al. 2012 and Franke 2017).

3.1 Short-Run

In the short-run, capital stock K, nominal wage W_X, prices P, labor productivity a_{I} and a_{N} for each type of firm as well as the firm distribution, y, are exogenous. Output-capital ratio and economic growth adjust to balance the goods market.

We assume that the costs of human capital accumulation are paid (at least in part) by the government through a constant tax rate τ on income. The aggregate investment in human capital by the government, normalized by the capital stock, is:

$$g_h=\frac{H}{K}=\tau\,(Y_{\mathrm{I}}+Y_{\mathrm{N}})\,u. \tag{22}$$

For simplicity, we assume that workers consume their entire salary, and firms use their entire profit to "invest" in capital. This is because "saving" is defined as the difference between the households' disposable income and consumption, and it should be reflected in the behavior of a household. Thus, the aggregate saving rate of the economy, g_{s}, is defined as the net profit after firm taxation:

$$g_{\mathrm{s}}=\frac{S}{K}=(1-\tau)\left(yr_{\mathrm{I}}^{*}+(1-y)r_{\mathrm{N}}^{*}\right)=(1-\tau)\left[y\pi_{\mathrm{I}}^{*}\,(y,h)+(1-y)\,\pi_{\mathrm{N}}^{*}\right]u. \tag{23}$$

Assume that the firms' expected investment in proportion to the capital stock is identical for both actions I and N and given by[16]

$$g_k=\eta_0+\eta_1(1-\tau)r^{\mathrm{e}}+\eta_2u^{\mathrm{e}}, \tag{24}$$

where r^{e} and u^{e} are the expected (average common) rates of profit and the expected degree of the output-capital ratio, respectively, and η_0, η_1, and η_2 are positive parameters. Silveira and Lima (2016) assume that the (average) rate of capital

16 This linear homogeneity assumption, although not necessarily realistic, allows to isolate the firm-level investment which is driven exclusively by expected future profit shares.

accumulation depends on the (average) expected profit rate, since the current rate of profit is treated as an index of expectations of future profitability. Therefore, we assume that the rate of accumulation positively depends on the (average) output-capital ratio. Assuming that current rates of profit and the output-capital ratio are proxies of their expected rates, it follows from $u_{\mathrm{I}} = u_{\mathrm{N}} = u = Q/K$ and thus,

$$u^{\mathrm{e}} = yu_{\mathrm{I}} + (1-y)u_{\mathrm{N}} = u, \tag{25}$$

The expected average rate of profit is then

$$r^{\mathrm{e}} = yr_{\mathrm{I}} + (1-y)r_{\mathrm{N}} = \left[y\pi_{\mathrm{I}}^{*}(y,h) + (1-y)\,\pi_{\mathrm{N}}^{*}\right] u. \tag{26}$$

Substituting the values of (25) and (26) into (24), the expected investment is

$$g_K = \eta_0 + \eta_1(1-\tau)\left[y\pi_{\mathrm{I}}^{*}(y,h) + (1-y)\,\pi_{\mathrm{N}}^{*}\right] u + \eta_2 u. \tag{27}$$

We assume that taxes are entirely spent on human capital accumulation, $T = \tau(Y_{\mathrm{I}} + Y_{\mathrm{N}}) = H$, and total saving is equal to capital stock investment, where $S = Q - T - C$, leading to $g_{\mathrm{s}} = g_k$. Equating expressions (23) and (27) and solving for u, we obtain the equilibrium value of the degree of the output-capital ratio in the short-run as a function of y and h

$$u^{*}(y,h) = \frac{\eta_0}{(1-\eta_1)(1-\tau)\left[y\pi_{\mathrm{I}}^{*}(y,h) + (1-y)\,\pi_{\mathrm{N}}^{*}\right] - \eta_2}, \tag{28}$$

where the denominator of the equation must be positive, which means that savings should be more sensitive to changes in u than investments. Therefore, we require $1 > \eta_1$ and $(1-\eta_1)\left[y\pi_{\mathrm{I}}^{*}(y,h) + (1-y)\,\pi_{\mathrm{N}}^{*}\right] - \eta_2 > 0$ as a stability condition for the model. By substituting (28) into (23), we obtain the equilibrium value of the economic growth rate

$$g^{*}(y,h) = (1-\tau)\left[y\pi_{\mathrm{I}}^{*}(y,h) + (1-y)\,\pi_{\mathrm{N}}^{*}\right] u^{*}(y,h). \tag{29}$$

Notice that the short-run economic growth rate, $g^{*}(y,h)$, depends on:

(i) the firms' profit sharing functions (after taxation) and the I—firms profile distribution. We have $\frac{\partial g^{*}(y,h)}{\partial y} > 0$

(ii) the optimal degree of the output-capital ratio of innovative firms (which is a proxy of the aggregate demand). We have $\frac{\partial g^{*}(y,h)}{\partial u^{*}} > 0$

(iii) the average quality of human capital. We have $\frac{\partial g^{*}(y,h)}{\partial h} > 0$

The economic growth rate in Eq. (29) increases with the share of innovators (i.e. R&D activity), the existing level of human capital, and the consequential output-capital ratio $u^{*}(y,h)$. To better understand the co-evolution of the firms' decision to innovate, y, and the average quality of human capital embedded in skilled workers, h, we consider both variables to be endogenous and develop an evolutionary model in the following. This will enable us to determine the factors that dynamically drive both y and h and consequently, economic growth in the long run.

4 The Evolutionary Dynamics

In the evolutionary model, the distribution of innovative and non-innovative firms varies according to the evolutionary dynamics based on the payoffs of innovative firms relative to non-innovative firms. We assume that a firm observes the payoffs of a random sample of firms in each period of time. If, on average, firms pursuing a different strategy obtain a higher payoff, the former firm switches to this strategy with positive probability and a firm's likelihood to switch increases with the difference in expected payoffs. Notice that we approximate the dynamics on the basis of the expected payoff for each strategy. However, since biased samples and matching noise, while being rare, may occur, we assume that firms choose to opt for a strategy that is not optimal with a very low probability. Consequently, we will assume in our stability analysis in Section 4.1 that the population of firms is slightly fluctuating around the predicted state. Since the frequency of non-innovative firms is defined by $1-y$, it suffices to study the frequency of innovative firms. Assuming that updating occurs slowly to guarantee that savings equal investment according to Eq. (28), the trajectory of y over time is given by the replicator dynamic:[17]

$$\dot{y} = y(1-y)\left[r_{\mathrm{I}} - r_{\mathrm{N}}\right] = y(1-y)\left[\pi_{\mathrm{I}}^{*}(y,h) - \pi_{\mathrm{N}}^{*}\right] u(\pi_{\mathrm{I}}^{*}, \pi_{\mathrm{N}}^{*}), \tag{30}$$

following Eqs. (17), (18), (20), (21) and (26). In other words, the replicator dynamic (30) is determined by the difference in the rate of profit. The replicator dynamic (30) states that the share of innovative firms increases (decreases) when the profit of I−firms is above (below) the average profit of N−firms. In the long-run, it is assumed that the short-run equilibrium values of the rates of capacity utilization (as a proxy for the aggregate demand) are maintained, whereas the levels of productivity varies with the frequency distribution of the innovative strategy according to (30). Substituting Eqs. (17) and (18) and the equilibrium value of

17 The general replicator dynamic is of the form $\dot{x}_i = x_i(f_{xi} - \phi)$, where ϕ denotes the average payoff in a population, x_i the frequency of some strategy i and f_{xi} the associated payoff. For two strategies, the replicator simplifies to the form given by (30).

$u^*(y,h)$ in (30), we obtain:

$$\dot{y} = y(1-y)\,\upsilon\left[1 - \frac{1}{(1-\beta h)\, f((1-y)(\rho X_{\mathrm{I}}))}\right] u^*(y,h). \tag{31}$$

The right hand side of (31) varies not only with y, but also with h.

Further, note that the growth rate of human capital, $\dot{h}$, is a function of the existing stock of human capital h, and the share of innovative firms y. We can write

$$\dot{h} = \gamma_0 - \gamma_1 h + \gamma_2 y, \tag{32}$$

where γ_0, γ_1, and γ_2 are positive constants. We have,

- γ_0 is simply the natural accumulation of human capital, $\dot{h}$, due to basic education and experiences (Barro 2001).
- γ_1 indicates the strength of the diminishing marginal return of human capital on $\dot{h}$ (i.e. human capital depreciation). This assumption is important and critical for the following results and therefore requires further elaboration. Human capital depreciates if skills are unused, i.e. when skilled workers are unemployed (Kehoe, Midrigan and Pastorino 2019; Ljungqvist and Sargent 1998; Neal 1995; Oreopoulos, Von Wachter and Heisz 2012).[18] Similarly, human capital illustrates diminishing returns due to obsolescence (Rebelo 1991; Psacharopoulos and Patrinos 2018). The latter is measured by experience-earnings profiles (Neuman and Weiss 1995). However, Weinberg (2002) found that new technologies may also complement the existing skills of the workers, which implies that having experience with obsolete technology may improve a worker's ability to use a new technology.
- γ_2 is the marginal impact of an increase in demand by I-firms on $\dot{h}$. Hicks-neutral technological change or innovation at firm-level leads to hiring more skilled workers (see the literature that considers firm-level skill demand and technological change, such as Caroli and Van Reenen 2001, Bresnahan et al. 2002).

Hence, an increase in the current level of human capital decreases its growth rate (since incremental growth becomes smaller if the existing stock is already developed) whereas innovative firms foster the accumulation of human capital.

18 See also Dinerstein et al. (2020) who show that macroeconomic models incorporating human capital depreciation tend to use a variety of parameterizations. They offer a depreciation rate estimate that leverages quasi-random variation in the periods workers are not employed.

4.1 Evolutionary Dynamics without Idiosyncratic Firm Choice

In this section, we study the long-term behaviour of the dynamic system. Firms are exposed to the decision of other firms and update their response to the previous innovation decision of other firms. In doing so, other firms are exposed to a changing environment and on their part, update the decision of whether to innovate. Firms will then update their strategy based on their observation of a limited number of other firms. Due to a firm's limited sample, we assume that it chooses a strategy that is not a best response to the entirety of all other firms with a very low probability. We will contrast the results of this section with a situation in which firms are sporadically motivated to innovate or to forgo innovation due to external idiosyncratic reasons. In the latter scenario, firms are exposed to a noisy environment in which idiosyncratic choices of other firms affect the individual decision of each firm and thus, the stability of the equilibria in the long-run.

The long-run equilibria require that $\dot{y} = 0$ and $\dot{h} = 0$. From Eq. (31), it follows that this occurs at $y = 0$ or $y = 1$. Solving for Eq. (32), we have two pure strategy equilibria:

1. $\left(0, \bar{h}_1\right)$ for $\bar{h}_1 = \frac{\gamma_0}{\gamma_1} > 0$ and $y = 0$,
2. $\left(1, \bar{h}_2\right)$ for $\bar{h}_2 = \frac{(\gamma_0+\gamma_2)}{\gamma_1} > 0$ and $y = 1$.

The nulls of Eqs. (30) and (31) define an interior equilibrium in coexisting strategies where (17) equals (18). For $u^*(\cdot) > 0$, this interior equilibrium is defined by the solution to $\xi\,(y^*) = 0$, given by:

$$\xi(y^*) = 1 - \frac{1}{\left(1 - \beta\bar{h}^*\right) f\left(\left(1 - y^*\right)\left(\rho X_{\mathrm{I}}\right)\right)} \tag{33}$$

for $\bar{h}^* = (\gamma_0 + \gamma_2 y^*)/\gamma_1$. If condition (33) is equal to zero, firms are indifferent between being innovative or non-innovative, since both strategies are equally profitable.

Lemma 1. *Given that* $(h, \gamma_0, \gamma_1, \gamma_2) > 0$, $(1 - \beta h) > 0$, $f((1 - y)\,\rho X_I) > 1$, $\frac{\partial \dot{h}}{\partial y} > 0$, *and* $f(.)$ *being strictly monotonically increasing in its argument,* $\forall y \in (0, 1)$, *i.e.* $\frac{\partial f(.)}{\partial y} < 0$. *Then, an equilibrium value* $y^* \in (0, 1) \subset \mathbb{R}$ *exists, such that*

$$f\left(\left(1 - y^*\right)\left(\rho X_{\mathrm{I}}\right)\right) = 1/\left(1 - \beta\bar{h}^*\right),$$

if and only if $\xi(y = 0) > 0$ *and* $\xi(y = 1) < 0$.

Proof. We have:

$$\xi(y=0) = 1 - \frac{1}{(1-\beta\gamma_0/\gamma_1)\, f(\rho X_{\mathrm{I}})}$$

$$\xi(y=1) = 1 - \frac{1}{(1-\beta(\gamma_0+\gamma_2)/\gamma_1)\, f(0)}$$

and thus, $\xi(0) > \xi(1)$. For the existence of the interior equilibria, we require two conditions:

$$\xi(0) \Rightarrow f(\rho X_{\mathrm{I}}) > \frac{\gamma_1}{\gamma_1 - \beta\gamma_0} \tag{34}$$

$$\xi(1) \Rightarrow f(0) < \frac{\gamma_1}{\gamma_1 - \beta(\gamma_0+\gamma_2)}. \tag{35}$$

□

In essence, since $f(.)$ is strictly decreasing in y and $1/(1-\beta\bar{h}^*)$ is strictly increasing in y, both functions must intersect at a single point y^*. If and only if $y^* \in (0,1)$, we obtain an interior equilibrium requiring that (33) intercepts the abscissa at $y^* \in (0,1)$.

Lemma 1 states that the maximum forgone potential (which occurs if none of the firms innovates) must be higher than the normalized marginal impact of an increase in human capital on itself, i.e. the benefits from innovation need to outperform the negative impact on future human capital growth at the low equilibrium. At the high equilibrium, this condition is reversed and the gains in the all I-firms equilibrium must be sufficiently small. If the latter is not the case, such that there is sufficient potential even if all but one firm innovate, the economy will converge to the all I-firms equilibrium. If, on the other hand, the potential of an economy is too low, it will remain in the low equilibrium. This leads to the following proposition:

Proposition 1. *For the dynamical system* (31) *and* (32) *of innovation and human capital and by conditions* (34) *and* (35), *it holds that:*

- *If* (34) *is violated, equilibrium* $(y,h) = (0,\bar{h}_1)$ *is stable while* $(y,h) = (1,\bar{h}_2)$ *is a saddle point. The economy evolves to a low-innovation trap.*
- *If* (35) *is violated, equilibrium* $(y,h) = (0,\bar{h}_1)$ *is a saddle point while* $(y,h) = (1,\bar{h}_2)$ *is stable. The economy evolves to a high-level equilibrium of sustained economic growth characterized by only innovative firms and a large amount of human capital accumulation.*
- *If neither* (34) *nor* (35) *are violated, both pure strategy equilibria are saddle points, and a stable interior equilibrium exists.*

Proof. The qualitative analysis of the two pure strategy equilibria is done by looking at the dynamic system composed of Eqs. (31) and (32) with state space:

$$\Theta = \left\{(y,h) \in \mathbb{R}^2 \colon 0 \le y \le 1, 0 < h < h^{\max}\right\}.$$

The condition for the boundedness of h is that:

$$\frac{1}{\beta} > h^{\max} > \frac{\gamma_0 + \gamma_2}{\gamma_1} \Rightarrow \dot{h}|_{h^{\max}} < 0.$$

The population of firms can either converge to one of the pure equilibria or to an interior attractor (i.e. a unique equilibrium or limit cycle). The local stability properties of the equilibria can be analysed by studying the qualitative behavior of the system's linearization near the equilibria. The eigenvalues of the respective Jacobian matrices then determine an equilibrium's stability. The Jacobian matrix at $\left(0, \bar{h}_1\right)$ is given by:

$$J\left(0, \bar{h}_1\right) = \begin{bmatrix} \upsilon\left[1 - \frac{1}{\left(1-\beta\bar{h}_1\right) f\left(\rho X_{\mathrm{I}}\right)}\right] u^*\left(0, \bar{h}_1\right) & 0 \\ \gamma_2 & -\gamma_1 \end{bmatrix}$$

with eigenvalues

$$E_{10} = \upsilon\xi(0)u^* \quad \text{and} \quad E_{20} = -\gamma_1 < 0. \tag{36}$$

The Jacobian matrix for $\left(1, \bar{h}_2\right)$ is given by:

$$J\left(1, \bar{h}_2\right) = \begin{bmatrix} -\upsilon\left[1 - \frac{1}{\left(1-\beta\bar{h}_2\right) f(0)}\right] u^*\left(1, \bar{h}_2\right) & 0 \\ \gamma_2 & -\gamma_1 \end{bmatrix}$$

with eigenvalues

$$E_{11} = -\upsilon\xi(1)u^* \quad \text{and} \quad E_{21} = -\gamma_1 < 0. \tag{37}$$

In the presence of an interior equilibrium, both E_{10} and E_{11} are positive, and thus, both pure strategy equilibria are saddle points. Following Hilborn (1994), the divergence of $\dot{y}$ and $\dot{h}$ is given by $\text{div}(\dot{y}, \dot{h}) = \frac{\partial \dot{y}}{\partial y} + \frac{\partial \dot{h}}{\partial h}$. Since $\frac{\partial \dot{y}}{\partial y} < \gamma_1$ and thus, $\text{div}(\dot{y}, \dot{h}) < 0$, the system must have at least one attractor. Since the pure equilibria are unstable and the interior equilibrium is unique, the dynamical system (31) and (32) is stable at the interior equilibrium. □

Note that since E_{20} and E_{21} are negative, the pure strategy equilibria are saddle points if a mixed strategy equilibrium exists. Saddle points are characterized by an unstable and a stable manifold. We ignore the latter, since only if the system initially starts at a point exactly on the stable manifold, it converges to the saddle point. However, it is very unlikely that this is initially the case in a continuous state space. Furthermore, we assumed that firms and workers sample their environment and therefore may base their choice on a biased sample with a very low probability. In this case, the system is fluctuating around the trajectory defined by the stable manifold of the unperturbed system, and therefore will eventually diverge from the manifold.

If (34) is violated and thus $\xi(0) < 0$, only $\left(0, \bar{h}_1\right)$ is an attractor, while if (35) is violated and thus $\xi(1) > 0$, only $\left(1, \bar{h}_2\right)$ is an attractor. Figure 5 shows a numerical simulation of the three possible dynamics.

The results in Proposition 1 have direct implications on the equilibrium growth rate in Eq. (29). If condition (34) is violated, the equilibrium values for human capital is $\bar{h}_1 = \gamma_0/\gamma_1 > 0$ and firms decide not to innovate. The resulting equilibrium growth rate is then given by

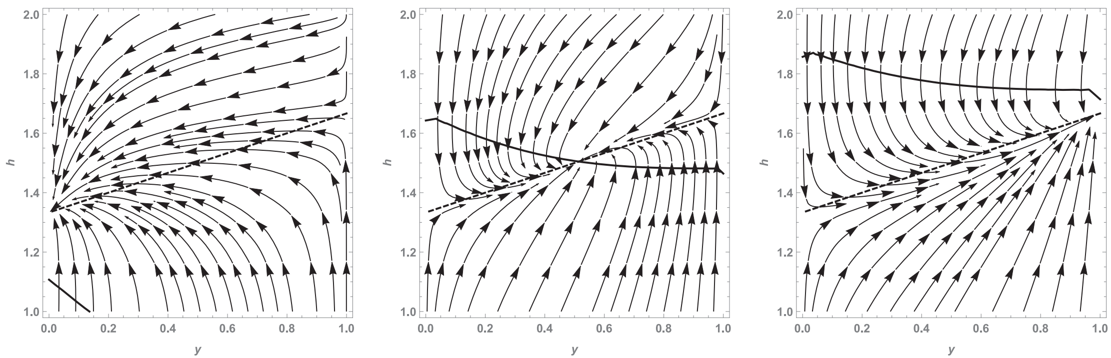
$$g^*\left(0, \bar{h}_1\right) = \frac{(1-\tau)\pi_N^*\eta_0}{(1-\eta_1)(1-\tau)\pi_N^* - \eta_2} = \frac{(1-\tau)(1-\upsilon)\eta_0}{(1-\eta_1)(1-\tau)(1-\upsilon) - \eta_2} \tag{38}$$

Equivalently, if condition (35) is violated, all firms innovate and the equilibrium value for human capital is $\bar{h}_2 = (\gamma_0 + \gamma_2)/\gamma_1 > \bar{h}_1$ and the ensuing equilibrium growth rate is

$$\begin{aligned} g^*\left(1, \bar{h}_2\right) &= \frac{(1-\tau)\pi_I^*\left(\bar{h}_2\right)\eta_0}{(1-\eta_1)(1-\tau)\pi_I^*\left(\bar{h}_2\right) - \eta_2} \\ &= \frac{(1-\tau)((1-\beta\bar{h}_2)f(0)-\upsilon)\eta_0}{(1-\eta_1)(1-\tau)((1-\beta\bar{h}_2)f(0)-\upsilon) - \eta_2(1-\beta\bar{h}_2)f(0)} \end{aligned} \tag{39}$$

and under the given parameter constraints, we have $g^*\left(1, \bar{h}_2\right) > g^*\left(0, \bar{h}_1\right)$. In the absence of an interior equilibrium, the economy converges either to a state in which it uses its full innovation potential, or is trapped in a state of no innovation. The state of full innovation is achieved if either the diminishing marginal returns of human capital, given by γ_1, are low, while the natural rate of human capital accumulation, γ_0, and the impact of an increase in innovative firms on human capital, γ_2, are strong, or if the forgone potential at $y = 0$ is high. Thus, the state is more likely to appear in friction-less markets with no significant recruiting costs, excess labor demand, and consequently strong incentives to innovate and higher skilled labor.

The opposite relation holds for the innovation trap which can be caused by a low innovative potential of an economy and existing human capital stock due to strong diminishing marginal returns of a skilled labor force. Other contributing factors are a very costly matching process between labor demand and supply, and a subsequent low usage of skilled workers and high unemployment rates of



(a) Low equilibrium: $c = 1.3$ (b) Mixed equilibrium: $c = 1.8$ (c) High equilibrium: $c = 2.1$

Figure 5: Dynamics for the three different equilibria for $f((1-y)\rho X_l) = 2\rho(1-y)^3 X_l + c$. Parameters: $\beta = 0.3, \eta_0 = 1, \eta_1 = 0.1, \eta_2 = 0, \gamma_0 = 0.4, \gamma_1 = 0.3, \gamma_2 = 0.1, \rho = 0.1, \tau = 0.1, \upsilon = 0.5, X_l = 1$. Solid line indicates $\dot{y} = 0$ loci, dashed line $\dot{h} = 0$ loci.

highly skilled employees. If the forgone potential of an economy is high, but is coupled with a lack of proper economic institutions as well as a low impact of innovations on human capital and a lack of suitable employments, a mixed state defined by both innovative and non-innovative firms will evolve.

4.2 Evolutionary Dynamics with Idiosyncratic Firm Choice

In the previous section, evolutionary dynamics are based exclusively on a profit comparison between firms choosing different strategies of innovation. Firms are, however, homogeneous otherwise. These dynamics therefore ignore firm specific idiosyncrasies and may eliminate plausible long-run equilibria with heterogeneous firm behavior. Recently, the replicator dynamics with idiosyncratic choice (in discrete time) has been studied by Bischi et al. (2018) modelling the case in which decisions are taken by using a convex combination of the current observation and some previous ones. In this section, we present the replicator dynamics (in continuous time) that account for idiosyncratic behavior, i.e. a convex combination of strategies, where a small share of firms randomly chooses a strategy without taking into account strategy payoffs.

Let $\theta \in (0, 1)$ be the share of firms that choose one of the strategies at random with equal probability. Hence, the share of I—firms is defined by θy and the share of N—firms by $\theta\,(1-y)$, with half of each share changing strategy. The net increase in I—firms through random choice is thus:

$$\theta\,(1-y)\,\frac{1}{2} - \theta y\frac{1}{2} = \theta y\,(1-y)\left(\frac{1}{2}-y\right) \tag{40}$$

At the same time $1-\theta$ firms update their strategy according to (31). We obtain the new replicator dynamics $\dot{\hat{y}}$ as a convex combination of the firms choosing a strategy at random, defined by (40), and the $1-\theta$ share behaving according to (31):

$$\dot{\hat{y}} = y\,(1-y)\left((1-\theta)\,\upsilon\left[1-\frac{1}{(1-\beta h)\,f\,((1-y)\,(\rho X_{\mathrm{I}}))}\right]u^{*}\,(y,h) + \theta\left(\frac{1}{2}-y\right)\right) \tag{41}$$

Consequently, the dynamical system given by Eqs. (41) and (32) is defined in the same state space $\Theta\left\{(y,h) \in \mathbb{R}^2 \colon 0 \le y \le 1, h > 0\right\}$ while the stability of the equilibria depends both on the shape of $f(.)$ and the size of θ.

Proposition 2. *The system's equilibria are defined by the nulls of* (41)*. We obtain the following:*

1. *Only if* $\xi(0) < 0$ *and* $\theta < 1 - \frac{1}{1-2u^{*}\upsilon\xi(0)}$*, equilibrium* $(0, \bar{h}_1)$ *is stable while* $(1, \bar{h}_2)$ *is a saddle point.*

2. *Only if* $\xi(1) > 0$ *and* $\theta < 1 - \frac{1}{1+2u^* \upsilon \xi(1)}$, *equilibrium* $(0, \bar{h}_1)$ *is a saddle point while* $(1, \bar{h}_2)$ *is stable.*

Proof. The proof is analogous to the proof of Proposition 1. The null of (41) are defined by $y = 0$, $y = 1$, and:

$$\Xi(y^*) = (1-\theta)\,\upsilon \left[1 - \frac{1}{(1-\beta h^*)\, f\left((1-y^*)\,(\rho X_{\mathrm{I}})\right)}\right] u^*\left(y^*, h^*\right) + \theta\left(\frac{1}{2} - y^*\right) = 0 \tag{42}$$

Since $\pi_{\mathrm{I}}^*(y,h) > \pi_N^*$, $\forall y \in (0,1)$ we have $\frac{\partial u^*(y,h)}{\partial y} < 0$, and thus, $\frac{\partial \tilde{\xi}(y^*)}{\partial y} < 0$. The existence of the interior equilibrium requires that

$$\Xi(0) = \frac{\theta}{2} + (1-\theta)\frac{\upsilon\left(1 - \frac{1}{(1-\beta h) f(\rho X_P)}\right)\eta_0}{(1-\eta_1)\,(1-\tau)(1-\upsilon) - \eta_2} > 0 \tag{43}$$

$$\Xi(1) = -\frac{\theta}{2} + (1-\theta)\frac{\upsilon(1-(1-\beta h) f(0))\eta_0}{(1-\eta_1)\,(1-\tau)\upsilon - (1-\beta h) f(0)\left((1-\eta_1)\,(1-\tau) - \eta_2\right)} < 0 \tag{44}$$

The eigenvalues of the Jacobian matrix of this dynamical system is given by

$$E_{10} = \frac{\theta}{2} + (1-\theta)\upsilon\left(1 - \frac{1}{(1-\beta h) f(\rho X_{\mathrm{I}})}\right) u^*(0, \bar{h}_1) = \frac{\theta}{2} + (1-\theta)\upsilon\xi(0)u^* \tag{45}$$

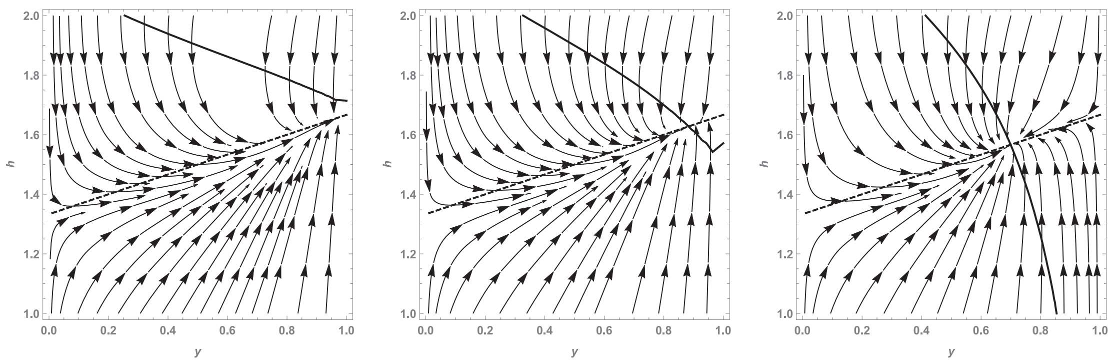
$$E_{11} = \frac{\theta}{2} - (1-\theta)\upsilon\left(1 - \frac{1}{(1-\beta h) f(0)}\right) u^*(1, \bar{h}_2) = \frac{\theta}{2} - (1-\theta)\upsilon\xi(1)u^* \tag{46}$$

$$E_{20} = E_{21} = -\gamma_1 \tag{47}$$

for $y = 0$ and $y = 1$, respectively. Since $\Xi(0) = E_{10}$ and $\Xi(1) = -E_{11}$, we know that the pure strategy equilibria are unstable in the presence of an interior equilibrium. □

The effect of idiosyncratic firm choice is illustrated in Figure 6. Defining $\underline{\theta} = \max\{1 - \left(1 - 2u^*\upsilon\xi(0)\right)^{-1}, 1 - \left(1 + 2u^*\upsilon\xi(1)\right)^{-1}\}$, we obtain that the stability of the pure strategy equilibria requires an additional condition to those derived in Proposition 1, i.e. signaling between firms must be sufficiently reliable. If signaling is noisy and idiosyncratic actions are ample, the economy converges to a mixed equilibrium in which innovative firms coexist along non-innovative firms.

Garcia-Rodriguez and Sanchez-Losada (2014) point out that growth depends on the R & D success probability, while imperfect information can cause a low engagement in R & D. Here, we show that if increasingly imperfect information



(a) $\theta = 0.2$ (b) $\theta = 0.3$ (c) $\theta = 0.5$

Figure 6: Dynamics with idiosyncratic choice and different degrees of θ following the same parameters as Figure 5(c). The critical value is $\theta^* \approx 0.222$.

is represented by higher values of idiosyncratic firm decisions about their level of innovation, the likelihood of a mixed equilibrium with coexisting innovative and non-innovative firms increases with the noise in the market, as long as a sufficiently high enough amount of human capital exists (see Figure 6 part (c)). Noisy signaling can be caused by the lack of well-defined property rights and public information about R & D levels and other firm characteristics while the impact of weak signaling is mitigated by a higher wage share and the output-capital ratio.

Proposition 2 further shows that the mixed equilibrium is more likely to occur if selection pressure (i.e. the speed of convergence given by the absolute value of $\dot{y}$) is weak, i.e. if competition between firms and the gains from innovation are limited. We observe that the relationship between human capital accumulation and technological innovation, and other aspect of our political economy is complex. It is difficult to determine if the accumulation of human capital or technological innovation drive economic growth (for further details, see for instance Risso and Carrera 2019) given their strategic complementarity and the misallocation of production factors.[19]

5 Conclusions

In this paper, we study the strategic interaction between innovation and human capital in advanced economies. Specifically, our model presents an economy populated by decentralized innovative and non-innovative firms in which being innovative involves hiring skilled workers. We demonstrate that even in initially rich and developed economies, low-innovation traps can emerge which – due to the complementarity between innovation, human capital and employment - can cause advanced economies to diverge from an optimal growth path.

The static version of our model in Section 3 shows that short-run growth rates depend on the prevalence of innovation among firms, the firms' profit sharing functions, the optimal output-capital ratio and the existing stock of human capital. In Section 4, we present a dynamic version of our model in which the

19 Jones (2017) illustrates a key result of the literature – the dispersion of marginal products of the same factors across firms in the US and in Europe is associated with the degrees of misallocation (e.g. Gopinath et al. 2017, highlighting deteriorating factor allocation in Italy and Spain). Note, however, that a recent paper by Bils et al. (2017) suggests that if one corrects for measurement error, the entire increase of allocative inefficiency in US manufacturing since the late 1970s disappears. It may therefore be advisable to wait until measurement errors are also accounted for in the literature on non-manufacturing sectors and on Europe before drawing clear-cut conclusions.

share of innovative firms and the level of human capital coevolve. We derive conditions that determine the self-reinforcing equilibrium in the long run and show that it is characterized either by (1) a low innovation trap, in which firms do not undertake innovation, (2) a steady state in which all firms engage in R & D, or (3) a mixed steady state where only a subset of firms innovate. In all these equilibria, innovation and human capital are strictly related to each other, while aggregate demand which contributes to determining the profits of firms and the decisions to innovate, plays a critical role. Since an equilibrium in our model is the system's unique attractor, firms spontaneously engage in or cease innovation and converge to the steady state once certain conditions are met. In other words, firms do not need to be actively encouraged to innovate but innovation and human capital co-evolve.

Proposition 1 in Section 4 defines these intuitive conditions. A high natural rate of human capital accumulation and marginal returns that are not diminishing too quickly avert the low-innovation trap. Similarly, an economy experiences high growth rates of human capital and innovative firms, if the latter undergo strong positive externalities from other innovative firms and encourage the creation of human capital. Contrary, a low-innovation trap emerges if human capital exhibits a low natural growth rate that can be caused by a lack of institutions that foster education, and suffers from low marginal returns of an increased skilled labor force eliminating the benefits from further education. In addition, Proposition 2 shows that the reinforcing nature of innovation by individual firms can be negated if signalling between firms is imperfect, thus limiting a firm's awareness of the decision of other firms and the benefits the latter obtain from innovation.

Our results indicate support for policies that enhance innovation and/or education, limit recruiting costs, improve matching between innovative firms and skilled labor, encourage positive spillover effects between innovative firms and increase the percentages of people employment in Knowledge Intensive Activities (KIA). Furthermore, low levels of innovation due to ill-defined property rights and a lack of public information on R & D levels can be counteracted by policies that improve the wage share and the output-capital ratio.

Consequently, our paper stresses the need for countries to exploit technological innovations via a sufficiently qualified workforce and a labor market that is flexible enough to shift jobs towards innovative firms. It further emphasizes the importance of a sufficiently transparent and high degree of market competition between firms. Future research should extend our investigation into the interplay of technological innovation, human capital accumulation, and economic growth to a broader context that includes climate change, sustainability and aging populations.

In addition, this research needs further study of the importance of market signals and transparent competition on the gains of higher education and knowledge transfer between highly skilled workers, and thus, the interaction of skilled employees within firms (e.g. via promoting employee engagement, high performance working, job security as well as a lower reliance on competency-based learning as opposed to a broader education with strong theoretical underpinnings, see also Toner 2011).

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References

Accinelli, E., and E. S. Carrera. 2011. "Strategic Complementarities between Innovative Firms and Skilled Workers: The Poverty Trap and the Policymaker's Intervention." *Structural Change and Economic Dynamics* 22 (1): 30–40.

Accinelli, E., and E. S. Carrera. 2012. "The Evolutionary Game of Poverty Traps." *The Manchester School* 80: 381–400.

Acemoglu, D. 1997. "Training and Innovation in an Imperfect Labor Market." *The Review of Economic Studies* 64: 445–64.

Acemoglu, D. 1998. "Why Do New Technologies Complement Skills? Directed Technical Change and Wage Inequality." *Quarterly Journal of Economics* 113 (4): 1055–89.

Acemoglu, D. 2002. "Directed Technical Change." *The Review of Economic Studies* 69: 781–809.

Acemoglu, D. 2003. "Patterns of Skill Premia." *The Review of Economic Studies* 70 (2): 199–230.

Acemoglu, D., U. Akcigit, H. Alp, N. Bloom, and W. Kerr. 2018. "Innovation, Reallocation, and Growth." *The American Economic Review* 108 (11): 3450–91.

Aghion, P., A. Bergeaud, R. Blundell, and R. Griffith. 2017. *Innovation, Firms and Wage Inequality*. Also available at https://scholar.harvard.edu/files/aghion/files/innovationsfirmsandwage.pdf.

Akao, K., T. Kamihigashi, and K. Nishimura. 2011. "Monotonicity and Continuity of the Critical Capital Stock in the Dechert—Nishimura Model." *Journal of Mathematical Economics* 47: 677—82.

Annicchiarico, B., and A. Pelloni. 2019. "Innovation, Growth, and Optimal Monetary Policy." *Macroeconomic Dynamics*: 1—24. https://doi.org/10.1017/S1365100519000610.

Archibugi, D., and A. Filippetti. 2018. "The Retreat of Public Research and its Adverse Consequences on Innovation." *Technological Forecasting and Social Change* 127: 97—111.

Asheim, B., L. Coenen, and J. Vang. 2007. "Face-to-face, Buzz, and Knowledge Bases: Sociospatial Implications for Learning, Innovation, and Innovation Policy." *Environment and Planning C: Government and Policy* 25: 655—70.

Askenazy, P., and C. Le Van. 1999. "A Model of Optimal Growth Strategy." *Journal of Economic Theory* 85: 24—51.

Autor, D., and A. Salomons. 2017. "Robocalypse Now: Does Productivity Growth Threaten Employment?" In *the ECB Forum on Central Banking, Investment and Growth in Advanced Economies*. June 26—28, Sintra: Portugal. https://www.ecb.europa.eu/pub/pdf/other/ecb.ecbforumcentralbanking2017.en.pdf?ca5f1762dbdf7ada8d8d6bc0789e14d8.

Autor, D., D. Dorn, L. F. Katz, C. Patterson, and J. V. Reenen. 2020. "The Fall of the Labor Share and the Rise of Superstar Firms." *Quarterly Journal of Economics* 135 (2): 645.

Azariadis, C., and A. Drazen. 1990. "Threshold Externalities in Economic Development." *Quarterly Journal of Economics* 105: 501—26.

Azariadis, C., and H. Starchuski. 2005. "Poverty Traps." In *Handbook of Economic Growth*, edited by P. Aghion, and S. Durlauf. Oxford: Elsevier.

Barro, R. J. 2001. "Human Capital and Growth." *The American Economic Review* 91 (2): 12—7.

Bils, M., P. Klenow, and C. Ruane. 2017. "Misallocation or Mismeasurement?" *NBER Working Papers 26711*, National Bureau of Economic Research, Inc. https://www.nber.org/papers/w26711.

Bischi, G., U. Merlone, and E. Pruscinia. 2018. "Evolutionary Dynamics in Club Goods Binary Games." *Journal of Economic Dynamics and Control* 91: 104—19.

Blackburn, K., T. Y. VictorHung, and A. F. Pozzolo. 2000. "Research, Development and Human Capital Accumulation." *Journal of Macroeconomics* 22 (Issue 2): 189—206.

Bondarev, A., and A. Greiner. 2019. "Endogenous Growth and Structural Change through Vertical and Horizontal Innovations." *Macroeconomic Dynamics* 23 (1): 52—79.

Bosco, M. G., and E. Valeriani. 2018. "The Road to Permanent Work in Italy: "It's Getting Dark, Too Dark to See"." *Italian Economic Journal* 4: 385—419.

Boyd, R., and P. J. Richerson. 2005. *The Origin and Evolution of Cultures*. Oxford: Oxford University Press.

Bresnahan, T. F., E. Brynjolfsson, and L. M. Hitt. 2002. "Information Technology, Workplace Organization, and the Demand for Skilled Labor: Firm-Level Evidence." *Quarterly Journal of Economics* 117 (1): 339—76.

Caroli, E., and J. Van Reenen. 2001. "Skill-biased Organizational Change? Evidence from a Panel of British and French Establishments." *Quarterly Journal of Economics* 116 (4): 1449—92.

Chu, A., L. Ning, and D. Zhu. 2019. "Human Capital and Innovation in a Monetary Schumpeterian Growth Model." *Macroeconomic Dynamics* 23 (5): 1875—94.

Colonna, F. 2014. *Innovation and Human Capital: Theory and Evidence from Italy*. Also available at https://www.bancaditalia.it/pubblicazioni/altri-atti-convegni/2014-innovazione-italia/Colonna.pdf.

Cyranoski, D., N. Gilbert, H. Ledford, A. Nayar, and M. Yahia. 2011. “Education: The PhD Factory.” *Nature* 472: 276–9.
Danilov, V., G. A. Koshevoy, and A. I. Sotskov. 1997. “Equilibrium Analysis of an Economy with Innovations.” *Journal of Mathematical Economics* 27 (2): 195–228.
Diebolt, C., and R. Hippe. 2018. “The Long-Run Impact of Human Capital on Innovation and Economic Development in the Regions of Europe.” *Applied Economics* 51: 1–22.
Dima, A. M., L. Begu, M. D. Vasilescu, and M. A. Maassen. 2018. “The Relationship between the Knowledge Economy and Global Competitiveness in the European Union.” *Sustainability* 10 (6): 1706.
Dinerstein, M., R. Megalokonomou, and C. Yannelis. 2020. “Human Capital Depreciation.” In *Working Paper No. 2020-146, National Bureau of Economic Research Working Paper Series 27925*. University of Chicago, Becker Friedman Institute for Economics. http://www.nber.org/papers/w27925.
Durlauf, S., and H. P. Young. 2001. *Social Dynamics*. Cambridge, Massachusetts, and London: The MIT Press.
Ermini, B., L. Papi, and F. Scaturro. 2017. “An Analysis of the Determinants of Over-education Among Italian Ph.D Graduates.” *Italian Economic Journal* 3: 167–207.
European Commission 2018. *Science, Research and Innovation Performance of the EU 2018 – Strengthening the Foundations for Europe’s Future*. Luxembourg: Publications Office of the European Union.
Eurostat 2019. *Population by Educational Attainment Level, Sex and Age* [online]. Also available at https://appsso.eurostat.ec.europa.eu/nui/submitViewTableAction.do.
Franke, R. 2017. “What Output-Capital Ratio to Adopt for Macroeconomic Calibrations?” *International Review of Applied Economics* 31 (2): 208–24.
Fukuda, S. 2008. “Knightian Uncertainty and Poverty Trap in a Model of Economic Growth.” *Review of Economic Dynamics* 11 (3): 652–63.
Ganotakis, P. 2012. “Founders’ Human Capital and the Performance of UK New Technology Based Firms.” *Small Business Economics* 39: 495–515.
García-Rodríguez, A., and F. Sánchez-Losada. 2014. “R & D Poverty Traps.” In *UB Economics Working Papers E14/307*. Also Available at SSRN https://ssrn.com/abstract=2405003.
Gintis, H., S. Bowles, R. Boyd, and E. Fehr. 2005. *Moral Sentiments and Material Interests – the Foundations of Cooperation in Economic Life*. Cambridge, Massachusetts, and London: The MIT Press.
Gopinath, G., Kalemli-Ozcan, L. Karabarbounis, and C. Villegas-Sanchez. 2017. “Capital Allocation and Productivity in South Europe.” *Quarterly Journal of Economics* 132 (4): 1915–67.
Grassetti, F., C. Mammana, and E. Michetti. 2018. “Poverty Trap, Boom and Bust Periods and Growth. A Nonlinear Model for Non-developed and Developing Countries.” *Decisions in Economics and Finance* 41: 145–62.
Guerrini, L. 2006. “The Solow–Swan Model with a Bounded Population Growth Rate.” *Journal of Mathematical Economics* 42: 14–21.
Henkin, G. M., and V. M. Polterovich. 1991. “Schumpeterian Dynamics as a Non-linear Wave Theory.” *Journal of Mathematical Economics* 20 (6): 551–90.
Hilborn, R. 1994. *Chaos and Nonlinear Dynamics*. Oxford: Oxford University Press.
Hritonenko, N., and Y. Yatsenko. 2010. “Technological Innovations, Economic Renovation, and Anticipation Effects.” *Journal of Mathematical Economics* 46 (6): 1064–78.

Ille, S. 2021. "The Evolution of Sectarianism." *Communications in Nonlinear Science and Numerical Simulation* 97: 105726.
Jones, C. 2017. "The Productivity Growth Slowdown in Advanced Economies." In *the ECB Forum on Central Banking Investment and Growth in Advanced Economies*. June 26–28, Sintra: Portugal, https://www.ecb.europa.eu/pub/pdf/other/ecb.ecbforumcentralbanking2017.en.pdf?ca5f1762dbdf7ada8d8d6bc0789e14d8.
Kehoe, P. J., V. Midrigan, and P. Elena. 2019. "Debt Constraints and Employment." *Journal of Political Economy* 127 (4): 1926–91.
Kikuchi, T., and G. Vachadze. 2015. "Financial Liberalization: Poverty Trap or Chaos." *Journal of Mathematical Economics* 59: 1–9.
Kopel, M., F. Lamantia, and F. Szidarovszky. 2014. "Evolutionary Competition in a Mixed Market with Socially Concerned Firms." *Journal of Economic Dynamics and Control* 48: 394–409.
Kuhlmann, S., and J. Edler. 2003. "Scenarios of Technology and Innovation Policies in Europe: Investigating Future Governance." *Technological Forecasting and Social Change* 70: 619–37.
Lesourne, J., A. Orléan, and B. Walliser. 2002. *Leşons de microéconomie évolutionniste*. Paris: Odile Jacob.
Ljungqvist, L., and T. J. Sargent. 1998. "The European Unemployment Dilemma." *Journal of Political Economy* 106 (3): 514–50.
Lundvall, B. A. 2009. "The Future of Innovation Is in the Learning Economy." In *The Future of Innovation*, edited by van Stamm, B., and Trifilova, A. 40–1. Surrey, UK: Gover UTEK.
Madsen, J., V. Mishra, and R. Smyth. 2012. "Is the Ouput-Capital Ratio Constant in the Very Long Run?" *The Manchester School* 80 (2): 210–36.
Maynard Smith, J. 1982. *Evolution and the Theory of Games*. Cambridge: Cambridge University Press.
Neal, D. 1995. "Industry-specific Human Capital: Evidence from Displaced Workers." *Journal of Labor Economics* 13 (4): 653–77.
Neuman, S., and A. Weiss. 1995. "On the Effects of Schooling Vintage on Experience-Earnings Profiles: Theory and Evidence." *European Economic Review* 39: 943–55.
Nowak, M. 2006. *Evolutionary Dynamics: Exploring the Equations of Life*. Cambridge, Massachusetts, and London: Belknap Press.
OECD. 2020. "OECD Main Science and Technology Indicators." In *R & D Highlights in the February 2020 Publication*. Directorate for Science, Technology and Innovation. www.oecd.org/sti/msti2020.pdf.
Oreopoulos, P., T. Von Wachter, and A. Heisz. 2012. "The Short-and Long-Term Career Effects of Graduating in a Recession." *American Economic Journal: Applied Economics* 4 (1): 1–29.
Page, K., and M. Nowak. 2002. "Unifying Evolutionary Dynamics." *Journal of Theoretical Biology* 219: 93–8.
Ponzano, F., and R. Ricciuti. 2018. "Growth and Inequality in an Experimental AK Model." *Italian Economic Journal* 4: 313–30.
Pritchett, L. 2001. "Where Has All the Education Gone?" *The World Bank Economic Review* 15 (3): 367–91.
Psacharopoulos, G., and H. A. Patrinos. 2018. "Returns to Investment in Education: A Decennial Review of the Global Literature (English)." In *Policy Research Working Paper*, no. WPS 8402. Washington, D.C.: World Bank Group. http://documents.worldbank.org/curated/en/442521523465644318/Returns-to-investment-in-education-a-decennial-review-of-the-global-literature.

Rebelo, S. 1991. "Long Run Policy Analysis and Long Run Growth." *Journal of Political Economy* 99: 500–21.
Redding, S. 1996. "The Low-Skill, Low-Quality-Trap: Strategic Complementarities between Human Capital and R & D." *Economic Journal* 106: 457–70.
Risso, A., and E. S. Carrera. 2019. "On the Impact of Innovation and Inequality in Economic Growth." *Economics of Innovation and New Technology* 28 (1): 64–81.
Romer, P. M. 1986. "Increasing Returns and Long-Run Growth." *Journal of Political Economy*: 1002–37. https://doi.org/10.1086/261420.
Romer, P. M. 1990. "Endogenous Technological Change." *Journal of Political Economy* 98: S71–S102.
Sanchez-Carrera, E. 2012. "Imitation and Evolutionary Stability of Poverty Traps." *Journal of Bioeconomics* 14: 1–20.
Sanchez-Carrera, E. 2019. "Evolutionary Dynamics of Poverty Traps." *Journal of Evolutionary Economics* 29: 611–30.
Santos-Rodrigues, H., P. F. Dorrego, and C. F. Jardon. 2010. "The Influence of Human Capital on the Innovativeness of Firms." *International Business & Economics Research Journal* 9 (9): 53–63.
Schneider, L., J. Gunther, and B. Brandenbury. 2010. "Innovation and Skills from a Sectoral Perspective: A Linked Employer-Employee Analysis." *Economics of Innovation and New Technology* 19: 185–202.
Silveira, J., and G. T. Lima. 2016. "Effort Elicitation, Wage Differentials and Income Distribution in a Wage-Led Growth Regime." *Metroeconomica* 67 (1): 44–75.
Slaper, T. F., N. R. Hart, T. J. Hall, and M. F. Thompson. 2011. "The Index of Innovation: A New Tool for Regional Analysis." *Economic Development Quarterly* 25: 36–53.
Soete, L., B. Verspagen, and T. Ziesemer. 2020a. "The Productivity Effect of Public R & D in the Netherlands." *Economics of Innovation and New Technology* 29: 31–47.
Soete, L., B. Verspagen, and T. Ziesemer. 2020b. "The Economic Impact of Public R & D: An International Perspective." In UNU-MERIT Working Paper 2020-014. Maastricht: UNU-MERIT.
Storper, M., and A. J. Scott. 2009. "Rethinking Human Capital, Creativity and Urban Growth." *Journal of Economic Geography* 9: 147–67.
Toner, P. 2011. *Workforce Skills and Innovation: An Overview of Major Themes in the Literature*. OECD Directorate for Science, Technology and Industry (STI), Centre for Educational Research and Innovation (CERI) (SG/INNOV(2011)1).
Tsiddon, D. 1992. "A Moral Hazard Trap to Growth." *International Economic Review* 33 (2): 299–321.
Turner, A. 2018. "Capitalism in the Age of Robots: Work, Income and Wealth in the 21st-Century." In *Lecture at School of Advanced International Studies*. Washington DC: Johns Hopkins University. https://www.ineteconomics.org/uploads/papers/Paper-Turner-Capitalism-in-the-Age-of-Robots.pdf (April 10th 2018).
Van Roy, V., D. Vértesy, and M. Vivarellicde. 2018. "Technology and Employment: Mass Unemployment or Job Creation? Empirical Evidence from European Patenting Firms." *Research Policy* 47 (9): 1762–76.
Weinberg, B. A. 2002. "New Technologies, Skills Obsolescence, and Skill Complementarity." In *The Economics of Skills Obsolescence, Research in Labor Economics*, Vol. 21, edited by A. De Grip, J. van Loo, and K. Mayhew. 101–18. Amsterdam/Boston: JAI Press.

Zheng, Z., C. Huang, and Y. Yang. 2019. "Inflation and Growth: A Non-monotonic Relationship in an Innovation-Driven Economy." *Macroeconomic Dynamics*: 1–28. https://doi.org/10.1017/S1365100519000622.

Ziesemer, T. 2019a. "Japan's Productivity and GDP Growth: The Role of GBAORD, Public and Foreign R & D." In *UNU-MERIT Working Papers 2019-029*, Maastricht: UNU-MERIT.

Ziesemer, T. 2019b. "The Impact of Mission-Oriented R & D on Domestic and Foreign Private and Public R & D, Total Factor Productivity and GDP." In *MERIT Working Papers 2019-047*.

Ziesemer, T. 2020. "Japan's Productivity and GDP Growth: The Role of Private, Public and Foreign R & D 1967–2017." *Economies* 8 (4): 77.