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Research Article

Eliciting Fairness in N-Player Network Games through Degree-Based Role Assignment

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From social contracts to climate agreements, individuals engage in groups that must collectively reach decisions with varying levels of equality and fairness. These dilemmas also pervade distributed artificial intelligence, in domains such as automated negotiation, conflict resolution, or resource allocation, which aim to engineer self-organized group behaviors. As evidenced by the well-known Ultimatum Game, where a Proposer has to divide a resource with a Responder, payoff-maximizing outcomes are frequently at odds with fairness. Eliciting equality in populations of self-regarding agents requires judicious interventions. Here, we use knowledge about agents' social networks to implement fairness mechanisms, in the context of Multiplayer Ultimatum Games. We focus on network-based role assignment and show that attributing the role of Proposer to low-connected nodes increases the fairness levels in a population. We evaluate the effectiveness of low-degree Proposer assignment considering networks with different average connectivities, group sizes, and group voting rules when accepting proposals (e.g., majority or unanimity). We further show that low-degree Proposer assignment is efficient, in optimizing not only individuals' offers but also the average payoff level in the population. Finally, we show that stricter voting rules (i.e., imposing an accepting consensus as a requirement for collectives to accept a proposal) attenuate the unfairness that results from situations where high-degree nodes (hubs) play as Proposers. Our results suggest new routes to use role assignment and voting mechanisms to prevent unfair behaviors from spreading on complex networks.

1. Introduction

Fairness has a profound impact on human decision-making and individuals often prefer fair outcomes over payoff-maximizing ones [1]. This has been evidenced through behavioral experiments, frequently employing the celebrated Ultimatum Game (UG) [2]. In the UG, one Proposer decides how to divide a given resource with a Responder. The game only yields payoffs associated with the proposed resource allocation to the participants if the Responder accepts the proposal. Human Proposers tend to sacrifice some of their share by offering high proposals, and Responders often prefer to earn nothing rather than accepting unfair divisions. These counterintuitive results motivated several lab experiments and theoretical models that aimed at justifying, mathematically and empirically, the emergence and maintenance of fair intentions in human behavior [3–7].

Most of these works, however, have neglected the fact that, in many situations, offers are made in the context of groups, instead of simpler pairwise interactions. This is the case in the negotiation of collective work contracts, environmental coalitions and policy making [8], human rights conventions, collective insurance [9], adoption of regulatory frameworks (e.g., in the use of technology [10]), exchange of flexibilities between local energy communities [11], or the simple act of scheduling a meeting with several participants, among other possible scenarios. Fairness and bargaining dilemmas occur within groups, in which group decisions emerge from the combination of each individual's assessment of what is perceived as a fair offer. Similarly, in engineering applications grounded on artificial intelligence and multiagent systems, fairness concerns are important in domains that go beyond pairwise interactions. Autonomous agents have to take part in group interactions that must decide between outcomes that may each favour a different part of the group. Examples of such domains are automated bargaining [12], conflict resolution [13], or multiplayer resource allocation [14].

To capture some of the dilemmas associated with fairness versus payoff maximization in these group interactions, one may resort to multiplayer extensions of the Ultimatum Game [15] (MUG) (see Figure 1). Here, a proposal is made by a Proposer to a group of $N-1$ Responders that, collectively, decide to accept or reject it. As in the pairwise UG, the strategy of a Proposer, p, is the fraction of resource offered to the Responders; the strategy of each Responder i, q_i, is the personal threshold used to decide between acceptance and rejection [5, 6]. Groups decide to accept and reject a proposal through functions of the individual acceptance thresholds, q. Group acceptance depends on a decision rule: if the fraction of acceptances equals or exceeds a minimum fraction of accepting Responders, M, the proposal is accepted by the group. In that case, the Proposer keeps what she did not offer $(1-p)$ and the offer is divided by the Responders—each receiving $p/(N-1)$. If the fraction of acceptances remains below M, the proposal is rejected by the group and no one earns anything. As in the UG, the *sub-game perfect equilibrium* of MUG consists of a very low value of proposal p and very low values of threshold q [16].

Previous studies with the UG [4–7, 17] and the MUG [15, 18, 19] assume that the roles of the Proposer and Responder are attributed following uniform probability distributions: each agent has the same probability of being selected to play as the Proposer. These assumptions are naturally at odds with reality. In real-life Ultimatum Games, being the Proposer or the Responder depends on particular agents' characteristics. Proposers, such as employers, investors, auction first-movers, and rich countries, are in the privileged position of having the material resources to decide upon which proposals to offer. This advantageous role is notorious if, again, one considers the theoretical prediction of payoff division in the UG (*sub-game perfect equilibrium*) posing that Proposers will keep the largest share of the resource being divided. The benefits of Proposers are more evident when proposals are made to groups, as Responders need to divide the offers—thus increasing the gap in gains between the single Proposer and the Responders. In this multiplayer context, punishing Proposers becomes harder: any attempt to punish unfair offers is only effective if there is

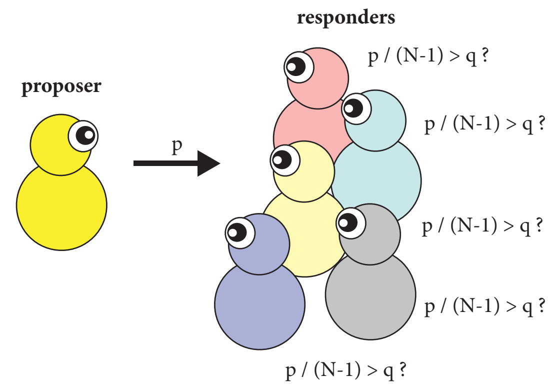

Figure 1: Setting of the Multiplayer Ultimatum Game. After a Proposer is selected, a proposal p is made to a group of $N-1$ Responders. Each Responder will compare its strategy q (translating the minimum acceptable offer) with the expected value to be received, $p/(N-1)$. For a given M, the proposal will be accepted if at least that fraction of Responders accepts the proposal.

a successful collective agreement—amongst Responders—to sacrifice individual gains and reject an offer. Asserting that these two roles are asymmetric, so should be the criteria to assign them, leading us to two main questions:

(i) How should a Proposer be selected within a group, in Multiplayer Ultimatum Games, to guarantee efficiency and fairness?

(ii) The fact that individuals are often embedded in networks makes it important to understand to which extent the network ties and the way groups are assembled influence overall fairness, exchanges, and cooperation. Given this networked context, which network-based role-assignment criteria can be used to maximize long-term efficiency and fairness?

Here, we introduce a model, based on evolutionary game theory (EGT) [20, 21] and complex networks, to approach the previous questions. We analyze multiplayer ultimatum games in heterogeneous complex networks through the network centrality-based role assignment. The fact that networks are heterogeneous allows us to test several node properties and centrality measures as base criteria for defining how to select Proposers in a group. We focus on degree centrality. We find that selecting low-degree Proposers elicits fairer offers and increases the overall fitness (average payoff) in a population.

1.1. Related Work. The questions we address in this work—and the model proposed to tackle them—lay on the interface between mechanisms for fairness elicitation in multiagent systems, multilayer bargaining interactions, dynamics on complex networks, and network interventions to sustain socially desirable outcomes.

Some of the most challenging contexts to elicit fairness involve the tradeoff between payoff-maximizing outcomes and fair outcomes. As stated, the UG [2] has been a fundamental interaction paradigm to study such dilemmas. In this context, reputations [5] and stochastic effects [7] were identified as mechanisms that justify fair behaviors. Page et al. found that, in a spatial setting, fairer proposals emerge as clusters of individuals proposing high offers are able to grow [6]. Also, in the realm of interaction networks, De Jong et al. concluded that scale-free networks allow agents to achieve fairer agreements; rewiring links also enhances the agents' ability to achieve fair outcomes [4]. A game similar to the UG assumes that Responders are unable to reject any proposal and Proposers unilaterally decide about a resource division. This leads to the so-called Dictator Game. In this context, reputations and mechanisms based on partner choice were also identified as drivers of fair proposals [22].

The previous works assume that all agents have the same probability of playing the role of Proposer or Responder. Going from well-mixed (i.e., all individuals are free to interact with everyone else) to complex networks, however, provides the opportunity to implement network-based role assignment that considers network measures. In this context, Wu et al. studied the pairwise UG in scale-free networks, with roles being attributed based on network degrees. The authors show that attributing the role of Proposer to high-degree nodes leads to unfair scenarios [23]. Likewise, Deng et al. studied the role assignment based on degree, concluding that the effect of degree-based role assignment depends on the mechanism of strategy update [24]. When considering a pairwise comparison based on accumulated payoffs and social learning (as we do in the present work), the levels of contribution in the population increase if lower-degree individuals have a higher probability of being the Dictators. Both works consider the pairwise Ultimatum Game.

In this work, we use a multiplayer version of the UG (MUG) proposed in [15]. Other forms of Multiplayer Ultimatum Games can be found in [19, 25, 26]. Santos et al. studied this game in the context of complex networks, showing that fairness is augmented whenever the networks, upon which the game is played, allow agents to exert a sufficient level of influence over each other, by repeatedly participating in each other's interaction groups. The authors also find that stricter group decision rules (i.e., high M in MUG) allow for fairer strategies to evolve under MUG. Here, we use networks to define group formation as suggested in the previously mentioned work (originally in [27]) and as exemplified in Figure 2.

Departing from previous works that study degree-based role assignment in pairwise Ultimatum Games [23, 24], we focus on a multiplayer game. As mentioned, this version highlights the asymmetries between the Proposer and Responder roles. By comparison with the UG, MUG Proposers are likely to receive an even higher share of payoffs than each Responder as the latter must divide any accepted offer between themselves. Moreover, in order to punish unfair MUG Proposers, Responders must act as a group which may naturally call for extra coordination mechanisms. Also, in contrast with [23, 24], here we combine the study of network-based role assignment with different voting mechanisms; we show that, whenever highly connected nodes are the natural candidates to play the role of Proposer, stricter voting rules (i.e., imposing an accepting consensus as requirement for collectives to accept a proposal) attenuate the emergent level of inequality.

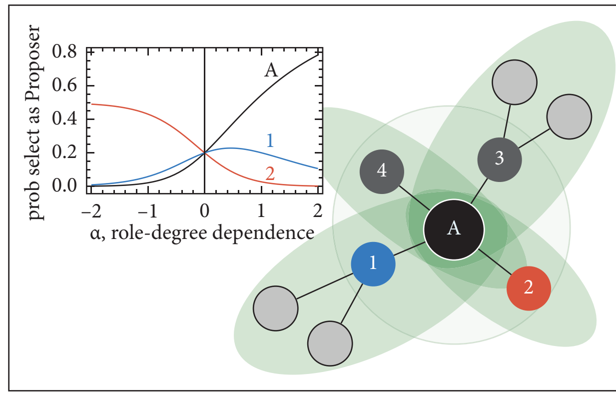

Figure 2: Example of group formation and Proposer selection based on the degree. Each node and its neighborhood define an interaction group. In the figure, node A plays in 5 groups and its fitness results come from the payoff sum after playing in all those groups. In general, a node plays in a number of groups equal to its degree plus one. For each group, the payoff is calculated after one individual is selected to be the Proposer. Proposer selection depends on the degree of each individual in the group, and a parameter α controls this dependence (see the Materials and Methods section). To exemplify this process, the inset graph represents the probability of each individual—A (high degree), 1 (medium degree), and 2 (low degree)—to be selected as a Proposer when playing in the group centered on A, as a function of α.

Finally, the approach we follow in this work is akin to testing network interventions for social good. Several works study social dilemmas on top of complex networks and stress the conditions leading, in this context, to socially desirable outcomes [28–31]. In this realm, we shall underline a recent work that employs EGT—as we do in the present paper—to study interventions that aim to sustain cooperation in complex networks [32]. The authors conclude that local interventions, i.e., based on information about the neighborhood of the affected node, outperform global ones. A similar conclusion is presented in [28].

2. Materials and Methods

Here, we detail the proposed evolutionary game theoretical model to evaluate the effect of degree-based role assignment on fairness under MUG. We start by providing details on the payoff calculation under MUG.

2.1. Multiplayer Ultimatum Games. In the 2-player UG, a Proposer has a resource and is required to propose a division with a Responder. The game only yields payoff to the participants if the Responder accepts the proposal [2]. Given a Proposer with strategy $p \in [0, 1]$ and a Responder with strategy $q \in [0, 1]$, the payoff for the Proposer yields

$$\Pi_P(p,q) = \begin{cases} 1-p, & p \geq q, \\ 0, & p < q, \end{cases} \tag{1}$$

and for the Responder,

$$\Pi_R(p,q) = \begin{cases} p, & p \geq q, \\ 0, & p < q. \end{cases} \quad (2)$$

In the MUG, proposals are made by one Proposer to the remaining $N-1$ Responders, who must individually reject or accept them [15, 18]. Since individuals may act both as Proposers and Responders (with a probability that will depend on node characteristics), we assume that each individual adopts a strategy $(p,\ q)$. When playing as the Proposer, individuals offer p to the Responders. Responders will individually accept or reject the offer having their q as a threshold: if the share of an offer p is equal or larger than q (i.e., $(p/N-1) \geq q$), the individual accepts the proposal. Otherwise, the Responder rejects that proposal. We can regard q as the minimum fraction that an individual is willing to accept, relative to the maximum to be earned as a Responder in a group of a certain size. Alternatively, we could assume that individuals ignore the group size and as such, when faced with a proposal, they must judge the absolute value of that proposal (an interpretation that also holds if we assume that individuals care about the whole group payoff).

Overall group acceptance will depend upon M, the minimum fraction of Responders that must accept the offer before it is valid. Consequently, if the fraction of individual acceptances stands below M, the offer will be rejected. Otherwise, the offer will be accepted. In this case, the Proposer will keep $1-p$ to himself and the group will share the remainder; that is, each Responder gets $p/(N-1)$. If the proposal is rejected, no one earns anything. All together, in a group with size N composed of 1 Proposer with strategy $p \in [0,1]$ and $N-1$ Responders with strategies $(q_1, \ldots, q_{N-1}) \in [0,1]^{N-1}$, the payoff of the Proposer is given by

$$\Pi_P(p, q_1, \ldots, q_{N-1}) = \begin{cases} 1-p, & \dfrac{\sum_{i=1}^{N-1} \Theta((p/N-1) - q_i)}{(N-1)} \geq M, \\ 0, & \text{otherwise}, \end{cases} \quad (3)$$

where $\Theta(x)$ is the Heaviside step function that evaluates to 1 when $x \geq 0$ and evaluates to 0 when $x < 0$. The payoff of any Responder in the group yields

$$\Pi_R(p, q_1, \ldots, q_{N-1}) = \begin{cases} \dfrac{p}{N-1}, & \dfrac{\sum_{i=1}^{N-1} \Theta((p/N-1) - q_i)}{(N-1)} \geq M, \\ 0, & \text{otherwise}. \end{cases} \quad (4)$$

We assume that MUG interactions are played on a complex network, in which individuals are assigned nodes and links define who can interact with whom. Following [27, 33], every neighborhood characterizes a N-person game, such that the individual fitness (or success) of an individual is determined by the payoffs resulting from the game centered on herself plus the games centered on her direct neighbors. We provide a visual representation of such group formation in Figure 2. Degree heterogeneity will create several forms of diversity, as individuals face a different number of collective dilemmas depending on their degree (and social position); groups where games are played may also have different sizes. Such diversity is introduced by considering two types of scale-free networks. One is generated with the Barabási–Albert algorithm (BA) of growth and preferential attachment [34] leading to a power-law degree distribution, high correlation in the degrees of neighboring nodes, and a low clustering coefficient. The clustering coefficient offers a measure of the likelihood of finding triangular motifs or, in a social setting, the likelihood that two friends of a given node are also friends of each other, a topological property of relevance in the context of fairness and N-person games [33]. In the second case, we consider the Dorogovtsev–Mendes–Samukhin (DMS) model [35], exhibiting the same power-law degree distributions, yet with large values of the clustering coefficient.

2.2. Networks Generated. In the BA model [34], at each time step, the network grows by adding a new node and connecting it to m other nodes already in the network. These connections are probabilistic, depending on the degree of the nodes to be connected with: having a higher degree increases the probability of gaining a new connection. This process results in heterogeneous degree distributions, in which older nodes become highly connected (creating the so-called *hubs*). This is the combination of two processes–*growth* and *preferential attachment*. In the DMS model [35], at each time step, a node is added; instead of choosing other nodes to connect with, it chooses one existing edge randomly and connects to both ends of the edge. The networks generated by the DMS model have higher clustering coefficient than those with the BA model, combining the high clustering and high heterogeneity that characterize real-world social networks.

2.3. Network-Based Role Selection. Previous works show that anchoring the probability of nodes being selected for the role of Proposer or Responder on their degree has a sizable and nontrivial effect on the evolving magnitude of proposals in traditional two-person Ultimatum Games [23, 24]. Considering multiplayer ultimatum games, however, opens space to study the interplay between group characteristics (such as group sizes) and network-based criteria to select Proposers in completely unexplored directions. So far, we assume that nodes are selected to be Proposers based on their degree. As such, in a group with N individuals, where each individual i has degree k_i, the probability that j is selected as the Proposer is given by $p_j = e^{\alpha k_j}/\sum_i e^{\alpha k_i}$, where α controls the influence of degree on role selection. One node is selected as the Proposer, and the remaining $N-1$ play as Responders.

2.4. Evolutionary Dynamics. We simulate the evolution of p and q in a population of size Z, much larger than the group size N. Initially, each individual has values of p and q drawn from a discretized uniform probability distribution in the interval $[0, 1]$. The fitness f_i of an individual i of degree k is determined by the payoffs resulting from the game instances occurring in $k+1$ groups: one centered on herself plus k others centered on each of her k neighbors (see Figure 2). Values of p and q evolve as individuals tend to imitate (i.e., copy p and q) the neighbors that obtain higher fitness values.

The numerical results presented in the following were obtained for structured populations of size $Z = 1000$. Similar results were obtained for $Z = 10000$. As already mentioned, we consider networks generated with both BA and DMS algorithms, with average degree $\langle k \rangle = \{4, 8, 16\}$. Simulations take place for 2×10^5 generations, considering that, in each generation, all the individuals have (on average) the opportunity to revise their strategy through imitation once.

At every (discrete and asynchronous) time step, two individuals A and B (neighbors) are selected from the population. Given the group setting of the MUG, B is chosen from one of the neighbors of A. Their individual fitness is computed as the accumulated payoff in all possible groups for each one, provided by the underlying structure (in each group, the role of Proposer or Responder is selected following the results); subsequently, A copies the strategy of B with a probability χ that is a monotonic increasing function of the fitness difference $f_B - f_A$, following the pairwise comparison update rule: $\chi = 1/1 + e^{-\beta(f_B - f_A)}$ [36].

The parameter β specifies the selection pressure ($\beta = 0$ represents neutral drift, and $\beta \longrightarrow +\infty$ represents a purely deterministic imitation dynamics). Imitation is myopic: the value of p and q copied will suffer a perturbation due to errors in perception, such that the new parameters will be given by $p' = p + \zeta_{p,\varepsilon}$ and $q' = q + \zeta_{q,\varepsilon}$, where $\zeta_{p,\varepsilon}$ and $\zeta_{q,\varepsilon}$ are uniformly distributed random variables drawn from the interval $[-\varepsilon, \varepsilon]$. This feature not only (i) models a slight blur in perception but also (ii) helps to avoid the random extinction of strategies and (iii) ensures a complete exploration of the strategy spectrum. To guarantee that new p and q are not lower than 0 or higher than 1, we implement reflecting boundaries at 0 and 1. Alternative options of mutation operators to test in the future include drawing mutations from normal distributions considering absorbing boundaries [37].

Furthermore, with probability μ, imitation will not occur and the individual will adopt random values of p and q, drawn from a uniform distribution over $[0, 1]$. This can either represent the adoption of a random strategy by an individual or a low rate of existing players being replaced by new naive players. We use $\mu = 1/Z$, $\beta = 10$, and $\varepsilon = 0.05$ throughout this work. The effect of varying μ is similar to the one verified when changing ε: an overall increase of randomness leads to higher chances of fairer offers (as in [7, 15]). For each combination of parameters, the simulations are repeated 100 times (10 times using 10 different networks from each class studied), whereas each simulation starts from a population where individuals are assigned random values of p and q drawn uniformly from $[0, 1]$. We provide a summary of the algorithm used to revise agents' strategies in Algorithm 1. The average values of p, q, and f (denoted by $\langle p \rangle$, $\langle q \rangle$, and $\langle f \rangle$) are obtained as a time and ensemble average, taken over all the runs (considering the last 10^5 generations, disregarding an initial transient period).

3. Results and Discussion

We run the proposed model and record the average strategies played by the agents over time and over different runs (starting from different initial conditions, see Materials and Methods). We find that attributing the role of Proposer to low-degree nodes (or *low-degree Proposer assignment*) increases the average level of proposal, p, adopted in the population of adaptive agents. This means that the payoff gap between Proposers and Responders is alleviated. Figure 3 shows that, for low α ($\alpha < 0$), we obtain higher levels of average proposal when considering both BA (low clustering coefficient) and DMS (high clustering coefficient) networks. We observe a steep decline in average proposals when the role of Proposer and Responder is attributed, regardless the degree of individuals ($\alpha = 0$). The low-proposal tendency is maintained if the role of Proposer is assigned to high-degree nodes ($\alpha > 0$).

We also confirm that the high-degree Proposer assignment leads to unequal (unfair) results within a population. Figure 4 depicts the average payoff gains for individuals with a certain degree. We can observe that, for $\alpha = 2$, high-degree nodes obtain much higher values of payoff than low-degree nodes. This situation is ameliorated if individuals with a lower degree are given a higher chance of becoming Proposers (lower α) and, to a lower extent, if more Responders are required to accept a proposal in order for it to be accepted (higher M, (d–f) in Figure 4).

We can further verify the effect of α on fairness through the so-called *Lorenz curves* [38], often used to compute the *Gini coefficients* [39] that quantify income inequality. In Figure 5, we represent the Lorenz curves associated with different role-assignment rules (α) and voting rules, M. Each curve is generated by ordering individuals by increasing the value of income plotting the corresponding cumulative distribution. A curve closer to the perfect equality line ($x = y$) represents a more egalitarian distribution of resources and a lower Gini coefficient. As we verify in Figure 5, the most unequal outcomes (higher Gini) are obtained for higher α. We further verify that, when fixing $\alpha = 2$, having stricter voting rules (high M; in this case, $M = 0.9$) attenuates the unfairness associated with having hubs as the Proposers.

Not only does the low-degree Proposer assignment reduce unfairness, it also sustains more efficient outcomes—taken as higher values of average fitness observed in the population. In Figure 6, we confirm that low values of α maximize the average fitness of populations. This occurs when considering heterogeneous networks with different average degrees ($\langle k \rangle$) and group decision rules (M). This effect is more evident when considering less-strict group decision rules (that is, lower M, meaning that less number of Responders are required to accept a proposal for the group to accept it) and networks with higher $\langle k \rangle$.

```
Initialize all p_i, q_i = X ~ 𝒰(0,1), i ∈ {1,...,Z}
For t⟵1 To Gens do Main cycle of interaction and strategy update:
  For j⟵1 To Z Select agent to update:
    /* Sample two neighbors of the population
    A⟵X ~ 𝒰({1,...,Z}) (agent to update)
    B⟵Y ~ 𝒰(neighbours(A)) (agent to be imitated)
    if X ~ 𝒰(0,1) < μ then Mutation:
      p_A⟵X ~ 𝒰([0,1])
      q_A⟵X ~ 𝒰([0,1])
    else Imitation:
      f_A⟵ fitness(A)
      f_B⟵ fitness(B)
      prob⟵ 1/(1 + e^{-β(f_B - f_A)})
      If X ~ 𝒰([0,1]) < prob then
        p_A⟵p_B + imitation error ~𝒰([-ε, ε])
        q_A⟵q_B + imitation error ~𝒰([-ε, ε])
```

Algorithm 1: Pseudo-code of the main cycle of our simulations. We perform 100 runs over 10 different networks of each type (BA and DMS) with 2×10^5 generations per run.

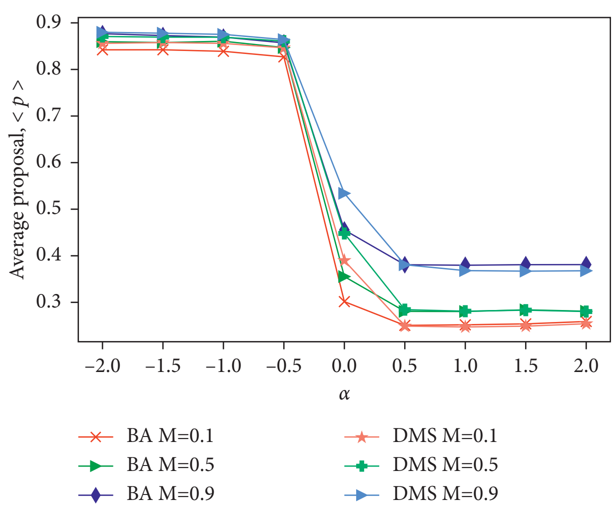

Figure 3: The average proposal played by agents in a population, $\langle p\rangle$, decreases with α. This means that attributing the role of Proposer to high-degree nodes reduces the overall fairness level in a population. We present results for BA and DMS networks with an average degree $\langle k\rangle = 4$. We verify that the low-degree Proposer assignment maximizes $\langle p\rangle$ for different group decision rules, $M = \{0.1, 0.5, 0.9\}$, i.e., the fraction of Responders that needs to accept a proposal, for it to be accepted by the group.

Finally, we confirm that the low-degree Proposer assignment maximizes the average proposal played in the population (and thus fairness) when considering networks with higher $\langle k\rangle$ and, as a result, larger average group sizes. As Figure 7 conveys, the higher values of average proposal, $\langle p\rangle$, are obtained for $\alpha < 0$. Notwithstanding, we are able to find parameter spaces where the dependence of $\langle p\rangle$ on α is seemingly affected by (*i*) the average connectivity of the network—and thus the average size of the groups in which MUG are played—and (*ii*) particular values of M. Also, we confirm that increasing M increases $\langle p\rangle$ for all values of α. Our results suggest that offering the first move to low-degree nodes balances the natural power of highly connected nodes in scale-free networks, leading to a significant increase in the global levels of fairness. Interestingly, we also find that particular voting rules (M) are able to attenuate the negative effect of high α (i.e., privileged high-degree nodes being selected to be Proposers) on fairness.

One can reach an additional intuition for the increase of fairness through the attribution of the role of Proposer to low-degree nodes if we approximate scale-free networks to a collection of heterogeneous starlike structures [27]. For simplicity, let us consider two hubs (H_1 and H_2, both with degree k) at the center of two stars, each with two prevalent p values (high p_h in the green star, around H_1, and low p_l in the blue star, around H_2) (see Figure 8). Under this configuration, we may ask which strategy (p_h or p_l) will prevail. For that, we note that, within each star, the strategies of the hubs are likely to locally prevail and thereby we focus

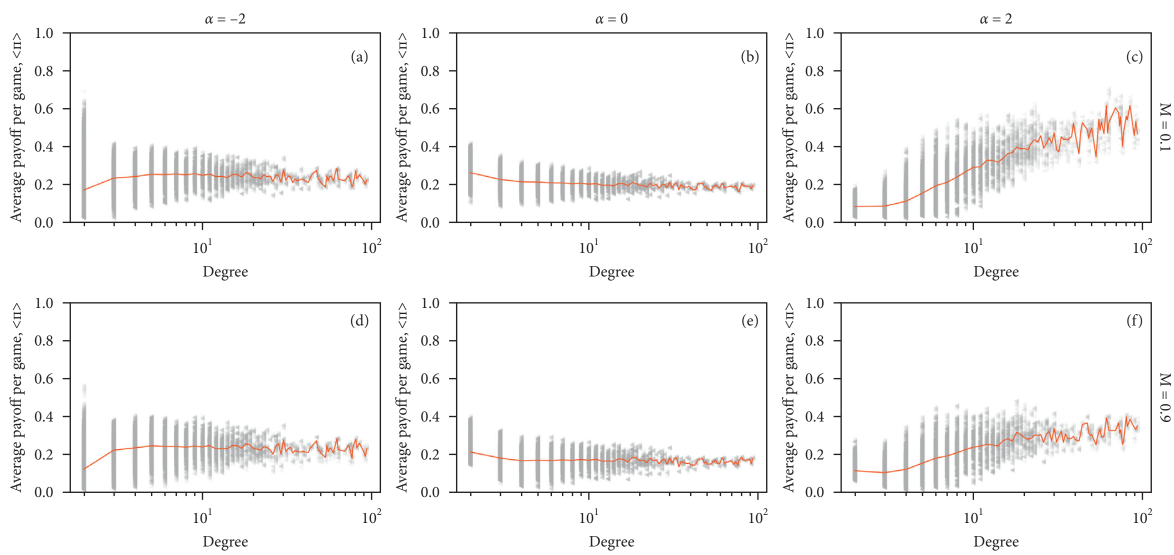

FIGURE 4: On top of decreasing the average level of proposal in the population, $\langle p \rangle$, we found that attributing the role of Proposer to highly connected nodes decreases the level of fairness and equality *within* the population. Here, we use scatter plots to observe the average payoff obtained per game, $\langle \Pi \rangle$, for individuals with a certain degree (horizontal axis). (a, d) A low-degree Proposer assignment scenario ($\alpha = -2$); (b, e) random—and degree-independent—role attribution ($\alpha = 0$); (c, f) a high-degree Proposer assignment scenario ($\alpha = 2$). Each gray cross represents a node in a degree-$\langle \Pi \rangle$ space; the orange line represents the mean taken over all nodes with a certain degree. (a–c) $M = 0.1$ and (d–f) $M = 0.9$. High α, i.e., high-degree Proposer assignment, implies that highly connected nodes earn (approximately) five times more payoff per game than low-connected nodes (c). This effect is alleviated for higher M; for $M = 0.9$, highly connected nodes earn (approximately) three times more payoff per game than low-connected nodes (f).

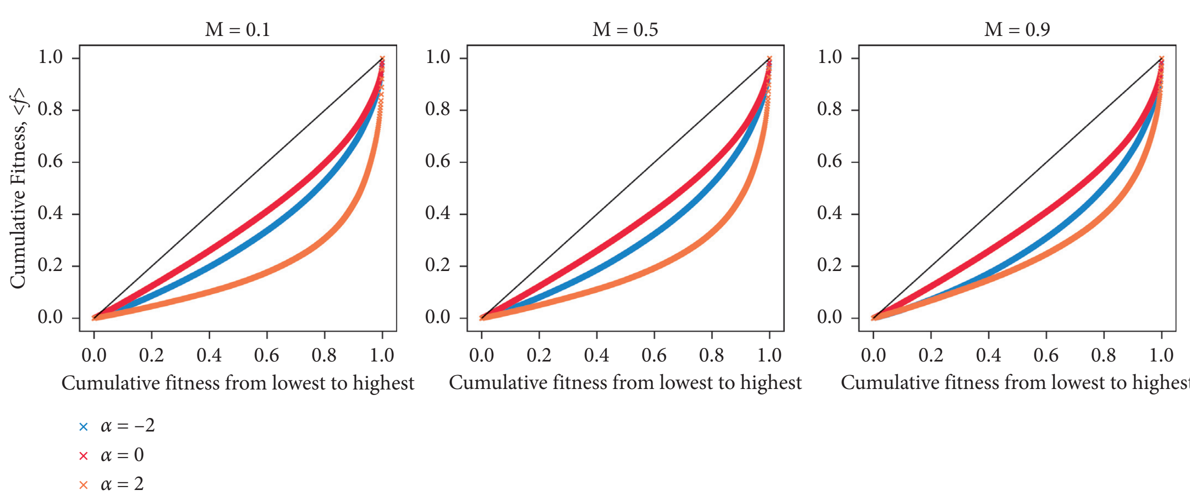

FIGURE 5: Selecting high-degree nodes as Proposers increases unfairness. Here, we represent the so-called Lorenz curves, often used to compute the Gini coefficients—a typical measure of income inequality. Each curve is generated by ordering individuals by increasing the value of income and plotting the corresponding cumulative distribution. Curves closer to the perfect equality line (45° line) represent more egalitarian outcomes. Here, we observe, yet again, that assigning the role of Proposer to high-connected nodes (alpha = 2) yields unfair outcomes (orange line). While this is evident for soft ((a), $M = 0.1$), medium ((b), $M = 0.5$), and strict ((c), $M = 0.9$) decision rules, we also verify that whenever hubs are the Proposers ($\alpha = 2$), having strict decision rules (high M) reduces unfairness. In all cases, the random Proposer assignment leads the most egalitarian outcomes.

on strategy invasion along the edge connecting both hubs (red/thick link); we further assume that H_1 is characterized by a higher value of p than H_2 ($p_h > p_l$). The question is will α impact the total payoff of H_1 and H_2 (f_h and f_l, respectively) such that under high α, H_2 is likely to be imitated by H_1 ($f_h < f_l$) and under low α, H_1 is likely to be imitated by H_2 ($f_h > f_l$)? The answer is yes, if high-degree nodes are preferentially selected as Proposers (high α), the total payoffs of H_1 and H_2 decrease with the value of p characterizing their stars, p_h and p_l,

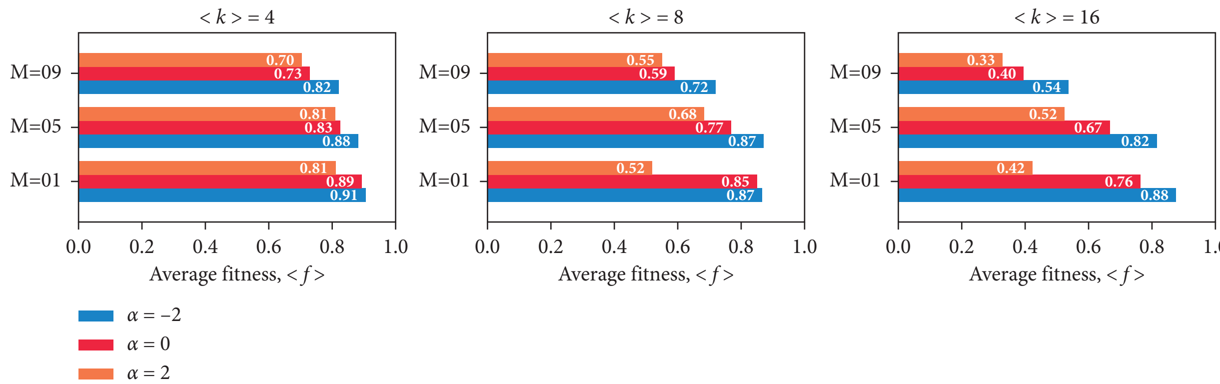

Figure 6: The low-degree Proposer assignment maximizes the average fitness (i.e., sum of payoffs taken over all games, see Figure 4) in a population. Here, we observe that the average fitness, $\langle f \rangle$, increases as α decreases. We show results for BA networks with different values of $\langle k \rangle$ and M. A similar conclusion is obtained when considering DMS networks with the same parameters. Note that increasing $\langle k \rangle$ implies that the average group size to play MUG also increases, which leads offers to be divided by larger groups (hence contributing to lower values of average payoff per game). On the contrary, increasing $\langle k \rangle$ means that more games are played, thus contributing to an increase in accumulated fitness (taken as the sum of payoffs in all games played).

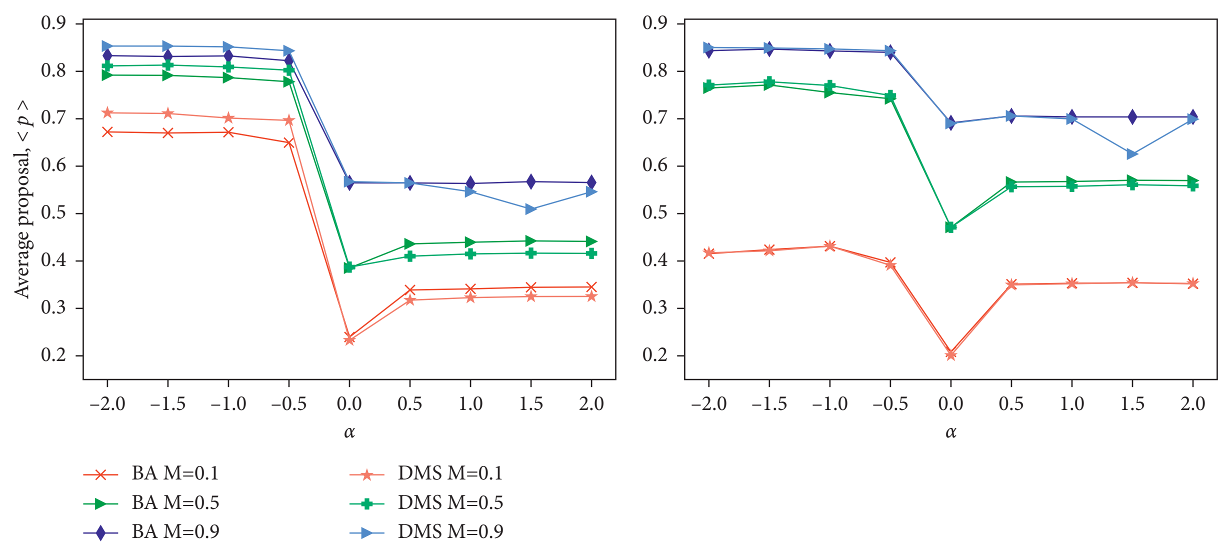

Figure 7: We confirm that the low-degree Proposer assignment maximizes populations' average level of proposal, $\langle p \rangle$, for both BA and DMS networks with higher average degree ($\langle k \rangle = 8$, (a) and $\langle k \rangle = 16$, (b)). For networks with higher $\langle k \rangle$—leading to MUG played in larger groups—and low M, random role attribution ($\alpha = 0$) configures the worst scenario in terms of fair proposals.

respectively. As a result, if $p_h > p_l$, it is likely that H_2 gets imitated by H_1 which contributes to decrease in the average value of p in both stars (Box 1). Conversely, if high-degree nodes are preferentially selected as Responders (low α), the fitness of H_1 and H_2 will increase with p_h and p_l, respectively. As a result, the hub associated with the star revealing a higher p is likely to be imitated which, if $p_h > p_l$, implies that H_1 will tend to be imitated by H_2; the average value of p in both stars thereby increases (Box 2). This intuition hinges on the assumption that all offers are accepted, which is only true for $M = 0$ or $p \geq q$ for all nodes. If M increases, it is harder for low proposals to get accepted as they require a higher number of accepting Responders to be validated which contributes for p to increase overall and to fairer proposals [15, 33]. This intuition remains valid if such heterogeneous structures portray a high clustering coefficient (e.g., when leaves of each starlike community are linked to each other), offering an additional intuition on why the DMS and the BA network models offer similar results.

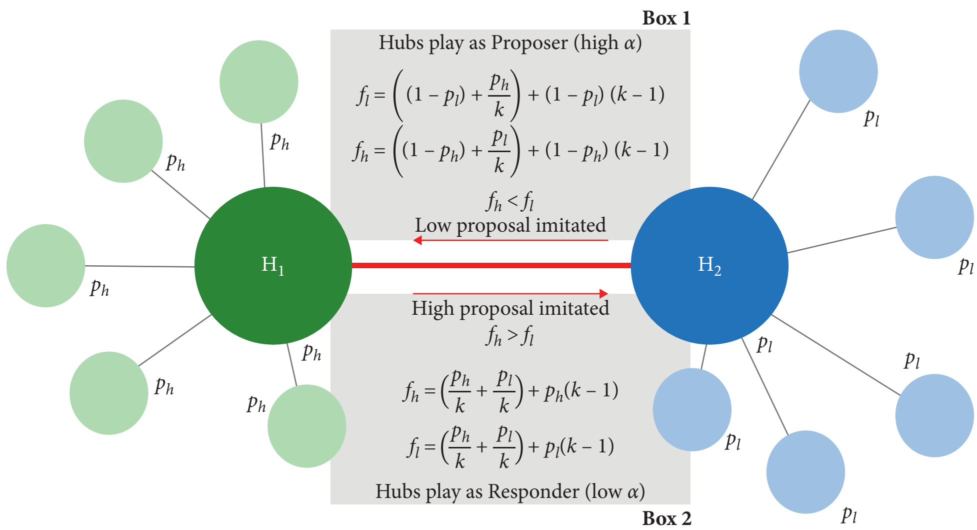

FIGURE 8: Dynamics of fairness on two stars, centered in hub nodes H_1 and H_2, characterized by values of proposals p_h and p_l. In Box 1, we present the total fitness of H_1 and H_2 (f_h and f_l, respectively), assuming that α is high and hubs play as Proposers. In Box 2, we present the total fitness, assuming that α is low and hubs play as Responders. In both cases, we assume that proposals are always accepted. Assuming that H_1 is a fairer node ($p_h > p_l$), we can conclude that H_1 is likely to be imitated for low α and H_2 is likely to be imitated for high α.

4. Conclusion

In this paper, we address the general problem of (1) deciding how to attribute bargaining roles in a social network and, in particular, (2) understanding the impact of different criteria on the emerging levels of fairness in Multiplayer Ultimatum Games. We verified that attributing the role of Proposer to low-degree nodes boost both fairness and overall fitness. This conclusion remains valid for different network structures (BA and DMS networks with average degrees ranging from 4 to 16) and interaction scenarios (in terms of group sizes and group decision rules).

We also find that the perils of having high-degree Proposers can be softened with strict group decision rules. This means that, whenever α is high, can be default, and cannot be lowered (e.g., hubs by having the needed resources to be the first movers in a bargaining situation), unfairness can be reduced by imposing that proposals need to be validated by a large fraction of Responders. The effect of M on eliciting fairer offers is similar to that found in the recent literature [15, 18]. By considering a higher M, more accepting Responders are required in order for a proposal to be accepted; as a result, it is harder for unfair Proposers (i.e., adopting lower p) to have their proposals accepted and increase their payoffs by keeping the largest sum of the initial endowment to themselves. As a result, there is a tendency for the average offer in the population, p, to increase. Also, our results are in line with works showing that selecting low-degree Proposers maximizes fairness in the context of pairwise Ultimatum Games [23] and Dictator Games [24]. Here, we confirm that the mechanisms contributing for fairness through adaptive role assignment in the pairwise UG are likely to extend to Multiplayer Ultimatum Games, a setting where (as discussed in Related Work) the asymmetry in payoffs between Proposers and Responders is exacerbated; extending the analysis of role assignment in MUG allows one to cover N-person interactions and, importantly, to test how voting mechanisms can curb the effects of particular role assignments.

This work can underlie several extensions of interest for social and engineering sciences. Here, we consider that role assignment is endogenously imposed. In reality, the tendency for certain nodes to be allocated with particular roles is likely to evolve side-by-side with individual strategies, being another self-organized property of the system, like fairness and wealth distributions. Other sources of heterogeneity known to influence the propensity to be fair, such as cultural [40] and socioeconomic [41] settings or individuals' engagement in institutions [42], may further influence how roles and power dependencies [43] are assigned. Moreover, the fact that network-based role assignment elicits fairness in rather complicated scenarios—as multiplayer bargaining games—suggests that such an approach could also be used within the broader context of active interventions aiming at fostering fairness in hybrid populations comprising humans and machines [18, 44–46]. In this context, it would be relevant to assess—both experimentally and through numerical simulations—the impact on human decision-making of having virtual regulators dynamically deciding the role to adopt by their group peers, depending on their position in the interaction structure.

Finally, we note that, while here we consider static networks, it is likely that dynamic networks [47–50] can offer extra means for degree and roles to become correlated over time. For example, if fair Proposers (or lenient Responders) attract a higher number of neighbors, their degree will increase as a by-product of their role and strategy, which may imply that effective values of α may emerge from the coevolution of strategies and social ties.

Despite these open questions, our present work already suggests that carefully selecting the role of each agent within a group—depending on their social position and without limiting their available options—can offer a long-term social benefit, both in terms of the overall levels of fairness, wealth inequality, and global wealth of a population comprised of self-regarding agents.

Data Availability

The methods to produce the data that support the findings of this study are included within the article.

Disclosure

A preliminary version of this work was presented in Abstracts from the Proc. of the 20th International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2021).

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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