

Unveiling individual and collective temporal patterns in the tanker shipping network

Kevin Teo¹, Naomi Arnold¹, Andrew Hone², Michael Coulon³,
Martin Ireland³, Mauricio Santillana⁴, István Z. Kiss^{*,1,5}

¹ Network Science Institute, Northeastern University London, London, E1W 1LP, United Kingdom

² School of Engineering, Mathematics & Physics, University of Kent, Canterbury CT2 7NF, United Kingdom

³ AlphaOcean.ai Ltd, Preston Park House, South Road, Brighton, East Sussex, United Kingdom, BN1 6SB

⁴ Machine Intelligence Group for the Betterment of Health and the Environment, Network Science Institute, Northeastern University, Boston, MA, USA

⁵ Department of Mathematics, Northeastern University, Boston, MA 02115, USA
February 10, 2026

Abstract

The global oil tanker shipping network emerges from individual ship and fleet decisions driven by economic, environmental, and operational efficiency factors. However, most existing analyses of shipping networks rely on static, time-aggregated representations, which overlook critical temporal patterns that connect individual vessel routing strategies with both operational efficiency and system-wide cargo flows. To address this gap, we introduce a dual-scale framework complementing sequential motif analysis—which captures recurring patterns in ships’ regional visit sequences—with Dynamic Mode Decomposition (DMD) to extract temporal dynamics spanning individual trajectories to global flow patterns. Using movement data from oil tankers across four vessel classes, we demonstrate that vessels exhibiting diverse regional exploration patterns spend up to 50% more time carrying rather than seeking cargo, indicating greater economic and environmental efficiency. At the system level, DMD analysis of regional cargo flows reveals distinct seasonal cycles with amplitudes averaging 16% between peak and trough periods. Major importing regions (Europe, East Asia) show synchronous annual demand cycles, while export regions (Middle East, South America) exhibit anti-synchronous patterns. These multi-scale temporal dependencies, undetectable through conventional static analysis, reveal actionable performance differentials that enable data-driven routing strategies to optimize both economic efficiency and environmental sustainability.

1 The global maritime network handles over 80% of inter-
2 national trade by volume, with crude oil and petroleum
3 products alone accounting for nearly 30% of this market [1,
4 2]. Despite its economic significance and substantial en-
5 vironmental impact [3–6], maritime shipping networks re-
6 main understudied compared to other transportation sys-
7 tems, particularly regarding their temporal operational dy-
8 namics [7]. This research gap is especially pronounced for
9 oil tanker networks, which exhibit fundamentally different
10 operational characteristics from the more extensively re-
11 searched container shipping networks where fixed schedules
12 and regular route structures have made traditional com-
13 plex network analysis particularly effective for understand-
14 ing system-level properties [8].

15 Current maritime network research has predominantly fo-
16 cused on static, time-aggregated analyses, with four main
17 analytical themes: trade and connectivity; hubs and cen-
18 trality; vulnerability and robustness; and communities and
19 spatial structure [9–11]. These studies established that
20 global shipping is dominated by hub-and-spoke structures,
21 where major ports are connected to numerous smaller ports,
22 while other important ports act as regional gateways con-
23 trolling the flow of goods between different geographic ar-
24 eas [8, 12–16]. Moreover, such analyses have also been used
25 to compare changes in network connectivity and regional

communities at different time periods, often before and af-
ter significant geopolitical events [17–19]. These studies are
commonly interpreted through the lens of economic and
ecological impacts [15, 20–22].

Despite these insights, static approaches that aggregate
temporal data obscure critical operational patterns. When
networks are analyzed as snapshots, essential sequential in-
formation is lost: the timing and order of vessel movements
between regions, and the dynamic variation in cargo flows
over time. This information loss masks valuable operational
insights, as vessels following identical aggregate movement
patterns between the same ports may employ fundamen-
tally different sequential strategies that significantly im-
pact their efficiency and market positioning. These different
temporal routing patterns create distinct operational out-
comes that remain invisible in static networks.

These temporal dynamics are especially critical in oil
tanker movements, which are operationally different com-
pared to container liners. Unlike container ships operating
on fixed schedules along established routes, tankers operate
in charter-based markets where individual vessels compete
for cargo, creating highly flexible and responsive movement
patterns [8]. These competitive dynamics generate precisely
the type of sequential, temporal dependencies that static
aggregation eliminates—as vessels dynamically adjust their

routing sequences based on market opportunities and cargo availability. Recent work on oil tanker networks has examined traffic density [23], hub-and-spoke structures [24], and influential port identification [14], yet these studies maintain the static network paradigm and do not capture the temporal operational patterns that may distinguish successful from unsuccessful competitive strategies.

To overcome these analytical limitations and uncover the hidden temporal structure, we demonstrate that sequential motif analysis (which identifies recurring patterns of regional visits) and Dynamic Mode Decomposition (DMD, a method for extracting temporal patterns from time series data) reveal previously undetected patterns spanning both individual vessel operations and global flow dynamics. This dual-scale approach links routing strategies to efficiency outcomes and cargo flows to seasonal cycles. Specifically, we show that ships exhibiting diverse regional exploration patterns—captured through sequential motifs—achieve significantly better laden-ballast ratios, a key efficiency measure closely related to the industry-standard Energy Efficiency Operational Indicator. In parallel, DMD applied to regional cargo flow time series reveals that different ship classes operate with distinct regional footprints and specialized seasonal trading patterns. We identify previously uncharacterized synchronous and anti-synchronous relationships between maritime regions that correlate with economic and climate cycles. While these analyses operate at different scales—individual vessel behavior versus collective fleet dynamics—they provide complementary insights into the multi-scale temporal structure of maritime operations.

Results

Data

We obtained proprietary data from our partner company AlphaOcean, covering voyages taken by 3026 medium to large crude-oil and petroleum tankers from 2016 January to 2020 February. This consists of 452 Panamax, 1141 Aframax, 610 Suezmax, and 823 Very Large Crude Carrier (VLCC), listed in order of their size in deadweight tonnage. The dataset contains a list of laden legs (single trips where the ship is loaded with cargo), detailing the origin/destination locations and their respective departure/arrival times, as well as an identifier of the ship that completed the leg. This enables the reconstruction of the entire historical journey of an individual ship, illustrated in fig. 1, typically consisting of alternating laden and ballast legs. The definition of ballast legs is in the Methods section; additional data processing details are provided in Supplementary Information section 1.1 to 1.3.

While the locations provided are accurate up to the port-level, our analysis condenses this to a *regional* level, grouping 1337 ports into 26 regions. This aligns with industry practices of pricing freight rate indices on a region-to-region basis and reflects the uncertainty that exact discharge ports are often not specified to operators until late in the voyage. In total, we recovered over 192,000 shipping legs (both laden and ballast), split amongst 33,509 Panamax, 94,840 Aframax, 34,401 Suezmax, and 29,418 VLCC legs.

The tanker ships in our study typically do not travel on fixed routes, nor do they service particular countries or regions exclusively. Instead, they are free to compete for cargo across different markets and regions, with their movements driven by economic considerations such as travel times, fuel and staffing costs, potential earnings, and other market opportunities. This operational flexibility presents a unique challenge for tankers: how do we evaluate the routing performance of ships?

A key challenge in evaluating tanker routing performance lies in the inaccessibility of comprehensive voyage and vessel data. We propose an accessible metric using temporal data—the *laden-ballast ratio* (LBR),

$$LBR = \frac{\sum_{l \in L} \Delta t_l}{\sum_{l \in L} \Delta t_l + \sum_{b \in B} \Delta t_b}, \quad (1)$$

where Δt_l is the at-sea duration of laden leg $l \in L$ (adjusted for port waiting times; see Methods for details and robustness checks) and Δt_b is the duration of ballast leg $b \in B$, where L and B are the sets of laden and ballast legs, respectively, of a ship’s trajectory. The ratio above simply measures the proportion of time spent laden at sea out of the total time at sea. The LBR is designed to capture the relative amount of time a ship spends performing useful work (transporting cargo) versus non-revenue generating operations (repositioning to load cargo). By focusing solely on temporal patterns of laden versus ballast operations, the LBR excludes ship-specific technical factors such as fuel consumption patterns or engine specifications. This design allows us to avoid scenarios where a vessel with efficient engines but poor routing outperforms another vessel with less efficient engines but superior routing strategy. This enables fleet-level comparisons where comprehensive operational data remains unavailable.

The stratification of our analysis by ship type (Panamax, Aframax, Suezmax, VLCC) controls for substantial inter-class variability in vessel characteristics and operational profiles. Within each class, multivariate analysis controls for vessel age, effectively isolating routing efficiency from vessel-specific factors. Furthermore, as demonstrated in Supplementary Information section 1.4, LBR captures fundamentally similar efficiency patterns to the International Maritime Organization’s Energy Efficiency Operational Indicator (EEOI) [25], which measures CO₂ emissions per unit of cargo work. While EEOI remains an industry gold standard, its calculation requires proprietary fuel consumption and precise route distance data typically unavailable in fleet-level research datasets. Our LBR measure, though simplified, provides a theoretically grounded and empirically useful measure that reflects revenue-optimizing incentives, enabling large-scale analysis of routing efficiency patterns across global shipping networks.

The laden-ballast ratio distributions in fig. 2(top) expose significant performance disparities across each ship class, with the upper quartile (top 25%) displaying substantially higher values compared to the lower quartile (bottom 25%). Comparing median values between these groups reveals that the upper quartile achieve laden-ballast ratios approximately 50% higher than the lower quartile. This clear difference in time spent carrying versus seeking cargo indicates significant heterogeneity in operational strategies within identical ship classes. Such performance differentials

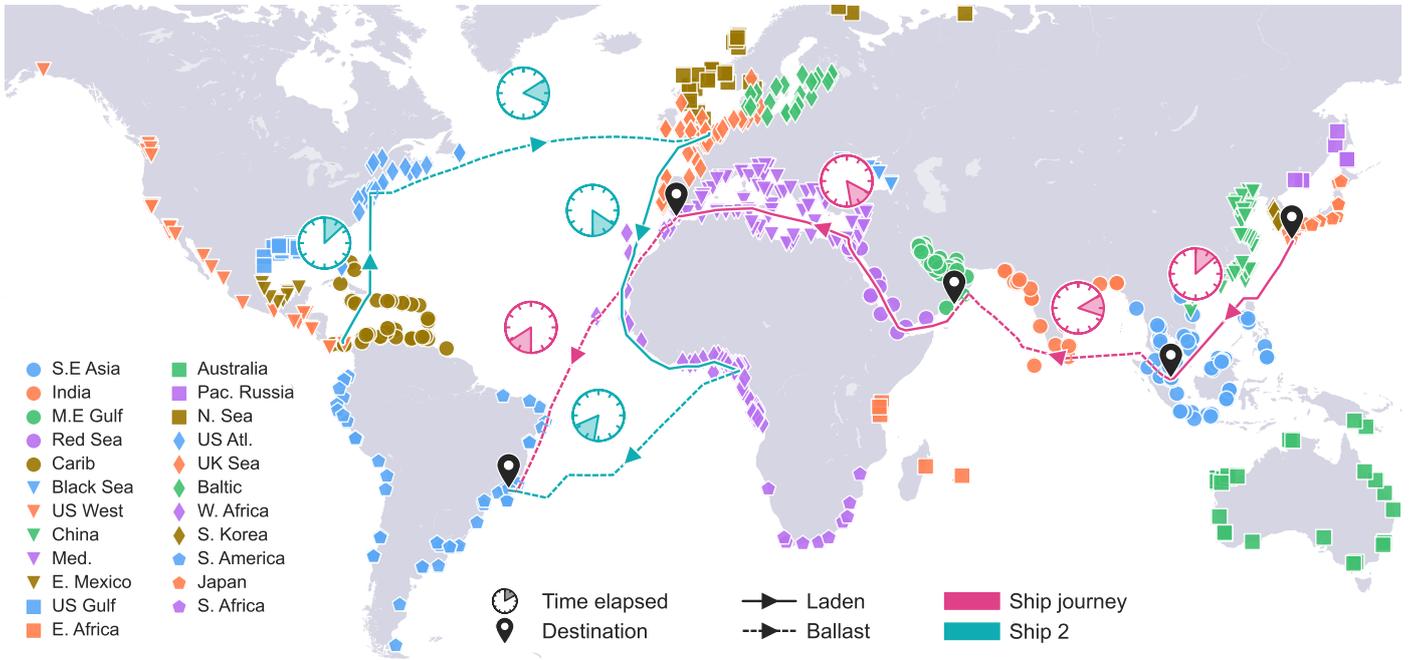


Figure 1: **Schematic representation of voyages in time and space of two ships.** Ships travel along directed port-to-port routes with journey time indicated by clock markers, alternating between laden and ballast voyages. Individual ports in the dataset are shown, with the regions they belong indicated by their marker shape and color.

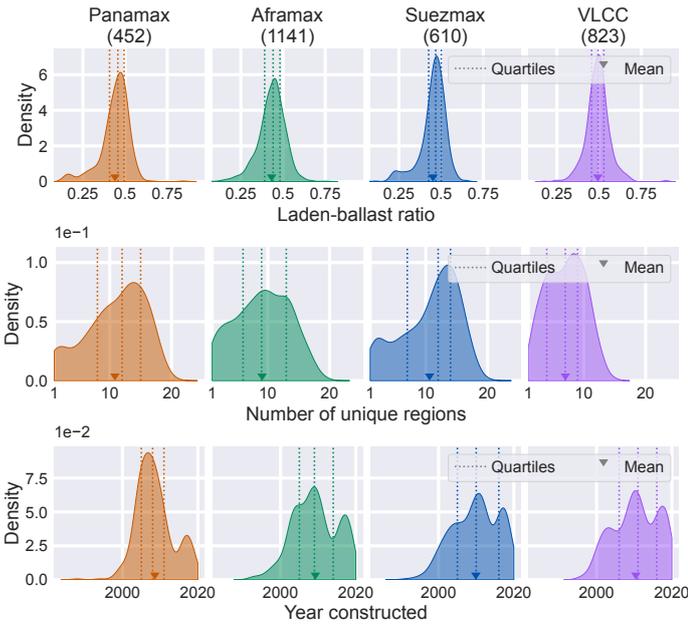


Figure 2: **Distribution of vessel measure data across classes.** Kernel density estimates for laden-ballast ratio (top), number of unique regions visited (middle), and year constructed (bottom) across four tanker classes. Dotted lines mark quartiles; triangles markers denote means. Sample sizes shown in parentheses: Panamax (452), Aframax (1141), Suezmax (610), VLCC (823).

class, VLCCs are often deployed on inter-regional long-haul routes inaccessible to smaller ships, maximizing their at-sea laden time while minimizing time spent waiting in port. These longer voyage durations enable easier fixture planning, reducing inter-voyage delays that would otherwise erode efficiency.

Tanker vessels exhibit remarkable freedom to explore diverse markets rather than operating from fixed regional bases. Analysis of total regional coverage, fig. 2 (middle), uncovers distinct operational strategies across ship classes that directly reflect their design constraints and market positioning. Panamax and Suezmax vessels demonstrate similar exploration capabilities, visiting a median of 12 regions, indicating comparable operational flexibility despite size differences. Aframax vessels show the greatest variation in regional diversity with broader distribution patterns (median: 9 regions), reflecting their widespread usage, superior versatility for different cargo types, and port accessibility. VLCCs operate under significant geographical constraints, visiting only 7 regions on average with markedly narrow distribution. This restricted mobility stems from their specialized infrastructure requirements—as the largest vessels in the global fleet, VLCCs can only access ports with sufficient depth and handling capacity for their extreme size.

Extracting sequential motifs

Ship routing decisions involve complex factors that create distinctive movement signatures, exposing the strategic thinking behind high-performance operations. Consider two ships visiting identical regions: Ship 1 follows (*Middle East, China, Southeast Asia, Middle East, China, Southeast Asia*)—systematically cycling through three regions—while Ship 2 alternates (*Middle East, China, Middle East, Southeast Asia, China, Southeast Asia*)—return journeys between pairs of regions. In this example, time-aggregated networks would show identical connectivity between these three major trading regions, yet their sequential trajec-

169 suggest that optimizing routing strategies could substantially improve both economic outcomes and environmental performance.

170
171
172 This performance landscape reveals distinct efficiency profiles that align with each vessel class’s operational niche.
173 Panamax, Aframax, and Suezmax vessels show comparable laden-ballast ratios with overlapping median ranges,
174 while VLCCs achieve marginally higher values due to their specialized operational advantages. As the largest vessel
175
176
177

214 ries reveal fundamentally different strategies: systematic regional diversification versus back-and-forth movements between trading partners. To better understand the routing patterns that arise from these decision processes, we analyze short repeating sequences known as sequential motifs.

219 Previous studies have shown that ship trajectories can be modeled statistically using higher-order Markov chains, where future locations depend not only on the current position but also on recent prior locations [26, 27]. Higher-order networks generalize this correlation structure of individual agent trajectories to a network representation where nodes represent trajectory subsequences rather than individual ports, embedding these sequential dependencies in the network, where the order indicates the correlation length captured [28]. Utilizing this framework on our tanker dataset, we systematically identified order-2 as the optimal order based on information-theoretic model selection criteria (see Methods). Intuitively, this means that routing decisions depend on both the previous location and current location, but extending knowledge further back adds little predictive value while dramatically increasing model complexity. Building on the insights of this framework, we examine ship movements through 2-hop motifs to uncover the relationship between routing patterns and operational routing performance.

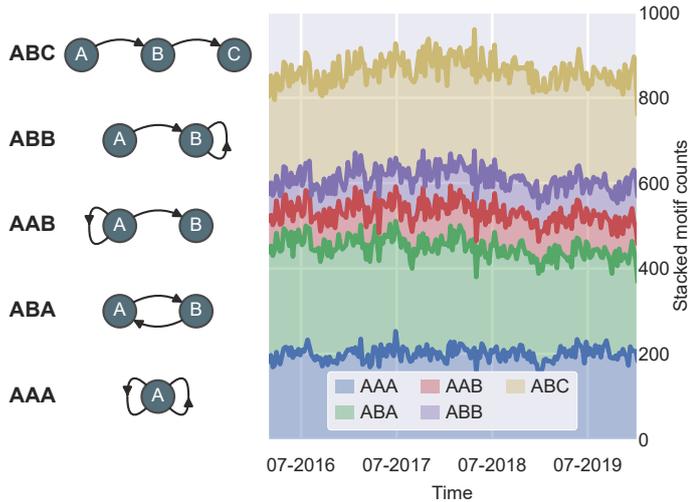


Figure 3: **Diagrammatic examples of 2-hop motifs and their observed counts over time.** (Left) Diagrams of 2-hop motifs. (Right) Stacked plot of motif counts over time in 1 week windows. Counts indicate the number of motifs that *started* within each 1 week window, to account for varying durations of 2-hop motifs.

239 Five distinct strategic patterns emerge from routing decisions. These are the five possible 2-hop motifs, which we call *AAA*, *ABA*, *AAB*, *ABB*, and *ABC*, illustrated in fig. 3 (left). The specific regional labels are not relevant; instead, motifs are identified by the topological relations between elements in the motif. For example, a journey sequence (*China, Japan, China*), (*Japan, China, Japan*), and (*UK, Mediterranean, UK*) all share the same underlying pattern: a ship visits region *A*, then region *B*, then returns to region *A*. This maps onto the motif *ABA*, regardless of the regions involved. Thus, *ABC* motifs represent maximally diverse routing (3 unique regions within 2 consecutive legs), while *AAA* motifs indicate routing focused within a single region.

We examine the absolute counts of different motifs over time, shown in the stacked time series in fig. 3 (right). Despite random fluctuations, autocorrelation analysis reveals no clear temporal patterns in any of the motif counts, suggesting that fleet-level routing behaviors remain stable and consistent over time (see Supplementary Information section 2.2 for autocorrelation plots). The motifs ranked based on occurrence frequency consistently reveal *ABC* as the most common motif, followed by *ABA*, and then *AAA*. This allows us to aggregate our motif counts across the whole duration of the data without losing critical information.

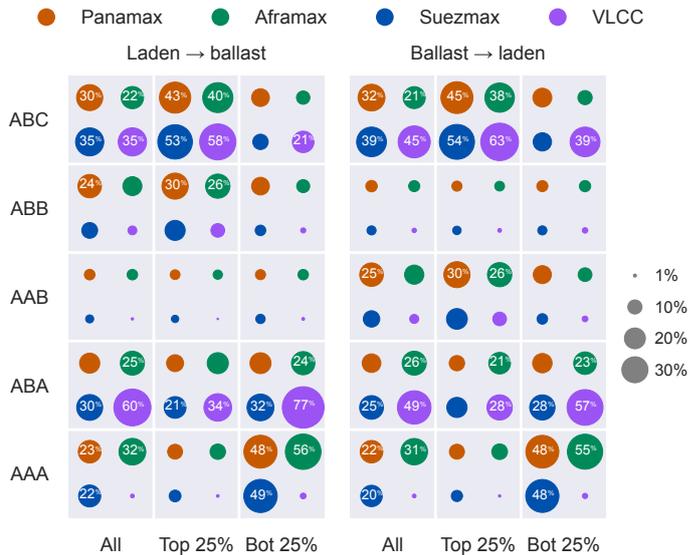


Figure 4: **Observed motif prevalence of each ship class based on laden-ballast ratio performance rank.** Motif prevalence by ship class for three groups: all ships, the top-ranked 25% by LBR performers, and the bottom-ranked 25% by LBR performers. Left panel shows laden-first motifs; right panel shows ballast-first motifs. Marker size indicates prevalence (in percentages).

To examine the relationship between routing patterns and LBR, we compared the motif prevalence between high-performing (top quartile by LBR) and low-performing (bottom quartile by LBR) ships, stratified by vessel class and cargo status (shown in fig. 4). Within each performance group, prevalence values are calculated as the weighted average of individual ship prevalence rates, where each ship is weighted by its total number of observed motifs; e.g. the weighted average prevalence of laden-first *ABC* motif among all Panamax ships is 30%.

Our analysis reveals a striking relationship: high-performing ships consistently exhibit greater prevalence of *ABC* motifs (visiting three different regions consecutively) and diminished prevalence of *AAA* motifs (remaining within the same region). Meanwhile, the inverse is true for low performing ships. This relationship holds across all ship classes, with top 25% performers displaying notably larger *ABC* prevalence and correspondingly smaller *AAA* prevalence compared to the bottom 25% performers (seen by the marker sizes in top 25% versus bot 25% columns).

The cargo status analysis reveals prominent usage of intra-regional legs for ballast operations. In laden→ballast motifs, *ABB* motifs (where the second leg represents intra-regional ballast movement) are more prevalent than *AAB* motifs overall, and especially in the top 25% group, consis-

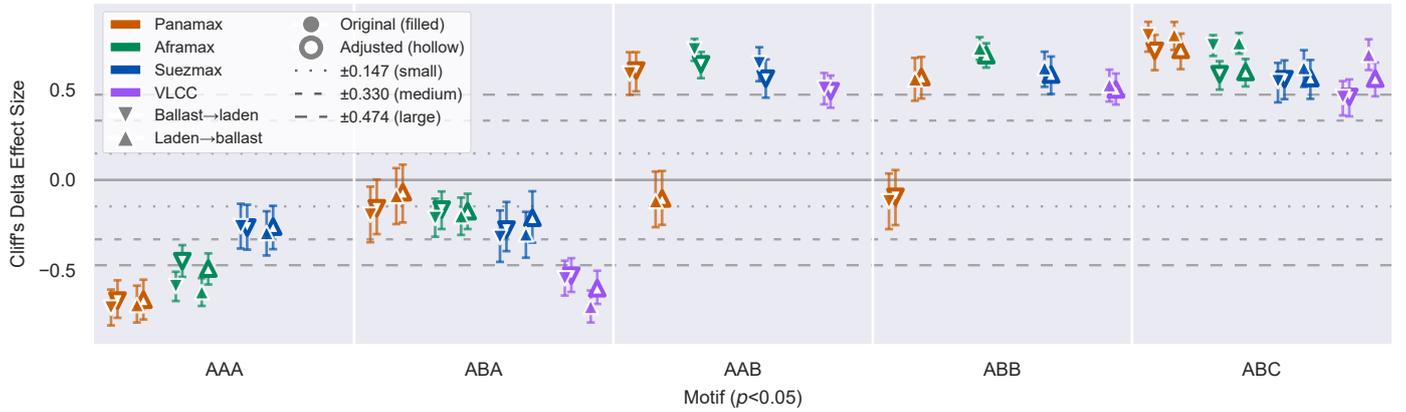


Figure 5: **Motif effect size (Cliff’s delta δ) comparing top- and bottom- performers by laden-ballast ratio.** Effect sizes (with bootstrapped 95% confidence intervals) of statistically significant associations measured between top-ranked 25% against bottom-ranked 25%, grouped by vessel type and cargo status. Colors indicate vessel type and shapes distinguish cargo status (\blacktriangle : laden \rightarrow ballast; \blacktriangledown : ballast \rightarrow laden). Filled markers (\blacktriangle , \blacktriangledown) indicate original δ_{orig} , while hollow markers (\triangle , \triangledown) indicate age-adjusted δ_{adj} . Dotted vertical lines denote effect size thresholds (small: ± 0.147 ; medium: ± 0.33 ; large: ± 0.474).

290 tent with the strategy of minimizing ballast travel times by
 291 seeking nearby cargo when available. The reverse applies to
 292 the ballast \rightarrow laden motifs, with increased *ABA* prevalence
 293 for high performance ships optimizing their cargo pickup
 294 location.

295 We strengthen our findings through comprehensive statisti-
 296 cal validation. We systematically control for ship age—
 297 distribution shown in fig. 2 (bottom)—as a potential con-
 298 founder and establish the statistical significance of observed
 299 trends through multiple approaches. Our stratified anal-
 300 ysis by vessel class and motif cargo status employs Beta
 301 regression with motif prevalence as the dependent variable
 302 and LBR and ship age as regressors, isolating the indepen-
 303 dent contribution of each predictor (detailed in Methods
 304 and Supplementary Information section 2.3). The regres-
 305 sion coefficients quantify each regressor’s unique effect on
 306 motif prevalence, with p-values confirming whether these
 307 effects are statistically significant. Additionally, we cap-
 308 ture the magnitude of group separation using Cliff’s delta
 309 δ [29, 30], a robust non-parametric effect size that reveals
 310 distributional shifts beyond what simple mean comparisons
 311 can detect.

312 The associations between movement patterns and vessel
 313 LBR discussed above are statistically significant, show sub-
 314 stantial effect sizes, and remain robust after controlling for
 315 vessel age. Effect sizes measured by Cliff’s delta, shown
 316 in fig. 5, reveal large positive effects for *ABC* motifs across
 317 all vessel classes, while *AAA* motifs display small to large
 318 negative effect sizes for Panamax, Aframax, and Suezmax
 319 vessels. The laden-first *ABB* and ballast-first *AAB* motifs
 320 also exhibit large effect sizes across all vessel classes. These
 321 patterns achieve statistical significance ($p < 0.05$ after con-
 322 trolling for false discovery rates [31]) with the sole exception
 323 of *AAA* motifs in VLCCs. Critically, adjusting for ship age
 324 produces negligible changes in effect sizes (mean absolute
 325 difference $|\delta_{orig} - \delta_{adj}| = 0.056$; 20 out of 32 comparisons dif-
 326 fer by less than 0.05), confirming that the motif-LBR asso-
 327 ciations represent genuine operational patterns rather than
 328 age-related artifacts. Qualitative interpretations of effect
 329 magnitude remain consistent whether or not age is explic-
 330 itly controlled, with only Aframax’s *AAA* ballast \rightarrow laden
 331 shifting from large to medium effect size after adjustment

(fig. 5).

332
 333 VLCCs present a slightly unique case due to their opera-
 334 tional constraint: unlike other ship classes, VLCCs show
 335 minimal *AAA* usage across both performance groups, re-
 336 flecting their specialized role in long-haul routes with few
 337 intra-regional routes available to them. Instead, high-
 338 performing VLCCs show a reduced *ABA* motif prevalence
 339 (representing round-trips between two regions), suggest-
 340 ing that even within their operational constraints, high-
 341 performing VLCCs benefit from route diversification rather
 342 than repeating the same trading route. While reduced *ABA*
 343 motif prevalence of the top 25% is also observed in other
 344 ship classes, their effect sizes are notably smaller than in
 345 VLCCs, suggesting a weaker reliance on *ABA* motifs across
 346 the general population of ships (see Supplementary Infor-
 347 mation section 2.4 for VLCC leg duration comparisons of
 348 *ABC* and *ABA* motifs).

349 Together, these findings indicate that route diversifica-
 350 tion represents a key component of operational efficiency,
 351 with more efficient ships consistently exhibiting varied mar-
 352 ket exposure while optimizing ballast destinations through
 353 intra-regional movements.

354 Identifying periodicity in shipping activity

355 While our motif analysis establishes a connection between
 356 individual ship efficiency and route diversification, these
 357 individual routing decisions aggregate to create system-
 358 level phenomena. The collective movements of many ships,
 359 driven by opportunities arising from fluctuations in the
 360 crude oil market, manifest as non-trivial temporal signals at
 361 the regional level. Although some fluctuations are a result
 362 of highly unpredictable events (i.e. geopolitical disruptions
 363 or accidents), others may be driven by predictable physical
 364 or social phenomena, ranging from climate patterns [32] to
 365 socioeconomic patterns in production, operation, or con-
 366 sumption [33, 34]. We therefore shift our focus from ship-
 367 centric trajectories to examine the macro-scale cargo flow
 368 dynamics that these aggregated movements generate across
 369 regions.

370 These macro-scale dynamics are captured by time-series
 371 $y(t)$ of cargo flows (measured in 10,000 deadweight ton-

372 nage) entering and leaving a region for each ship class,
 373 yielding 8 time series per region. Each unique combina-
 374 tion [region, direction, ship class] is referred to as a *re-*
 375 *gional flow segment*, indexed with subscript n . To cap-
 376 ture the complex dynamics within this collective time series
 377 $\mathbf{y}(t) = [y_1(t), \dots, y_n(t), \dots]$, we employ bagging-optimized
 378 Dynamic Mode Decomposition (DMD) [35–37], a powerful
 379 and noise-robust tool for extracting spatio-temporal pat-
 380 terns from a vector time series, with regional flow segments
 381 serving as spatial coordinates. Detailed methodology on
 382 the time series construction and DMD reconstructions is in
 383 the Methods section.

384 A rank R DMD yields R complex spatial components
 385 $\phi_{n,j}$, scalar amplitudes A_j , and a temporal frequency com-
 386 ponent ω_j , where n is the regional flow segments index and
 387 j is the index of the R modes. The DMD reconstructed
 388 time series for region flow segment n can be expressed in the form
 389

$$\hat{y}_n(t) = \sum_j A_j |\phi_{n,j}| \cos(\omega_j t + \arg(\phi_{n,j})). \quad (2)$$

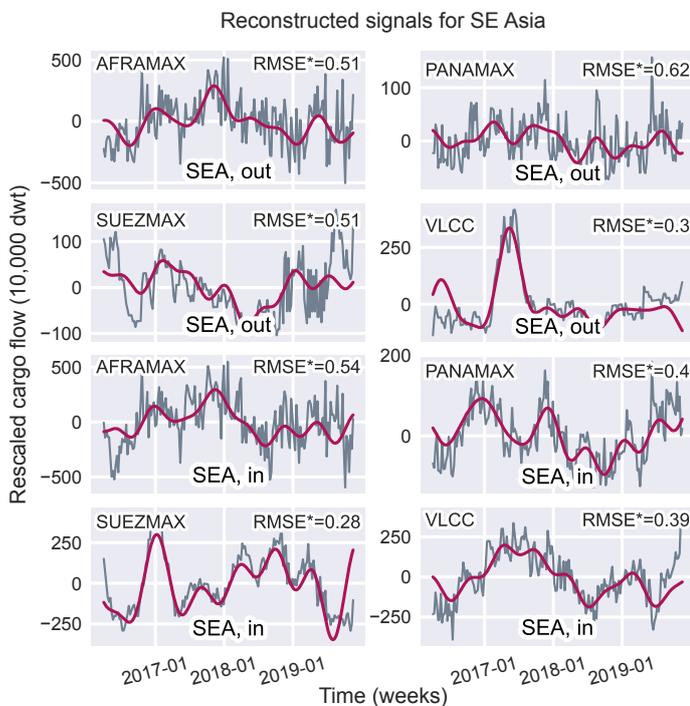


Figure 6: **Time series of cargo flow in South East Asia.** The original data is shown in gray, and the reconstructed signal in magenta. Each plot is labeled with the vessel class and the flow direction. The normalized root mean squared error $RMSE^*$ is shown for each time series.

390 Despite the noisy data, the DMD method is able to con-
 391 sistently recover periodic oscillatory patterns; examples for
 392 the reconstruction $\hat{y}_n(t)$ for South East Asia are shown
 393 in fig. 6. The reconstruction errors are measured using the
 394 normalized root mean squared error $RMSE^*$ [38], which
 395 lies in the range of $[0, 1]$ where 0 represents a perfect
 396 reconstruction. We see that for $RMSE^* \leq 0.5$ the recon-
 397 structions visually fit the peaks and troughs, although much
 398 of the shorter-scale fluctuations are not captured.

399 The amplitude A_j is interpreted as the overall contribu-
 400 tion of mode j to the dynamics; modes with larger A_j
 401 are more important. The spatial components $\phi_{n,j}$ repre-
 402 sent the individual contributions of the regional flow seg-

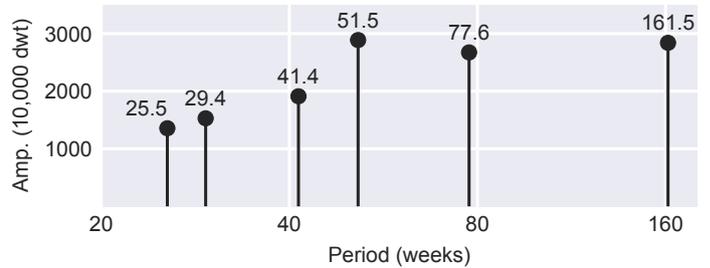


Figure 7: **Amplitude A_j against period τ_j from DMD decomposition.** Twelve modes were chosen for the decomposition, which were merged into 6 as each mode has a complex conjugate pair.

403 ments n to mode j , and the complex argument of $\phi_{n,j}$ in-
 404 troduces a phase shift that controls when the peaks in each
 405 cycle occur. Thus, the combined term $A_j |\phi_{n,j}|$ in eq. (2)
 406 is interpreted as the amplitude of regional flow segment n .
 407 The temporal frequencies are more easily understood as co-
 408 sine oscillation periods $\tau_j = 2\pi/\omega_j$. Note that, since DMD
 409 modes come in complex conjugate pairs with identical con-
 410 tributions to the reconstruction, we interpret our results in
 411 terms of these pairs.

412 Our decomposition uncovers a dominant annual cycle
 413 that governs global tanker movements. The scalar ampli-
 414 tudes and oscillation periods from the decomposition are
 415 shown in fig. 7, where the mode index j are ordered in
 416 decreasing period lengths. The dominant mode-pair ex-
 417 hibits a striking 51.5-week oscillation period with 16% peak-
 418 to-trough amplitude variations across regional flows. To
 419 strengthen the validity of this finding, we applied two al-
 420 ternative periodicity extraction methods: Fourier transform
 421 and Multi-channel Singular Spectrum Analysis (MSSA) [39,
 422 40]. Both methods independently recovered dominant peri-
 423 ods of 48.0 weeks (95% CI: 47.1 – 49.9 weeks) via Fourier
 424 analysis and 49.5 weeks via MSSA, confirming the valid-
 425 ity of our findings (see Supplementary Information section
 426 3.2).

427 This yearly seasonal cycle firmly matches our prior expec-
 428 tations, as the natural seasons shape physical and socio-
 429 economic forces that drive the maritime oil trade. There
 430 are several direct factors that affect the movement of ships.
 431 In particular, the economic supply and demand of crude
 432 oil is driven primarily by the transportation sector, with
 433 secondary influences from the petrochemical industry and
 434 the energy sector [41, 42]. Analysis of global crude oil
 435 consumption [33] also confirms annual seasonality in con-
 436 sumption patterns. Our primary focus lies on interpreting this
 437 mode-pair; we present the rest our results in Supplementary
 438 Information section 3.3.

439 Our results reveal that different ship classes operate with
 440 distinct regional footprints, creating specialized seasonal
 441 trading patterns across the globe. Analysis of regional con-
 442 tributions to annual seasonality in each ship class, visual-
 443 ized in fig. 8, identifies where each vessel class concentrates
 444 its seasonal operations, with incoming and outgoing flow di-
 445 rections revealing the critical import and export hubs driv-
 446 ing global trade cycles. The most significant patterns are
 447 described below.

448 For incoming flows, VLCCs exhibit the most re-
 449 gional concentration in seasonal amplitude variation, seen
 450 in fig. 8a, with China dominating the annual oscillations.
 451 This concentration reflects both the dedicated infrastruc-

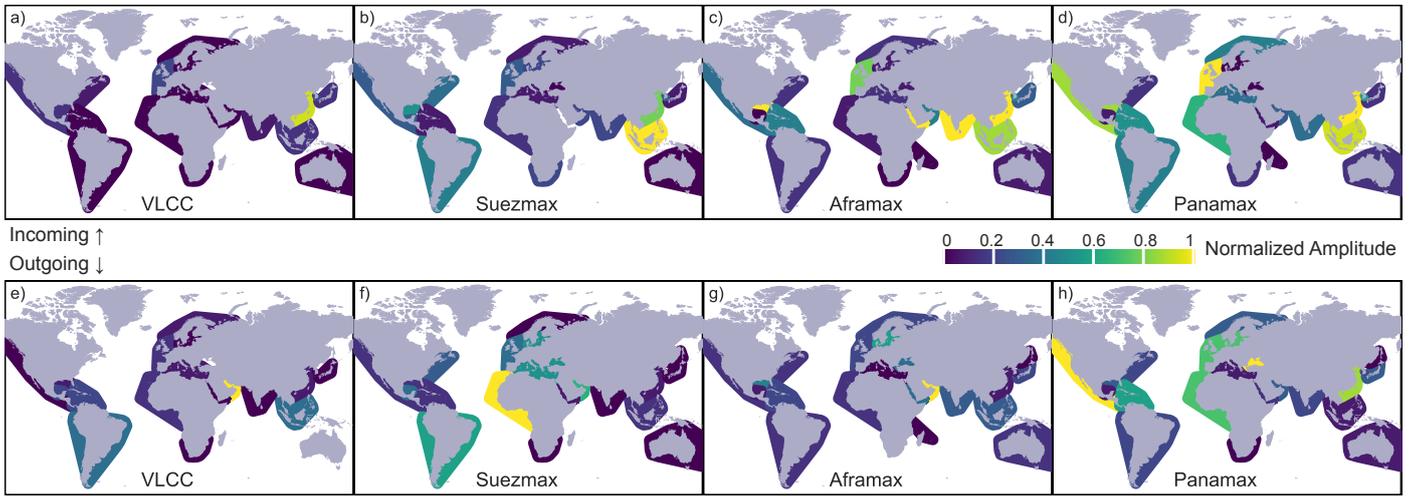


Figure 8: **Normalized regional amplitudes by ship class and flow direction.** Amplitudes, indicated by hue intensity, are normalized in each map by dividing actual amplitudes by the maximum observed amplitude within the same map. Maximum observed amplitudes by vessel class: VLCC (291.7 incoming, 238.7 outgoing), Suezmax (102.9 incoming, 99.0 outgoing), Aframax (99.6 incoming, 124.8 outgoing), and Panamax (38.2 incoming, 34.4 outgoing). Amplitude units are in 10,000 deadweight tonnage.

452 ture required for the largest vessels and their substantial
 453 cargo capacity. These factors draw VLCCs to regions with
 454 the highest demand—primarily economically developed areas
 455 in the northern hemisphere with port facilities capable of
 456 accommodating these massive ships.

457 The geographical distribution of high incoming flow amplitude
 458 regions becomes increasingly dispersed for smaller vessels,
 459 reflecting their greater operational flexibility and broader
 460 port accessibility. Suezmax vessels (fig. 8b) maintain
 461 strong seasonal amplitudes around China but show their
 462 most prominent activity in Southeast Asia, with notable
 463 contributions from South America. Aframax vessels (fig. 8c)
 464 demonstrate further geographical spread encompassing
 465 much of Asia and central America, while Panamax vessels
 466 (fig. 8d) exhibit the most even distribution of amplitudes
 467 across the globe. The prominence of Western European
 468 and US ports for Panamax operations may reflect port
 469 size limitations in these regions. The amplitude scale
 470 factors (VLCC: 292, Suezmax: 103, Aframax: 100, Panamax:
 471 38) reveal that a significant portion of seasonal demand
 472 is supplied by VLCCs, particularly toward China, while
 473 Aframax and Panamax vessels support most of the remaining
 474 global seasonal demand.

475 The outgoing flow patterns reveal complementary geographical
 476 specializations shaped by both resource distribution and
 477 maritime infrastructure constraints. VLCC exports (fig. 8e)
 478 show dominant seasonal amplitudes primarily in the Middle
 479 East followed by South America and Southeast Asia. While
 480 the Middle East and South America represent major crude
 481 oil production centers, Southeast Asia’s prominence reflects
 482 its role as a major crude oil trading and storage hub rather
 483 than production, serving as redistribution centers for the
 484 Far East market. Suezmax (fig. 8f) exports are primarily
 485 driven by West Africa, the Mediterranean, and the Middle
 486 East—a pattern reflecting how Suezmax vessels, designed
 487 as the largest ships capable of Suez Canal transit, can
 488 efficiently serve seasonal Asian demand.

489 Aframax (fig. 8g) exports remain concentrated in the
 490 Middle East, while Panamax (fig. 8h) exports show peak
 491 amplitudes in the western United States, followed by West-

493 ern Europe, West Africa, and China. The western United
 494 States dominance in Panamax exports suggests these ves-
 495 sels transited the Panama Canal eastward to access Atlantic
 496 regions [43]. These amplitude distributions demonstrate
 497 how size constraints interact with port infrastructure lim-
 498 itations and geographical resource distributions to create
 499 distinct seasonal flow patterns across the global shipping
 500 network.

501 Additionally, the DMD analysis on phase shifts uncovers
 502 patterns of synchronous and anti-synchronous oscilla-
 503 tions between ship classes and between regions, illus-
 504 trated in fig. 9. We find that few regions exhibit strong
 505 synchronicity across the ship classes, as observed through
 506 the clustering (or lack thereof) of markers along rows
 507 in fig. 9. Notably, only China has the phase shift of
 508 all ship classes being aligned around a seasonal peak in
 509 February/March, or $\pi/2$ corresponding to an offset of three
 510 months, (see fig. 9a). Other notable regions (Middle East
 511 and South East Asia) show moderate synchronicity, where
 512 only the VLCC outgoing flows display a distinctly different
 513 phase shift, highlighted in fig. 9b and fig. 9c.

514 We also observe clustering of multiple regions around the
 515 start of the calendar year, i.e. February/March (see fig. 9d),
 516 corresponding to the winter months of the Northern hemi-
 517 sphere. Indeed, these include prominent developed econ-
 518 omic northern regions in Europe and Asia, which explains
 519 their ability to import in large quantities. Interestingly,
 520 South East Asia, while not a northern region, is largely
 521 in sync with this particular seasonal cycle, potentially due
 522 to their strong trading relationship with their northern
 523 Asian partners. Meanwhile, contributors to the outgoing
 524 flow (such as the Middle East) experience a peak around
 525 June/July or $-\pi/2$, anti-synchronous to the winter peaks.
 526 American seasonal patterns (USG, USW, and USA) do not
 527 appear to be in sync with the rest of the northern regions,
 528 potentially explained by their increasing independence in
 529 crude oil production [41].

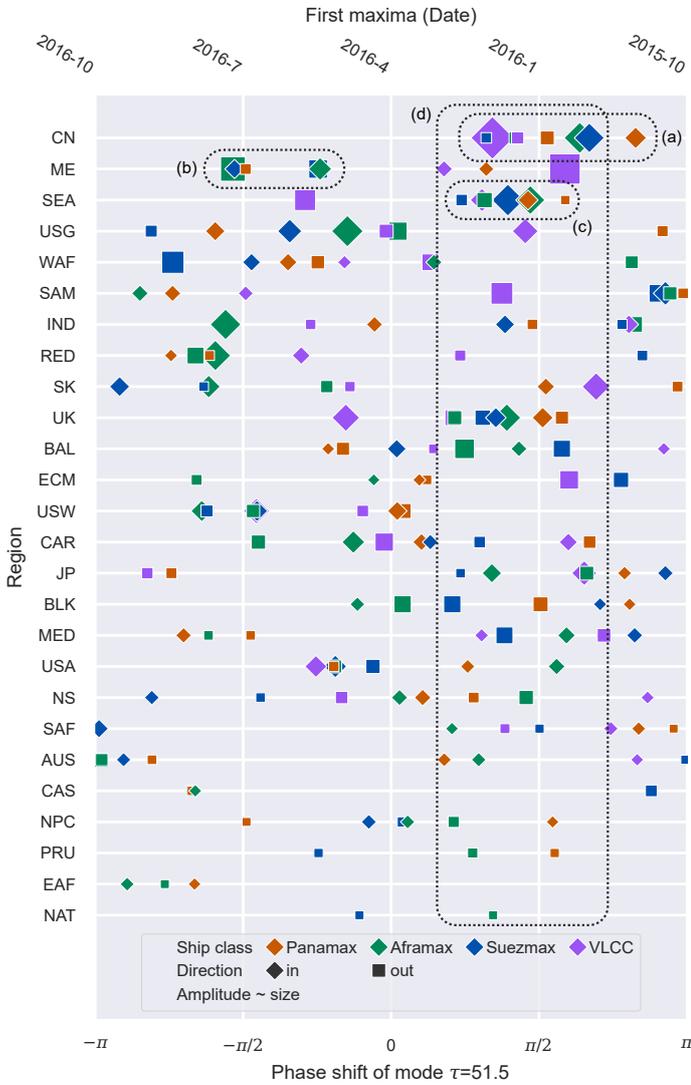


Figure 9: **Phase-amplitude plots for each region.** The color and marker size indicate the ship class and amplitude, respectively. The phase can be interpreted as where in time the first peak is by reading the top axis (note the direction of time is right to left), or the angular shift θ in $\cos(\theta + 2\pi/\tau)$ for $\tau = 51.5$ weeks by reading the bottom axis. Dotted outlines represent areas of interest.

Discussion

We demonstrated that moving beyond time-aggregated static network analysis reveals previously undetected patterns in maritime tanker networks at the individual vessel and fleet level. By applying sequential motif analysis, we uncover links between individual routing strategies and operational efficiency, while Dynamic Mode Decomposition shows seasonal spatio-temporal patterns in the dynamic global flow of cargo.

At the individual vessel level, our sequential motif analysis reveals that strategic route diversification correlates with superior operational routing performance across all ship classes. Top-performing ships favor *ABC* motifs (regional exploration) over *AAA* motifs (single-region focus) and *ABA* motifs (repeating round trips), even after controlling for ship age and stratifying by ship class, suggesting genuine strategic advantages beyond economies of scale or vessel-specific technical characteristics. While a definitive causal relationship warrants further investigation, we interpret this correlation as reflecting responsiveness to regional

price differentials and cargo availability. Unlike container ships with fixed routes, tankers operate in charter-based markets where strategic flexibility enables exploitation of market opportunities across different regions, often manifesting as *ABC* motifs. Our findings suggest that fleet operators practicing regional specialization may improve performance through greater strategic diversification and market responsiveness. We note that *AAA* motifs encompass heterogeneous routing behaviors (i.e. from repetitive shuttles between specific port pairs to diverse movements across multiple ports within a region) which our regional aggregation cannot distinguish; future sub-regional analysis could clarify whether diversification benefits persist at finer spatial scales.

At the system level, our DMD analysis identifies pronounced seasonal patterns in regional cargo flows with quantifiable periods and phase relationships between maritime regions. These seasonalities represent strong underlying structural trends; however, they can also be obscured by short-term fluctuations week-to-week due to non-linear drivers—including market volatility, port congestion, weather delays, or disruptive events—which manifest as high frequency noise. Our findings should therefore inform strategic planning over longer horizons. For fleet operators, this enables strategic positioning towards regions experiencing increasing seasonal supply or demand, opening opportunities for vessels willing to explore markets beyond their traditional operational areas. For port and governmental authorities, these patterns enable proactive planning for quarterly import/export variations, including timing infrastructure upgrades during off-peak periods, allocating seasonal workforce and berth capacity, and coordinating logistical resources to match anticipated cargo.

These findings establish strong ties between routing performance incentives and routing diversification; the preference for exploration over regional concentration among high-performing vessels provides actionable insights for fleet optimization strategies that can outperform competitors in revenue generation versus costs incurred. Furthermore, our new framework closes methodological gaps explicitly identified by Álvarez et al. [9], who emphasized the need for “deeper comparison among connected ports in terms of performance indicator” and urged researchers to “check the interactions between economy evolution in different regions and the maritime routes”.

However, several limitations should be acknowledged. In the absence of ship-specific operational data, we constructed a performance indicator using the durations of laden and ballast legs. While these durations are recorded precisely in the data, they include port waiting times stemming from factors outside operator control (e.g., port congestion, administrative delays), which ideally would be isolated to strengthen inferences about routing strategy efficiency. With the availability of additional data—by including route-specific details such as the distance traveled and freight rate realized, ship-specific details such as the average speed and fuel consumption rate, or environmental conditions such as wind speeds or wave heights [44]—researchers could construct alternative performance indicators or EEOI proxies [45], including CO₂-based indicators, enabling more direct evaluation of individual vessel routing decisions’ environmental impact alongside economic efficiency. Alternatively, such enriched datasets would enable fleet-level comparative analysis through multivariate approaches that si-

614 multaneously control for technical, environmental, and op- 672
615 erational factors while isolating routing strategy effects. 673

616 While the Bagging-Optimized DMD method incorporates 674
617 sub-sampling to improve robustness against noise, our ap- 675
618 proach assumes an additive linear seasonal model [46], and 676
619 deliberately excludes nonlinear drivers such as oil price 677
620 shocks, policy changes, and geopolitical tensions or con- 678
621 flicts. Nonetheless, our primary objective was to establish 679
622 whether underlying seasonal components exist and can be 680
623 recovered despite system noise—and indeed we successfully 681
624 demonstrate both. Future research could develop more pre- 682
625 dictive models that explicitly incorporate disruptive events 683
626 alongside the baseline seasonal patterns we have estab- 684
627 lished. 685

628 Our methodological choices, together with the pre- 686
629 COVID time frame of our dataset (2016 January to 2020 687
630 February), limit the direct generalizability of the specific 688
631 spatial and temporal patterns we report. Nevertheless, our 689
632 dual-scale framework is fully deployable on post-COVID 690
633 data, where comparisons to pre-COVID data can reveal 691
634 whether major disruptions have fundamentally reshaped 692
635 seasonal trading behavior or whether the system has re- 693
636 verted to its pre-disruption state. 694

637 Our methodology extends beyond oil tanker applications. 695
638 Sequential motif analysis applies directly to dry bulk car- 696
639 riers and other charter-based shipping or transport sectors 697
640 with flexible routing patterns. The DMD component could 698
641 be applied to any transportation system with constrained 699
642 capacity and regional flows, including trucking networks or 700
643 airline cargo operations, while the general framework of 701
644 linking individual behavioral patterns to system-wide ef- 702
645 ficiency outcomes has broad applicability across complex 703
646 networked systems. 704

647 Methods

648 Ballast leg definition

649 Ballast legs (where a ship is not carrying cargo) are not 705
650 explicitly recorded in our dataset; instead, we infer their 706
651 occurrence between two consecutive laden legs of the same 707
652 ship: given that a particular ship performs two laden legs 708
653 from regions $a \rightarrow b$ and then $c \rightarrow d$, this implies a ballast 709
654 leg from $b \rightarrow c$. Consequently, our definition of ballast legs 710
655 may include other activities not typically related to reach- 711
656 ing the next load port. Note that, as the only timestamps 712
657 recorded are the departure and arrival times of each laden 713
658 leg, the duration of ballast legs are consequently on average 714
659 artificially longer than that of laden legs, as the additional 715
660 amount of time spent on loading and discharging is not 716
661 known. Nonetheless, including these port waiting times is 717
662 logical from an economic perspective, as delays of this type 718
663 incur further costs to operators without adding to revenues, 719
664 much like ballast legs do. 720

665 Port waiting days adjustment

666 Our dataset records laden voyage departure and arrival 721
667 times, but arrival timestamps correspond to berthing rather 722
668 than port arrival, systematically conflating at-sea time with 723
669 port waiting time. Since port waiting times vary randomly, 724
670 this introduces randomly distributed positive bias that dis- 725
671 proportionately affects shorter voyages. 726

We corrected this systematic bias using a reference 672
dataset with complete temporal information (including sep- 673
arate port arrival and berthing times) covering different 674
vessels and time periods. Analysis of this reference data re- 675
vealed that port waiting times are approximately independ- 676
ent of voyage duration and appear randomly distributed 677
within each vessel class. Empirically, these distributions 678
are well-characterized by lognormal models, which we fit- 679
ted using maximum likelihood estimation to obtain vessel- 680
class-specific parameters (μ, σ) in log-space. 681

To ensure conservative corrections that avoid introduc- 682
ing bias in the opposite direction, we applied lower-bound 683
estimates based on the mode of each lognormal distribution 684
 $\exp(\mu - \sigma^2)$. This approach identifies, with 95% confidence, 685
the minimum port waiting time that vessels spend at the 686
end of each laden leg. The conservative adjusted at-sea 687
laden durations are obtained by subtracting the correction 688
values from observed laden leg durations. See Supplemen- 689
tary Information section 1.3 for detailed results. 690

Validation analysis demonstrates that LBR performance 691
rankings remain stable across correction levels: mean rank 692
changes are ≤ 2 position for VLCCs, Suezmax, and Pana- 693
max, and ≤ 5 positions for Aframax vessels under median 694
corrections (Figure S3 in Supplementary Information), with 695
smaller changes for modal correction. For multi-event voy- 696
ages (multiple loads or multiple discharges), we applied the 697
same correction methodology at intermediate port visits to 698
the total port duration from arrival to departure. 699

700 Comparison of laden-ballast ratio to EEOI

Performance indicators are useful tools for evaluating the 701
efficiency of a ship’s operations. The ideal performance in- 702
dicator would accurately pick out the most efficient ships 703
(e.g. high revenue yield, low emissions rate, low incurred 704
costs, etc) and the least efficient ships accurately. The In- 705
ternational Maritime Organization (IMO) encourages the 706
use of the Energy Efficiency Operational Indicator (EEOI), 707
which acts as an industry standard indicator. The EEOI is 708
defined as 709

$$EEOI = \frac{\sum_i C_i}{\sum_i T_i D_i}, \quad (3)$$

where C is the carbon emissions, T is the cargo tonnage, D 710
is the distance traveled, and i is the index of a leg (which 711
can be either laden or ballast). The carbon emissions C 712
are calculated as $\sum_\alpha F_\alpha c_\alpha$, where F_α is the quantity of fuel 713
consumed and c_α is the CO_2 conversion factor for fuel type 714
 α . However, such detailed data are often only available to 715
ship or fleet owners, making direct calculations and com- 716
parisons of the EEOI challenging on an industry level. Ad- 717
ditionally, the EEOI is particularly sensitive to ship-specific 718
parameters, and as such is not as useful in comparing ship- 719
independent processes. 720

To tackle this challenge, we proposed an alternative route 721
performance indicator, the laden-ballast ratio (*LBR*). In 722
our data, the key parameters that are available to us are (i) 723
the load departure time. and (ii) discharge arrival times. 724
Given trajectory $J = \{(t_i, s_i)\}$ where t_i are port visit times 725
and $s_i \in \{\text{load, discharge}\}$, we define the set of laden legs L 726
and ballast legs B as 727

$$L = \{(t_i, t_{i+1}) : (s_i, s_{i+1}) = (\text{load}, \text{discharge})\}, \quad (4)$$

$$B = \{(t_i, t_{i+1}) : s_i = \text{discharge}, s_{i+1} = \text{load}\}. \quad (5)$$

728 Using these definitions, we compute the durations of the
729 laden legs and ballast legs as

$$\text{For } l = (t_1, t_2) \in L: \Delta t_l = t_2 - t_1 - \text{PWT}_c, \quad (6a)$$

$$\text{For } b = (t_1, t_2) \in B: \Delta t_b = t_2 - t_1, \quad (6b)$$

730 where PWT_c is the lower-bound estimated port waiting
731 time for vessel class c . The *LBR* for a given ship is then
732 defined as

$$\text{LBR} = \frac{\sum_{l \in L} \Delta t_l}{\sum_{l \in L} \Delta t_l + \sum_{b \in B} \Delta t_b}, \quad (7)$$

733 where Δt_l is the at-sea duration of laden leg l and Δt_b is
734 the duration of ballast leg b .

735 The laden-ballast ratio measures the proportion of time a
736 ship spends carrying cargo during its journey. This measure
737 captures both the revenue-generating aspect of laden dura-
738 tions and the cost-savings associated with shorter ballast
739 durations. By focusing on temporal patterns—time spent
740 laden versus ballast—it isolates the effectiveness of routing
741 decisions independent of vessel-specific factors such as en-
742 gine efficiency or fuel-type. Ships with higher laden-ballast
743 ratios spend more time performing useful work transport-
744 ing cargo, or waste less time and energy traveling ballast, or
745 both. As the industry increasingly adopts data-driven opera-
746 tions, this measure provides a practical tool for evaluating
747 and comparing route-based performance between ships or
748 fleets across different ownership groups.

749 Furthermore, the laden-ballast ratio shares similar prop-
750 erties with the EEOI. First, longer laden legs are beneficial.
751 In the EEOI calculations, the average ratio of fuel consumed
752 per unit distance for laden vs. ballast legs is much smaller
753 than the ratio of tonnage carried (ballast legs have 0 useful
754 tonnage carried). Thus, while laden legs emit more CO_2 ,
755 longer laden legs tend to be more efficient in the long run due
756 to the tonnage carried being higher. Second, longer ballast
757 legs are penalized. The ballast leg duration is defined as any
758 time between consecutive laden leg durations. Thus, port
759 waiting times are included in our ballast leg durations. As
760 such, our performance indicator penalizes ballast legs more
761 heavily than the EEOI—this approach reflects a financial
762 perspective where time spent waiting at ports represents
763 the same opportunity cost as time spent traveling ballast.
764 See Supplementary Information section 1.4 for further de-
765 tails on the EEOI.

766 Sequential Motif Mining

767 Sequential motifs represent short recurring patterns within
768 trajectory sequences that capture higher-order dependen-
769 cies in movement data [47]. Unlike traditional network mo-
770 tifs that focus on structural relationships between ports [8,
771 13, 48–50], sequential motifs analyze the ordered trajectory
772 patterns of individual ships navigating through the mar-
773 itime network. Selecting an appropriate motif length (i.e.
774 the chosen correlation length) is closely linked to identi-
775 fying the optimal order in higher-order networks [26–28],
776 where ℓ -hop sub-sequences serve as fundamental nodes of
777 the network.

778 We determined optimal motif length by evaluating
779 higher-order network models at orders 1 to 4 (beyond which
780 model complexity becomes computationally expensive) us-
781 ing information-theoretic criteria that balance explanatory
782 power against model complexity: Akaike Information Crite-
783 rion (AIC) [51], Bayesian Information Criterion (BIC) [52],

and likelihood ratio tests [53]. Each of these criteria applies
different penalties, offering complementary perspectives on
the optimal order selection. Parameters were estimated via
maximum likelihood fitting of transition matrices on the
full dataset for each vessel class.

789 Across all vessel classes, both AIC and likelihood ratio
790 tests consistently selected order-2 as optimal (see Supple-
791 mentary Information section 2.1). While BIC’s stricter
792 complexity penalty favored order-1, the order-2 BIC val-
793 ues remained substantially closer to order-1 than to order-
794 3, indicating that order-2 provides meaningful improve-
795 ment without excessive parameterization. Intuitively, this
796 means that routing decisions depend on the previous loca-
797 tion and current location, but extending knowledge further
798 back adds little explanatory power at the cost of dramati-
799 cally increasing model complexity. For order $\ell = 2$, the
800 higher-order network edges represent 3-node trajectory sub-
801 sequences, which our 2-hop motifs directly capture. This
802 convergence of independent model selection criteria estab-
803 lishes that 2-hop motifs represent the minimal sequence
804 length necessary to capture statistically significant sequen-
805 tial dependencies in tanker routing patterns.

806 Having established the optimal length, we now formally
807 define sequential motifs. A sequential motif M of length ℓ
808 is defined as a sequence of alphabets $(m_1, m_2, \dots, m_{\ell+1})$ where
809 $m_i \in \mathcal{A}$ for a finite alphabet set $\mathcal{A} = \{A, B, C, \dots\}$. Denot-
810 ing regions as v_i and the set of regions as V , a regional se-
811 quence $S = (v_1, v_2, \dots, v_{\ell+1})$ constitutes an instance of mo-
812 tif M if there exists a unique mapping function $f: V \rightarrow \mathcal{A}$
813 such that $f(v_j) = m_j$ for $1 \leq j \leq \ell + 1$, with the con-
814 straint that $m_i = m_j \Rightarrow v_i = v_j$. This formulation ensures
815 that motifs capture topological relationships independent
816 of specific regional identities.

817 At length 2, there are exactly five possible motif types:
818 *AAA*, *ABA*, *AAB*, *ABB*, and *ABC*. For example, the
819 sequence (*China* \rightarrow *Japan* \rightarrow *China* \rightarrow *SEA* \rightarrow *SEA* \rightarrow
820 *Japan*) contained the following 2-hop motifs: *ABA*, *ABC*,
821 *ABB*, *AAB*.

822 The total number of possible motifs of length ℓ follows
823 the Bell numbers, representing unique partitions of $\ell + 1$
824 labels — see <https://oeis.org/A000110>.

825 Multivariate motif model

826 To control for potential confounding by vessel age, we em-
827 ployed multivariate Beta regression for each motif variant
828 across all vessel classes. Beta regression is appropriate for
829 modeling proportional data (e.g. motif prevalence $\in [0, 1]$)
830 where observed values are close to the boundary [54, 55].
831 For each motif-class combination, we model motif preva-
832 lence ρ_M using Beta regression with logit link:

$$\text{logit}(\mu) = \beta_0 + \beta_{\text{LBR}} \cdot \text{LBR} + \beta_{\text{age}} \cdot \text{age}, \quad (8)$$

833 where $\mu = \mathbb{E}(\rho_M | \text{LBR}, \text{age})$ is the conditional mean mo-
834 tif prevalence. The regression yields coefficients β_{LBR} and
835 β_{age} quantifying the strength of association between each
836 regressor and motif prevalence ρ_M . Statistical significance
837 was assessed via hypothesis tests for $\beta_{\text{LBR}} \neq 0$ after apply-
838 ing Benjamini-Yekutieli correction across 40 tests (10 motif
839 variants \times 4 vessel classes) [31].

840 To isolate age effects from routing performance effects, we
841 computed age-adjusted LBR residuals by regressing LBR
842 against vessel age. These residuals capture the performance
843 variation unexplained by age alone. Complete results on the

844 Beta regression coefficients, p -values, and age-adjusted ef- 894
 845 fect sizes are detailed in Supplementary Information section 895
 846 2.3. 896

847 Motif mining algorithm

848 For each vessel journey $S = (v_1, v_2, \dots, v_{n+1})$ representing a 900
 849 sequence of regions visited: 901

- 850 1. Extract all consecutive 3-region subsequences $S_{i:i+2} =$ 902
 851 (v_i, v_{i+1}, v_{i+2}) for $i = 1, \dots, n - 1$.
- 852 2. Apply the topological mapping function to classify 903
 853 each subsequence. 904

854 Letting $\kappa_M(S) = \text{count of motif } M \text{ in } S$, for a collection of 905
 855 journeys S_i , the prevalence of motif M is calculated as 906

$$\rho_M = \frac{\sum_i \kappa_M(S_i)}{\sum_i (|S_i| - 2)}. \quad (9)$$

856 Dynamic Mode Decomposition for shipping 857 dynamics

858 To uncover coherent patterns in our regional shipping 907
 859 data, we employ the Dynamic Mode Decomposition (DMD) 908
 860 method [35], a powerful data-driven approach to studying 909
 861 dynamics of spatio-temporal data. DMD has been extended 910
 862 from its origins in fluid dynamics to a range of network data 911
 863 analysis—such as disease spreading dynamics and brain dy- 912
 864 namics [56–58]—yet remains unexplored in maritime net- 913
 865 work analysis despite its suitability for spatiotemporal pat- 914
 866 tern extraction in constrained systems. 915

867 DMD is able to treat spatial and temporal modes simul- 916
 868 taneously, making it particularly effective for identifying 917
 869 system-wide trends, something that can be leveraged in net- 918
 870 works where the underlying topological connections act as 919
 871 a latent space for the spatial modes. We specifically use 920
 872 bagging-optimized DMD [36, 37]—implemented in Python
 873 with the PyDMD package [59, 60]—which improves robust-
 874 ness against noisy data through ensemble averaging. We
 875 provide a summary of the DMD output next; see [36, 37]
 876 for full details and theory.

877 Given a vectorized time series $\mathbf{Y} =$ 921
 878 $[\mathbf{y}(1), \mathbf{y}(2), \dots, \mathbf{y}(m)]$ over N regional flow segments 922
 879 and m time steps, the R rank DMD finds a complex-valued 923
 880 reconstruction 924

$$\hat{\mathbf{y}}(t) = \sum_{j=1}^R A_j \Phi_j \exp(i\omega_j t) \quad (10)$$

881 where A_j is a scalar amplitude, $\Phi_j = (\phi_{1,j}, \phi_{2,j}, \dots, \phi_{N,j}) \in$ 925
 882 \mathbb{C}^N is a complex vector, and $\omega_j \in \mathbb{R}$ is the frequency 926
 883 of the j^{th} mode. The A_j , Φ_j , and ω_j terms are parameters 927
 884 to be fitted. In this case, the real part of $\hat{\mathbf{y}}$ is the best- 928
 885 fitted reconstruction of the data obtained by minimizing 929
 886 the Frobenius norm [37].

887 We can decompose the terms inside the right-hand side 930
 888 of eq. (10) into more intuitive forms. In particular, the real- 931
 889 component of the n^{th} region of the j^{th} mode can be written 932
 890 as 933

$$\Re[A_j \phi_{n,j} \exp(i\omega_j t)] = A_j |\phi_{n,j}| \cos(2\pi t/\tau_j + \theta_{n,j}), \quad (11)$$

891 where $\theta_{n,j}$ is obtained from the polar form of the complex 934
 892 number $\phi_{n,j} = |\phi_{n,j}| \exp(i\theta_{n,j})$, and $\tau_j = 2\pi/\omega_j$ is the pe- 935
 893 riod. 936

In order to apply DMD to our study, we must first get 894
 our shipping data in the form of $\mathbf{y}(t)$. Our data comes in 895
 the form of a set of laden legs. For a given vessel class, 896
 a laden leg describes a voyage from region u to region v 897
 between the times t_u and t_v with cargo weight w . This 898
 can be represented as a temporal edge (u, v, t_u, t_v) . We 899
 then define a time step δt and rolling window interval ΔW , 900
 where $t_{i+1} = t_i + \delta t$. A temporal edge (u, v, t_u, t_v) is then 901
 considered active in interval $t + \Delta W$ if 902

$$\begin{aligned} t \leq t_u \leq t + \Delta W & \quad \text{edge begins in window} \\ t \leq t_v \leq t + \Delta W & \quad \text{edge ends in window} \\ t_u \leq t \text{ and } t_v \geq t + \Delta W & \quad \text{edge overlaps window} \end{aligned} \quad (12)$$

Denoting the set of edges active in window $t + \Delta W$ as 903
 $\mathcal{E}(t, \Delta W)$, the time series of incoming or outgoing cargo 904
 flow for region i in time window t is the sum of the cargo 905
 weights w_e of all the active voyages $e \in \mathcal{E}(t, \Delta W)$: 906

$$\begin{aligned} y_{(i,in)}(t) &= \sum_{(u,v,t_u,t_v) \in \mathcal{E}(t,\Delta W)} \delta_{i,u} w_e \\ y_{(i,out)}(t) &= \sum_{(u,v,t_u,t_v) \in \mathcal{E}(t,\Delta W)} \delta_{i,v} w_e \end{aligned} \quad (13)$$

where $\delta_{i,u} = 1$ if $i = u$ else 0 and w_e is in units of 10,000 907
 deadweight tonnage (dwt). By definition then, ballast legs 908
 are not included. Thus, $y_{(i,in)}(t)$ is the total cargo arriv- 909
 ing at region i at time t , and $y_{(i,out)}(t)$ is the total cargo 910
 departing from region i at time t . 911

An additional pre-processing step is to remove linear 912
 trends from $\mathbf{y}(t)$, as the DMD method can only recover 913
 sinusoidal and exponential terms. While the DMD algo- 914
 rithm may still converge without this step, it tends to fit 915
 eigenmodes with period \gg the duration of the data set to 916
 approximate a linear term. This can obscure the function- 917
 ality and interpretability of results, and thus it is better to 918
 remove it beforehand. We apply a linear regression in time 919
 to each time series, such that 920

$$\tilde{y}_i(t) = y_i(t) - r_i t - k_i \quad (14)$$

where r_i and k_i are gradient and constant terms from a 921
 least-squares straight line fit. This brings our work in line 922
 with an additive seasonal model [46], where the original sig- 923
 nal $\mathbf{y}(t) = \mathbf{L}(t) + \mathbf{I}(t) + \tilde{\mathbf{y}}(t)$, where $\mathbf{L}(t)$ is a long-term lin- 924
 ear trend and $\mathbf{I}(t)$ are irregular terms. The component $\tilde{\mathbf{y}}(t)$ 925
 is composed of both the “seasonal” and “cyclical” terms, 926
 which in [46] corresponds to periodicities of < 1 year and 927
 > 1 year, respectively. However, in our case, we do not 928
 distinguish between the two. 929

930 Peak-to-trough relative amplitudes

To quantify the relative significance of an individual mode 931
 to each time-series, we need a standardized measure that 932
 accounts for the varying scales of cargo flows across different 933
 regional flow segments. To do this, we define the peak- 934
 to-peak relative amplitude, which normalizes each mode’s 935
 peak-to-peak amplitude by the total variability observed in 936
 the original time-series: 937

$$\text{Relative Amplitude}_{n,j} = \frac{2A_j |\phi_{n,j}|}{\max(y_n) - \min(y_n)} \quad (15)$$

where $A_j |\phi_{n,j}|$ represents the amplitude of mode j in re- 938
 gional flow segment n , and $\max(y_n) - \min(y_n)$ is the peak- 939
 to-peak range of the original time series for regional flow 940

segment n . The factor of 2 converts the cosine amplitude to peak-to-peak amplitude. An additional factor of 2 is introduced when considering the relative amplitude of a mode-pair, since DMD modes come in complex conjugate pairs that both contribute to the reconstructed signal.

A larger value in the relative peak-to-peak amplitude indicates that the mode contributes more significantly to the observed fluctuations in that regional flow segment. The standardized nature of this measure also enables calculation of the average significance of each mode across all regional flow segments.

Choice of parameters

Dynamic Mode Decomposition requires the selection of several key parameters: SVD rank, time interval, rolling window duration, and number of delay embeddings. We employed a two-stage selection strategy utilizing error-based optimization (for SVD rank) and stability-based selection (for other temporal parameters) to avoid overfitting to data-specific noise.

For time interval (1 week), rolling window duration (6 weeks), and delay embeddings (22), we ensured that the parameter values chosen were statistically stable under partial variation. We systematically scanned parameter ranges while computing eigenvalue distributions for each combination, then calculated pairwise statistical distances (variational distance and Hellinger distance) between eigenvalue distributions across parameter sweeps [61, 62]. Regions of low statistical distance correspond to stable parameter regimes where extracted eigenvalues remain consistent across parameter perturbations (Figure S7 in Supplementary Information section 3.1). For time intervals and rolling window duration, we selected small parameter values to balance eigenvalue stability with maximizing the number of time points available for decomposition. For delay embeddings, we selected the smallest value within the stable regime that ensures the data matrix is tall (more rows than columns), which is required for stable SVD decomposition.

We determined the SVD rank through a two-step process. First, we examined the explained variance, identifying an “elbow point” where explained variance plateaus. Then we evaluated the root-mean-squared error (RMSE) around this elbow point, corresponding to candidate ranks 6 – 20. The final rank was chosen such that it provided the largest average marginal improvement in RMSE across a range of delay-embedding. SVD rank 12 provided the largest average marginal improvement, while higher ranks showed inconsistent performance and diminishing returns. This approach balances reconstruction performance against model complexity to prevent overfitting to noisy signals.

The final parameter choices produced stable modes and eigenvalues that were well within the expected ranges under neighboring parameter choices. In particular, across all parameter combinations, we recovered the dominant annual mode with period 362 ± 10 days. Similar results were found for the other 5 modes. See our Supplementary Information section 3.1 for further details.

Reconstruction errors

The reconstruction errors for the $\tau = 51.5$ mode was evaluated for each combination of region, direction, and ship class using the normalized root mean squared error

$RMSE^*$ [38], shown in the scatterplot in fig. 10. The $RMSE^*$ lies within the range of $[0, 1]$, with 0 indicating a perfect fit and 1 a maximally erroneous fit. We find that regions with larger amplitudes tend to have lower $RMSE^*$, indicating that our interpretations should be focused on high amplitude regions.

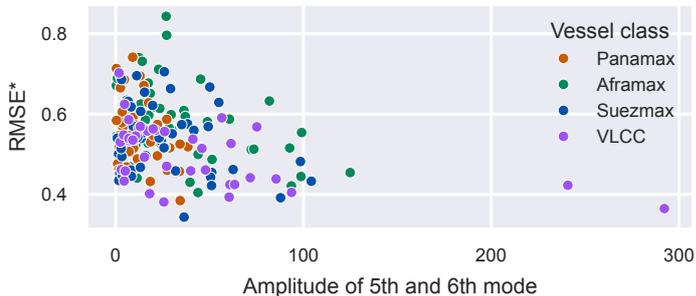


Figure 10: **Scatter-plot of reconstruction errors $RMSE^*$ against 4th and 5th mode amplitude.** Lower values of $RMSE^*$ indicate a better fit to the data.

Data availability

Due to legal restrictions, the data used to generate the main results in this article will be available only by contacting the author Kevin Teo.

References

1. United Nations Conference on Trade and Development. *Review of Maritime Transport 2017* 2017. <https://unctad.org/publication/review-maritime-transport-2017>.
2. U.S. Energy Information Administration. *World Oil Transit Chokepoints* 2024. https://www.eia.gov/international/analysis/special-topics/World_Oil_Transit_Chokepoints.
3. Rahim, M. M., Islam, M. T. & Kuruppu, S. Regulating global shipping corporations’ accountability for reducing greenhouse gas emissions in the seas. *Marine Policy* **69**, 159–170. ISSN: 0308-597X. <https://www.sciencedirect.com/science/article/pii/S0308597X16301907> (2016).
4. Brancaccio, G., Kalouptsidi, M. & Papageorgiou, T. Geography, transportation, and endogenous trade costs. *Econometrica* **88**, 657–691 (2020).
5. Walker, T. R. *et al.* in *World seas: an environmental evaluation* (ed Sheppard, C.) Second, 505–530 (Academic Press, 2019). ISBN: 978-0-12-805052-1. <https://www.sciencedirect.com/science/article/pii/B9780128050521000309>.
6. Shi, Z. *et al.* Perspectives on shipping emissions and their impacts on the surface ocean and lower atmosphere: An environmental-social-economic dimension. *Elementa: Science of the Anthropocene* **11**, 00052. ISSN: 2325-1026. eprint: <https://online.ucpress.edu/elementa/article-pdf/11/1/00052/792716/elementa.2023.00052.pdf>. <https://doi.org/10.1525/elementa.2023.00052> (Oct. 2023).

7. Ducruet, C. The geography of maritime networks: A critical review. *Journal of Transport Geography* **88**, 102824. ISSN: 0966-6923. <https://www.sciencedirect.com/science/article/pii/S0966692320305895> (2020).
8. Kaluza, P., Kölzsch, A., Gastner, M. T. & Blasius, B. The complex network of global cargo ship movements. *Journal of the Royal Society Interface* **7**, 1093–1103 (2010).
9. Álvarez, N. G., Adenso-Díaz, B. & Calzada-Infante, L. Maritime Traffic as a Complex Network: a Systematic Review. *Networks and Spatial Economics* **21**, 387–417 (June 2021).
10. Faure, M.-A. & Ducruet, C. Shipping Network Research: A Systematic and Quantitative Review. working paper or preprint. <https://hal.science/hal-05106768> (Mar. 2025).
11. Ducruet, C. in *Port Systems in Global Competition* 300–333 (Routledge, 2023).
12. Hu, Y. & Zhu, D. Empirical analysis of the worldwide maritime transportation network. *Physica A: Statistical Mechanics and its Applications* **388**, 2061–2071. ISSN: 0378-4371. <https://www.sciencedirect.com/science/article/pii/S0378437108010273> (2009).
13. Ge, J., fu, Q., Zhang, Q. & Wan, Z. Regional operating patterns of world container shipping network: A perspective from motif identification. *Physica A: Statistical Mechanics and its Applications* **607**, 128171. ISSN: 0378-4371. <https://www.sciencedirect.com/science/article/pii/S0378437122007294> (2022).
14. Peng, P., Poon, J. P., Yang, Y., Lu, F. & Cheng, S. Global oil traffic network and diffusion of influence among ports using real time data. *Energy* **172**, 333–342 (2019).
15. Liu, Q., Yang, Y., Ke, L. & Ng, A. K. Structures of port connectivity, competition, and shipping networks in Europe. *Journal of Transport Geography* **102**, 103360. ISSN: 0966-6923. <https://www.sciencedirect.com/science/article/pii/S0966692322000837> (2022).
16. Zhang, Q., Pu, S., Luo, L., Liu, Z. & Xu, J. Revisiting important ports in container shipping networks: A structural hole-based approach. *Transport Policy* **126**, 239–248. ISSN: 0967-070X. <https://www.sciencedirect.com/science/article/pii/S0967070X22002086> (2022).
17. Ducruet, C. & Notteboom, T. The worldwide maritime network of container shipping: spatial structure and regional dynamics. *Global networks* **12**, 395–423. <https://onlinelibrary.wiley.com/doi/abs/10.1111/j.1471-0374.2011.00355.x> (2012).
18. González Laxe, F., Jesus Freire Seoane, M. & Pais Montes, C. Maritime degree, centrality and vulnerability: port hierarchies and emerging areas in containerized transport (2008–2010). *Journal of Transport Geography* **24**. Special Section on Theoretical Perspectives on Climate Change Mitigation in Transport, 33–44. ISSN: 0966-6923. <https://www.sciencedirect.com/science/article/pii/S0966692312001561> (2012).
19. Guerrero, D., Letrouit, L. & Pais-Montes, C. The container transport system during Covid-19: An analysis through the prism of complex networks. *Transport Policy* **115**, 113–125. ISSN: 0967-070X. <https://www.sciencedirect.com/science/article/pii/S0967070X21003036> (2022).
20. Asgari, N., Farahani, R. Z. & Goh, M. Network design approach for hub ports-shipping companies competition and cooperation. *Transportation Research Part A: Policy and Practice* **48**. Psychology of Sustainable Travel Behavior, 1–18. ISSN: 0965-8564. <https://www.sciencedirect.com/science/article/pii/S0965856412001590> (2013).
21. Seebens, H., Schwartz, N., Schupp, P. J. & Blasius, B. Predicting the spread of marine species introduced by global shipping. *Proceedings of the National Academy of Sciences* **113**, 5646–5651. eprint: <https://www.pnas.org/doi/pdf/10.1073/pnas.1524427113>. <https://www.pnas.org/doi/abs/10.1073/pnas.1524427113> (2016).
22. Seebens, H., Gastner, M. T. & Blasius, B. The risk of marine bioinvasion caused by global shipping. *Ecology Letters* **16**, 782–790. eprint: <https://onlinelibrary.wiley.com/doi/pdf/10.1111/ele.12111>. <https://onlinelibrary.wiley.com/doi/abs/10.1111/ele.12111> (2013).
23. Yan, Z. *et al.* Analysis of global marine oil trade based on automatic identification system (AIS) data. *Journal of Transport Geography* **83**, 102637 (2020).
24. Peng, P., Yang, Y., Cheng, S., Lu, F. & Yuan, Z. Hub-and-spoke structure: Characterizing the global crude oil transport network with mass vessel trajectories. *Energy* **168**, 966–974 (2019).
25. International Maritime Organization. *Guidelines for Voluntary Use of the Ship Energy Efficiency Operational Indicator (EEOI)* Circular MEPC.1/Circ.684 (International Maritime Organization, London, Aug. 2009). <https://wwwcdn.imo.org/localresources/en/OurWork/Environment/Documents/Circ-684.pdf>.
26. Xu, J., Wickramaratne, T. L. & Chawla, N. V. Representing higher-order dependencies in networks. *Science Advances* **2**, e1600028. eprint: <https://www.science.org/doi/pdf/10.1126/sciadv.1600028>. <https://www.science.org/doi/abs/10.1126/sciadv.1600028> (2016).
27. Teo, K., Arnold, N., Hone, A. & Kiss, I. Z. Performance of higher-order networks in reconstructing sequential paths: from micro to macro scale. *Journal of Complex Networks* **13**, cnae050. ISSN: 2051-1329. eprint: <https://academic.oup.com/comnet/article-pdf/13/1/cnae050/61399620/cnae050.pdf>. <https://doi.org/10.1093/comnet/cnae050> (Jan. 2025).
28. Scholtes, I. *When is a network a network? Multi-order graphical model selection in pathways and temporal networks* in *Proceedings of the 23rd ACM SIGKDD international conference on knowledge discovery and data mining* (2017), 1037–1046.

- 1161 29. Cliff, N. Dominance statistics: Ordinal analyses to answer ordinal questions. *Psychological Bulletin* **114**, 494–509. <https://doi.org/10.1037/0033-2909.114.3.494> (1993). 1162 1163 1164
- 1165 30. Hess, M. R. & Kromrey, J. D. *Robust confidence intervals for effect sizes: A comparative study of Cohen'sd and Cliff's delta under non-normality and heterogeneous variances in annual meeting of the American Educational Research Association* **1** (2004). 1166 1167 1168 1169
- 1170 31. Benjamini, Y. & Yekutieli, D. The control of the false discovery rate in multiple testing under dependency. *The Annals of Statistics* **29**, 1165–1188 (2001). 1171 1172
- 1173 32. Stopa, J. E. & Cheung, K. F. Periodicity and patterns of ocean wind and wave climate. *Journal of Geophysical Research: Oceans* **119**, 5563–5584. <https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1002/2013JC009729> (2014). 1174 1175 1176 1177
- 1178 33. Inchauspe, J., Li, J. & Park, J. Seasonal patterns of global oil consumption: Implications for long term energy policy. *Journal of Policy Modeling* **42**, 536–556 (2020). 1179 1180 1181
- 1182 34. Raju, T. B., Chauhan, P., Tiwari, S. & Kashav, V. Seasonality in Freight Rates. *Journal of International Logistics and Trade* **18**, 149–157 (2020). 1183 1184
- 1185 35. Schmid, P. J. Dynamic mode decomposition of numerical and experimental data. *Journal of fluid mechanics* **656**, 5–28 (2010). 1186 1187
- 1188 36. Askham, T. & Kutz, J. N. Variable Projection Methods for an Optimized Dynamic Mode Decomposition. *SIAM Journal on Applied Dynamical Systems* **17**, 380–416. eprint: <https://doi.org/10.1137/M1124176>. <https://doi.org/10.1137/M1124176> (2018). 1189 1190 1191 1192 1193
- 1194 37. Sashidhar, D. & Kutz, J. N. Bagging, optimized dynamic mode decomposition for robust, stable forecasting with spatial and temporal uncertainty quantification. *Philosophical Transactions of the Royal Society A* **380**, 20210199 (2022). 1195 1196 1197 1198
- 1199 38. Müller-Plath, G. & Lüdecke, H.-J. Normalized coefficients of prediction accuracy for comparative forecast verification and modeling. *Research in Statistics* **2**, 2317172 (2024). 1200 1201 1202
- 1203 39. Hassani, H. A brief introduction to singular spectrum analysis. *Optimal decisions in statistics and data analysis* (2010). 1204 1205
- 1206 40. Hassani, H. & Mahmoudvand, R. Multivariate singular spectrum analysis: A general view and new vector forecasting approach. *International Journal of Energy and Statistics* **1**, 55–83 (2013). 1207 1208 1209
- 1210 41. U.S. Energy Information Administration. *Monthly Energy Review September 2024* tech. rep. Accessed: 2024-08-27 (U.S. Department of Energy, 2024). <https://www.eia.gov/totalenergy/data/monthly/>. 1211 1212 1213
- 1214 42. Wu, X. & Chen, G. Global overview of crude oil use: From source to sink through inter-regional trade. *Energy Policy* **128**, 476–486. ISSN: 0301-4215. <https://www.sciencedirect.com/science/article/pii/S0301421519300229> (2019). 1215 1216 1217 1218
43. U.S. Energy Information Administration. *Petroleum Supply Annual, Volume 1* tech. rep. Accessed: 2024-09-27 (U.S. Department of Energy, 2024). <https://www.eia.gov/petroleum/supply/annual/volume1/>. 1219 1220 1221 1222 1223
44. Chen, X. *et al.* Intelligent ship route planning via an A* search model enhanced double-deep Q-network. *Ocean Engineering* **327**, 120956 (2025). 1224 1225 1226
45. Chen, X. *et al.* Ship energy consumption analysis and carbon emission exploitation via spatial-temporal maritime data. *Applied Energy* **360**, 122886. ISSN: 0306-2619. <https://www.sciencedirect.com/science/article/pii/S0306261924002691> (2024). 1227 1228 1229 1230 1231
46. Hylleberg, S. *Seasonality in regression* (Academic Press, 2014). 1232 1233
47. LaRock, T., Xu, M. & Eliassi-Rad, T. A path-based approach to analyzing the global liner shipping network. *EPJ Data Science* **11**, 18. <https://doi.org/10.1140/epjds/s13688-022-00331-z> (2022). 1234 1235 1236 1237
48. Si, R., Jia, P., Li, H. & Zhao, X. Assessing the structural resilience of the global crude oil maritime transportation network: A motif-based approach from network to ports. *Journal of Transport Geography* **123**, 104123. ISSN: 0966-6923. <https://www.sciencedirect.com/science/article/pii/S0966692325000146> (2025). 1238 1239 1240 1241 1242 1243 1244
49. Xu, M., Deng, W., Zhu, Y. & LÜ, L. Assessing and improving the structural robustness of global liner shipping system: A motif-based network science approach. *Reliability Engineering & System Safety* **240**, 109576 (2023). 1245 1246 1247 1248 1249
50. Wei, X. *et al.* Resilience analysis of container port networks based on motif dynamics in 2023 7th International Conference on Transportation Information and Safety (ICTIS) (2023), 263–266. 1250 1251 1252 1253
51. Akaike, H. in *Selected papers of hirotugu akaike* 199–213 (Springer, 1998). 1254 1255
52. Neath, A. A. & Cavanaugh, J. E. The Bayesian information criterion: background, derivation, and applications. *Wiley Interdisciplinary Reviews: Computational Statistics* **4**, 199–203 (2012). 1256 1257 1258 1259
53. Wilks, S. S. The large-sample distribution of the likelihood ratio for testing composite hypotheses. *The annals of mathematical statistics* **9**, 60–62 (1938). 1260 1261 1262
54. Ferrari, S. & Cribari-Neto, F. Beta Regression for Modelling Rates and Proportions. *Journal of Applied Statistics* **31**, 799–815. eprint: <https://doi.org/10.1080/0266476042000214501>. <https://doi.org/10.1080/0266476042000214501> (2004). 1263 1264 1265 1266 1267
55. Geissinger, E. A., Khoo, C. L., Richmond, I. C., Faulkner, S. J. & Schneider, D. C. A case for beta regression in the natural sciences. *Ecosphere* **13**, e3940 (2022). 1268 1269 1270 1271
56. Proctor, J. L. & Eckhoff, P. A. Discovering dynamic patterns from infectious disease data using dynamic mode decomposition. *International Health* **7**, 139–145. ISSN: 1876-3413. eprint: <https://academic.oup.com/inthealth/article-pdf/7/2/139/16837591/ihv009.pdf>. <https://doi.org/10.1093/inthealth/ihv009> (Feb. 2015). 1272 1273 1274 1275 1276 1277 1278

- 1279 57. Griffith, T. D. & Hubbard Jr, J. E. System identification methods for dynamic models of brain activity.
1280 *Biomedical Signal Processing and Control* **68**, 102765
1281 (2021).
1282
- 1283 58. Brunton, B. W., Johnson, L. A., Ojemann, J. G.
1284 & Kutz, J. N. Extracting spatial-temporal coherent
1285 patterns in large-scale neural recordings using dy-
1286 namic mode decomposition. *Journal of Neuroscience*
1287 *Methods* **258**, 1–15. ISSN: 0165-0270. <https://www.sciencedirect.com/science/article/pii/S0165027015003829> (2016).
1288
1289
- 1290 59. Demo, N., Tezzele, M. & Rozza, G. PyDMD: Python
1291 dynamic mode decomposition. *Journal of Open Source*
1292 *Software* **3**, 530 (2018).
- 1293 60. Ichinaga, S. M. *et al.* PyDMD: A Python Package
1294 for Robust Dynamic Mode Decomposition. *Journal of*
1295 *Machine Learning Research* **25**, 1–9. <http://jmlr.org/papers/v25/24-0739.html> (2024).
1296
- 1297 61. Vaart, A. W. v. d. in *Asymptotic Statistics* 5–24 (Cam-
1298 bridge University Press, 1998).
- 1299 62. Vaart, A. W. v. d. in *Asymptotic Statistics* 192–214
1300 (Cambridge University Press, 1998).

1301 Acknowledgments

1302 Kevin Teo acknowledges the PhD studentship support from
1303 Northeastern University.

1304 Author contributions

1305 K.T, N.A, M.C, M.I, I.Z.K designed research; K.T, N.A
1306 performed research; M.S. contributed analytic tools; K.T,
1307 N.A, A.H, M.C, M.S, I.Z.K analyzed data; K.T, N.A, A.H,
1308 M.C, M.S, I.Z.K wrote the paper.

1309 Additional information

1310 Correspondence

1311 Correspondence should be addressed to Kevin Teo or István
1312 Kiss. Materials request should be addressed to Kevin Teo.