Integrating Statistical Correlation

with

Discrete Multi-Criteria Decision Making

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Abstract-This paper analyses two hypotheses that considers a correlation between the number of alternatives and the number of criteria considered in a Multiple Criteria Decision Making (MCDM) problem with the minimum percentage change required in the lowest criterion weight to change the outcome of a method. Two MCDM methods are considered, The Analytical Hierarchy Process (AHP) and The Preference Ranking Organization METHod for Enrichment of Evaluations II (PROMETHEE II) were applied to the same sets of criteria weights and performance measures. More than two thousand randomly generated sets of criteria weights and performance measures are considered. The minimum percentage change in the lowest criterion weight required to change the outcome of a method is calculated. Pearson's r parametric test is used to test the hypotheses. Results from parametric test were statistically significant and shows a weak negative correlation for hypothesis one and weak positive correlation for hypothesis two.

Keywords: Multiple Criteria Decision Making; AHP; PROMETHEE II; Correlation; Criteria; Pearson's r parametric test, Statistical analysis.

1. Introduction

The novel work presented in this paper identifies the effect of the redundancy of alternatives and criteria on the stability of Multiple Criteria Decision Making (MCDM) methods. For the first time, the correlation between the number of alternatives considered and the number of criteria considered vs the minimum percentage change in criteria weights is investigated. That analysis could assist decision makers in achieving a more robust outcome.

It is important for the decision makers to avoid adding irrelevant alternatives and criteria to their problem and understand the effect of alternatives and criteria redundancy. This paper will explore that and present two Hypotheses linking between MCDM methods sensitivity and the number of alternatives and criteria considered in a problem.

There is no unique and well-defined methodology that could be followed step-by-step from the beginning to the end of a decision making process. Decisions could be normative, descriptive or naturalistic (Mahaffey, 2015).

Normative decision-making is based on evidence, logic and analysis. These decisions focus on choosing the best-fit alternative from a set of alternatives using mathematical calculations and analysis (Mahaffey, 2015). Moreover, Mahaffey (2015) claimed that normative decision-making techniques might be preferred to other decision-making techniques due to the use of an organized collection process to gather information directly related to a problem, for analysis and assessment. In addition, they usually produce rational decisions.

Unlike normative decision-making, descriptive decision-making focuses on a perception of reality, personal experience and emotions (Dane and Pratt, 2007). Descriptive decision-making is based on intuition and experience. These techniques focus on the way people process information and make judgments.

Naturalistic decision-making is a hybrid decision-making technique formed by the combination of normative and descriptive processes. This hybridized decision-making technique has the advantage of both rapid analysis of information combined with the personal experience of a decision-maker (Cummings, 2004; Weber & Johnson, 2009).

MCDM are often considered as normative decision making, MCDM is an important part of operational research and decision theory, MCDM methods help decision makers to identify the best compromise solution by assessing a set of alternatives with respect to a set of multiple and often conflicting quantitative and/or qualitative real-world criteria to select the most suitable alternative that fulfilled the desired goal (Ishizaka & Siraj, 2018). MCDM methods are a set of methods and procedures by which multiple and conflicting criteria can be incorporated into a decision process. MCDM methods are not optimization methods, MCDM methods aim at providing decision makers with the best compromise solution.

MCDM methods could be divided into two types with respect to nature of alternatives set considered: continuous and discrete. Multi-objective decision making (MODM) methods are used to deal with a continuous set of alternatives. Multi-attribute decision-making (MADM) methods are used to deal with a discrete set of alternatives (Zavadskas *et al.*, 2014).

Since there development and during the past four decades many MCDM methods have been developed, the most popular of which are:

- Analytical Hierarchy Process (AHP).
- Preference Ranking Organization METHod for Enrichment Evaluations (PROMETHEE).
- Elimination Et Choix Traduisant la Realite or Elimination and Choice Expressing Reality (ELECTRE)
- Weighted Sum Model (WSM).
- Weighted Product Model (WPM).
- Weighted Aggregated Sum Product Model (WASPAS).
- Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS).
- VlseKriterijumska Optimizacija I Komopromisno Resenje (VIKOR).

These methods were applied to a large number of MCDM problems in different fields of science such as:

- Engineering.
- Economics.
- Financial.
- Real estate.

- Supplier selection and many more.

Experience showed that there is no MCDM method able to deal with all MCDM problems (Haddad and Sanders, 2018; 2019). Different MCDM methods could hold specific strengths and weaknesses (Haddad *et al,* 2019). Ozernoy (1992) stressed that there was no "perfect" MCDM method because decision-makers were often unable to provide all the necessary information and/or different decisional problems require different algorithms to deliver their intended outcomes. Moreover, information needed for making a decision can often be vague and uncertain. Vafaei *et al.* (2018) claimed that the outcome of MCDM methods depended on both the method used and the normalization technique applied. Therefore, careful judgment in selecting a MCDM method with regards to its strengths and weaknesses could provide a better outcome. Roy (2011) stressed on taking into account *"vague approximations and areas of ignorance"* in order to provide a suitable outcome in a MCDM problem, moreover, he classified these factors under the umbrella of uncertainty. Aljumaili *et al*. (2018) related between the quality provided and the outcome of the decision process, they asserted that poor input data could often lead to poor decisions.

Uncertainty could be sources of distortion in making decisions. Uncertainty is present throughout all phases of the decision-making process, many researchers stressed the need to consider uncertainty in making decisions (Butler *et al.*, 1997; Durbach & Stewart, 2011, 2012; French, 2003; Stewart, 2005), however, it was not often considered in practice. There are many definitions for uncertainty, Stewart (2005) identified it as "*At most fundamental level, uncertainty relates to a state of human mind, i.e. lack of complete knowledge about something",* Walker *et al.* (2003) stated that uncertainty was "*any departure from the unachievable ideal of complete determinism"*. Stewart (2005) classified uncertainty into two general categories based on their location:

- Internal uncertainty associated with decision makers' preferences and judgments
- External uncertainty associated with consequences of the outcome

Vanderpas *et al.* (2010) described four levels of uncertainties at both locations having two extremes: from determinism to total ignorance. Different methods were used to deal with different levels of uncertainties ranging from handling uncertainty probabilistically to deep uncertainty. Deep uncertainty is related to level 3 and level 4.

Comes (2013) distinguished between two types of decision-making using the type of uncertainty involved:

- Decision-making under ignorance where severe uncertainties were characterized by ignorance
- Decision-making under risk where probability functions were known

It is important for decision makers to understand the nature of uncertainty in order to enhance their ability to make decisions and to reduce the level of risk associated with their decisions. Decision makers are encouraged to use more complex scientific decision-making techniques that are less vulnerable to distortion in such environments. MCDM methods might provide a good example of such techniques and could provide a suitable outcome. Many MCDM methods had gained fuzzy versions based on Fuzzy Set Theory to address uncertainty. Stewart (2005) suggested proper problem structuring, appropriate sensitivity and risk analysis to deal with internal uncertainty and stressed that deep internal uncertainties cannot be reduced by proper problem structuring since it is irresolvable and encouraged using sensitivity and robustness analysis to deal with it. Saaty and Ergu (2015) stressed the need to conduct sensitivity analysis to check for robustness and validate feasibility of MCDM outcomes, most MCDM problems conduct sensitivity analysis at the end. This paper is concerned with the uncertainties that could be captured using sensitivity analysis.

Two new hypotheses have been proposed and tested. These hypotheses aim to help potential MCDM methods users to predict the behaviour of different MCDM methods in the presence of risk and uncertainty. These hypotheses were tested on two different MCDM methods, the Analytical Hierarchy Process (AHP) and the Preference Ranking Organization METHod for Enrichment of Evaluations II (PROMETHEE II). Randomly generated sets of criteria weights and performance measures were considered, the randomly generated sets were classified as follows:

- Three different sets of criteria each consisted of three criteria applied to 3, 4, 5, 6 and 10 alternatives.
- Three different sets of criteria each consisted of four criteria applied to 3, 4, 5, 6 and 10 alternatives.
- Three different sets of criteria each consisted of five criteria applied to 3, 4, 5, 6 and 10 alternatives.
- Three different sets of criteria each consisted of six criteria applied to 3, 4, 5, 6 and 10 alternatives.
- Three different sets of criteria each consisted of ten criteria applied to 3, 4, 5, 6 and 10 alternatives.

AHP and PROMETHEE II were applied to these sets. AHP was applied to a total of 1053 randomly generated sets of criteria and performance measures, PROMETHEE II was applied to a total of 1118 randomly generated sets of criteria and performance measures.

Sensitivity analysis was conducted and the minimum percentage change in the lowest criterion weight required to change the outcome of a MCDM method was calculated. Section 2 will briefly explain AHP and PROMETHEE II, Section 3will present the two new hypotheses, briefly describe the correlations considered, the parametric test used and will present the results, Section 4 will discuss these results, and Section 5 will provide some concluding remarks.

2. AHP and PROMETHEE II

This section will briefly explain two MCDM methods used to test the proposed hypotheses. The Analytical Hierarchy Process (AHP) and the Preference Ranking Organization METHod for Enrichment of Evaluations II (PROMETHEE II)

AHP is a MCDM method developed by Thomas L. Saaty in 1971- 1975 while at the Wharton School (Saaty, 1987). Since its development, AHP was applied to almost all fields of decision making.

AHP help decision makers in solving multiple conflicting subjective criteria (Ishizaka & Labib 2009) by breaking down a complex problem into simpler sub-problems then, aggregating the solutions of all sub-problems into one solution (Saaty, 1994). AHP uses expert judgments to derive priorities, apply pairwise comparisons to measure how much one alternative dominates another with respect to a certain criterion (Saaty, 2008). Using a hierarchical structure of the criteria, AHP could allow users to focus on specific criteria and sub-criteria when providing judgments. Figure 1 shows a simple Analytical Hierarchy Process hierarchy model composed of three levels, the goal of the decision process is on the first level, set of criteria on the second level by which alternatives are assessed, alternatives on the third level (Saaty, 2012). Moreover, AHP could incorporate group decision making (Vaidya & Kumar, 2006).

Figure 1: Simple three level decision hierarchy (Saaty, 2012)

AHP allows a level of inconsistency among judgments, Saaty (2004) proposed that inconsistency could be “one order of magnitude less important than consistency or 10% of the total concern with consistent measurement." If inconsistency was larger than 10% it could disturb the decision process.

Ishizaka and Labib (2009) identified seven steps for a decision making process using AHP:

1. Problem modelling: identify goals, criteria and alternatives.
2. Pairwise comparisons conducted on each part of the hierarchy.
3. Judgments scale, AHP can evaluate quantitative and qualitative criteria and alternatives using the same preference scale of nine levels.
4. Priorities derivation, traditional AHP used eigenvalue method.
5. Consistency check.
6. Aggregation of local priorities with respect to all criteria to calculate the global priorities of each alternative using Equation (1).

   $$P_i = \sum_j w_j \,.\, l_{ij} \qquad (1)$$

   Where: P_i: global priority of the alternative i

   w_j: weight of the criterion j

   l_{ij}: local priority
7. Sensitivity analysis.

According to Al-Shabeeb (2015) the Analytical Hierarchy Process often generates good results, provides a good approach to define and evaluate alternatives, and presents a powerful hierarchy model to visualize the problem, but considering large number of alternatives and criteria makes the application of AHP more time and effort consuming due to a large number of pairwise comparisons conducted.

PROMETHEE methods were developed by Jean-Pierre Brans and presented for the first time in 1982 at a conference at the Université Laval, Québec, Canada. PROMETHEE methods have been extensively studied since then. Their applications attracted the attention of many researchers and practitioners.

PROMETHEE methods are outranking MCDM methods with PROMETHEE I partial ranking and PROMETHEE II total ranking of alternatives. PROMETHEE methods generally consist of a preference function representing each criterion and weights describing their relative importance. The main idea of the PROMETHEE methods was to conduct pairwise comparisons among alternatives regarding each criterion then comprehensively comparing them with respect to all criteria (Xiaohann *et al.,* 2013).

According to Brans (1982), PROMETHEE methods apply the following steps:

- Identify the problem.
- Identify a set of criteria.
- Identify information between criteria (criteria weights) Identify Information within criteria (pairwise comparisons and preference functions).
- Identify a set of alternatives.
- Evaluate overall score of alternative.

Brans (1982) identified six types of preference functions, Usual criterion preference function was used in this paper shown in Figure 2 and stressed that efficient alternatives were the alternatives that were non-dominated by other alternatives.

Figure 2: Usual criterion preference functions (Brans, 1982)

Each preference function identified by Brans (1982) required a number of parameters (*q, p, or s*) to be identified:

- *q:* Indifference threshold.
- *p:* Strict preference threshold.

- *s:* Intermediate value between q and p.

Moreover, Brans (1982) calculated the Preference Indices using Equations (2) and (3):

Let a,b ЄA and:

$$\pi(a,b) = \sum_j P_i(a,b).w_i \tag{2}$$

$$\pi(b,a) = \sum_j P_i(b,a).w_i \tag{3}$$

Where, π (a,b) expressed the degree by which alternative a was preferred to alternative b, and π (b,a) express the degree by which alternative b was preferred to alternative a.

And

$\pi(a,a) = 0$

$0 \leq \pi(a,b) \leq 1$

$0 \leq \pi(b,a) \leq 1$

$0 \leq \pi(a,b) + \pi(b,a) \leq 1$

$\pi(a,b) \approx 0$ weak global preference of a over b.

$\pi(a,b) \approx 1$ strong global preference of a over b.

And calculated the Positive, Negative and Net outranking flows using Equations (4), (5) and (6).

- Positive outranking flow:

$$\emptyset^+ (a) = \frac{1}{n-1}\sum_{x \in A} \pi\ (a,x) \tag{4}$$

- Negative outranking flow:

$$\emptyset^- (a) = \frac{1}{n-1}\sum_{x \in A} \pi\ (x,a) \tag{5}$$

- Net outranking flow:

$$\emptyset(a) = \emptyset^+ (a) - \emptyset^+ (a) \tag{6}$$

Rather than pointing out a *"right"* decision, the PROMETHEE methods aids decision makers in finding the alternative that best suits their goal and their understanding of the problem. It provides a comprehensive and rational framework for structuring a decision problem, identifying and quantifying its conflicts and synergies, clusters of actions, and highlight the main alternatives and the structured reasoning behind.

3. Correlation and MCDM

This Section will present two new hypotheses, briefly describe the correlations considered and the parametric test used.

Hypothesis one suggested a correlation between the number of alternatives in a MCDM problem and the minimum percentage change required in the lowest criterion weight to change the outcome of a MCDM method.

Null Hypothesis one ($H^1{}_0$): There is no effect between the number of alternatives considered in a MCDM problem and the minimum percentage change required in the lowest criterion weight to change the outcome of a MCDM method.

Alternative Hypothesis one ($H^1{}_1$): There is effect between the number of alternatives considered in a MCDM problem and the minimum percentage change required in the lowest criterion weight to change the outcome of a MCDM method.

Hypothesis two suggested a correlation between the number of criteria considered in a MCDM problem and the minimum percentage change required in the lowest criterion weight to change the outcome of a MCDM method.

Null Hypothesis two (H^2_0): There is no effect between the number of criteria considered in a MCDM problem and the minimum percentage change required in the lowest criterion weight to change the outcome of a MCDM method.

Alternative Hypothesis two (H^2_1): There is effect between the number of criteria considered in a MCDM problem and the minimum percentage change required in the lowest criterion weight to change the outcome of a MCDM method.

To test Hypothesis one and two AHP was applied to a total of 1053 randomly generated sets of criteria and performance measures, PROMETHEE II was applied to a total of 1118 randomly generated sets of criteria and performance measures. Sensitivity analysis was conducted and the minimum percentage change required in the lowest criterion weight to change the outcome of a MCDM method was calculated.

AHP and PROMETHEE II were applied to the same sets of criteria weights and performance measures. In some sets the minimum percentage change required in the lowest criterion weight to change the outcome of a MCDM method was not feasible, these sets were excluded from the analysis.

Hypothesis one and two suggested a correlation between two variables using an interval/ratio scale, Figure 3 was used to select the appropriate parametric test to test this correlation.

The parametric test used to test Hypotheses one and two was Pearson’s r parametric test. Statistical Package for Social Science (SPSS) was used to conducting Pearson’s r parametric test to study and analyse the hypotheses. Pearson’s r correlation coefficient can be used to check for the direction and the strength of the correlation under consideration. Correlation values could range from - 1 to + 1. A correlation value of 0 indicates no correlation between the two variables, the type and strength of correlation values are shown in Tables 1 and 2.

Figure 3: Choosing a Statistical Test – Decision Tree (Field, 2013; Dancey & Reidy, 2004)

Table 1: Pearson's r correlation coefficient sign vs. type of correlation

Table 2: Pearson's r correlation coefficient magnitude vs. strength of correlation

Results of Pearson's r parametric test conducted to test Hypothesis one applied to AHP and PROMETHEE II were shown in Tables 3 and 4 respectively.

Results of Pearson's r parametric tests conducted to test Hypothesis two applied to AHP and PROMETHEE II were shown in Tables 5 and 6 respectively.

Table 3: Correlation between number of alternatives and minimum percentage change in the lowest criterion weight required to change the outcome of AHP

Table 4: Correlation between number of alternatives and minimum percentage change in the lowest criterion weight required to change the outcome of PROMETHEE II

Table 5: Correlation between number of criteria and minimum percentage change in the lowest criterion weight required to change the outcome of AHP

Table 6: Correlation between number of criteria and minimum percentage change in the lowest criterion weight required to change the outcome of PROMETHEE II

4. Discussion and Results

Using tables 1 and 2 to interpret the results of Pearson's correlation tests shown in tables 3- 6, results from Tables 3 and 4 rejected null Hypothesis one and showed that there was a weak negative correlation between the number of alternatives considered in a MCDM problem and the minimum percentage change in the lowest criterion weight required to change the outcome of AHP and PROMETHEE II. Moreover, results from both tests were statistically significant at a 0.01 (2-tailed) significance level.

Results from Tables 5 and 6 rejected null Hypothesis two and showed that there was a weak positive correlation between the number of criteria considered in a MCDM problem and the minimum percentage change in the lowest criterion weight required to change the outcome of AHP and PROMETHEE II. Moreover, results from both tests were statistically significant at a 0.01 (2-tailed) significance level.

Expert Choice and Visual-PROMETHEE software were used to apply AHP and PROMETHEE II to the randomly generated data. Sensitivity analysis was applied, the software used graphical representation to conduct sensitivity analysis. The minimum percentage change in the lowest criterion weight to change the ranking of the alternatives was calculated based on the values read from the graphical representations provided by the software. Expert Choice provided an accuracy of 0.1%, Visual-PROMETHEE provided an accuracy of 1% when reading the new criterion weights. These levels of accuracy limited the calculation of the minimum percentage change required in the lowest criterion weight since the minimum percentage change required in the lowest criterion weight was based on two factors: the lowest criterion weight and the new lowest criterion weight which changed the ranking of the alternatives.

Another limitation to the data analysis was due to rounding errors, Expert choice rounded all input data used for pairwise comparisons to three decimal places. Visual-PROMETHEE accepted only integers and did not accept fractions or decimal numbers as inputs, all data needed to be multiplied by factors of 10 to be converted to integers.

This paper applied AHP and PROMETHEE II to a coherent sets of alternatives, identified a-priori, both methods suffered from rank reversal when a new alternative was introduced to the set of alternatives under consideration. Cases of rank reversal were not considered in this paper. Moreover, all sets of criteria considered in this paper were usual criteria and had linear value functions.

Since the quality of outcomes were highly related to the quality of inputs provided (Aljumaili *et al,* 2018). It is important for decision makers to understand the nature of uncertainty in order to provide appropriate and stable decisions. Durbach and Stewart (2011; 2012) claimed that the most popular way to model uncertainty was using probabilities, this paper aimed at modelling uncertainty as percentage probabilities of criteria weights. The lowest weight criterion was selected in each case to test and analyse the correlation between number of alternatives and the number of criteria considered in a MCDM problem with the minimum percentage change, the lowest weight criterion often required the biggest change in its value to alter the outcome of a method, that big change provided the required breadth to analyse the behaviour of MCDM methods when risk and uncertainty could affect criteria weights.

5. Conclusions

This paper proposed two new hypotheses suggesting a correlation between the number of alternatives considered in a MCDM problem, the number of criteria considered in a MCDM problem and the minimum percentage change in the lowest criterion weight required to change the outcome of a MCDM method. More than two thousand randomly generated sets of criteria weights and performance measures were considered. AHP and PROMETHEE II were applied to these randomly generated sets. The correlation between the number of alternatives and weights of criteria were not considered in this paper.

Testing Hypothesis one by applying AHP and PROMETHEE II showed there was a week negative correlation between the number of alternatives considered in a MCDM problem and the minimum percentage change in the lowest criterion weight required to change the outcome of AHP or PROMETHEE II.

Testing Hypothesis two by applying AHP and PROMETHEE II, showed there was a week positive correlation between the number of criteria considered and the minimum percentage change in the lowest criterion weight required to change the outcome of AHP or PROMETHEE II.

Two Hypotheses were presented to help decision makers in understanding the effect of adding irrelevant alternatives and criteria to their problem and the effect of alternatives and criteria redundancy.

Tests provided satisfactory results and showed that there was a statistically significant correlation between the number of alternatives and the number of criteria considered in a problem and the stability of the outcome of MCDM methods in the presence of risk and uncertainty in criteria weights. Pearson's correlation test (a statistical parametric test) was used to prove that these relationships were statistically significant.

6. Future Work

The authors are now considering Technique for Order of Preference by similarity to Ideal Solution (TOPSIS) method, the Weighted Sum Model (WSM), the Weighted Product Model (WPM), the Weighted Aggregated Sum Product ASsessment (WASPAS) method, Additive Ratio ASessment method (ARAS), Complex PRoportional ASsessment (COPRAS) method, the Multiplicative Exponent Weighting (MEW) method, Simple Additive Weighting (SAW) method, and PROMETHEE II using different values of λ for WASPAS and different types of preference functions: U-shaped criterion, V-shaped criterion, Level criterion, V-shape with indifference criterion, and Gaussian criterion for PROMETHEE II. Future work will consider different MCDM methods such as ELECTRE family methods.

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List of Figures:

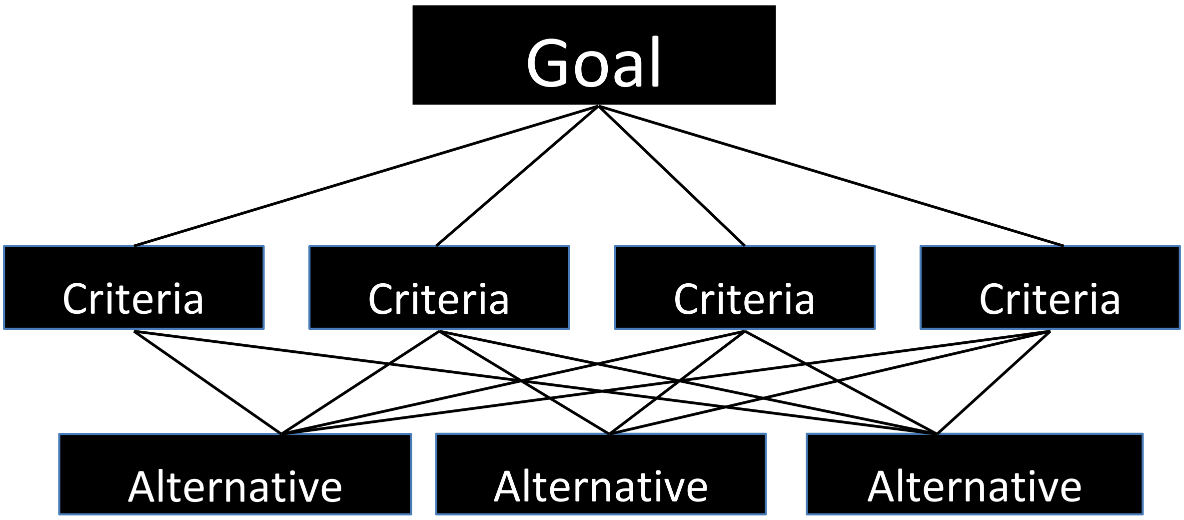

Figure 1: Simple three level decision hierarchy (Saaty, 2012)

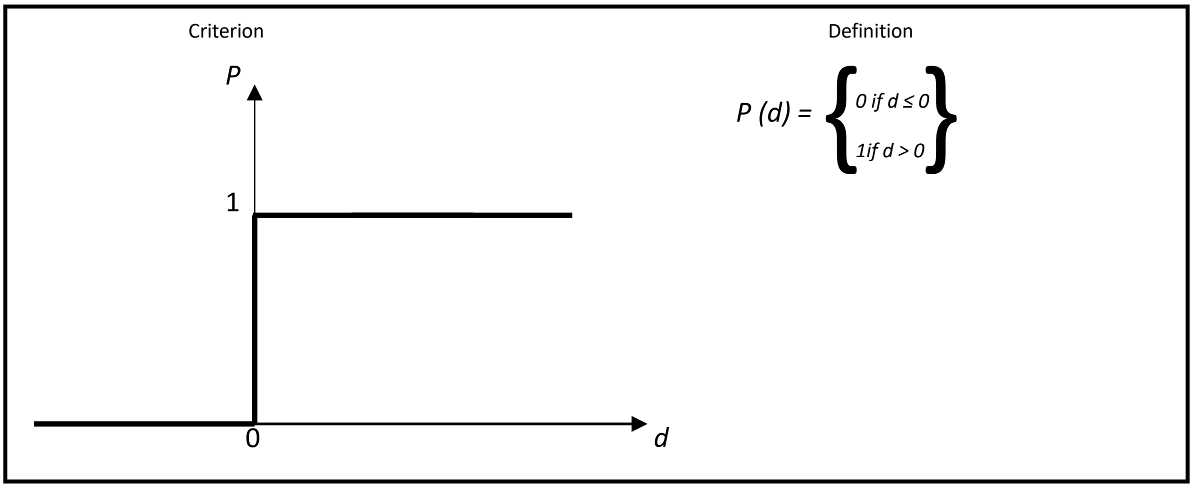

Figure 2: Usual criterion preference functions (Brans, 1982)

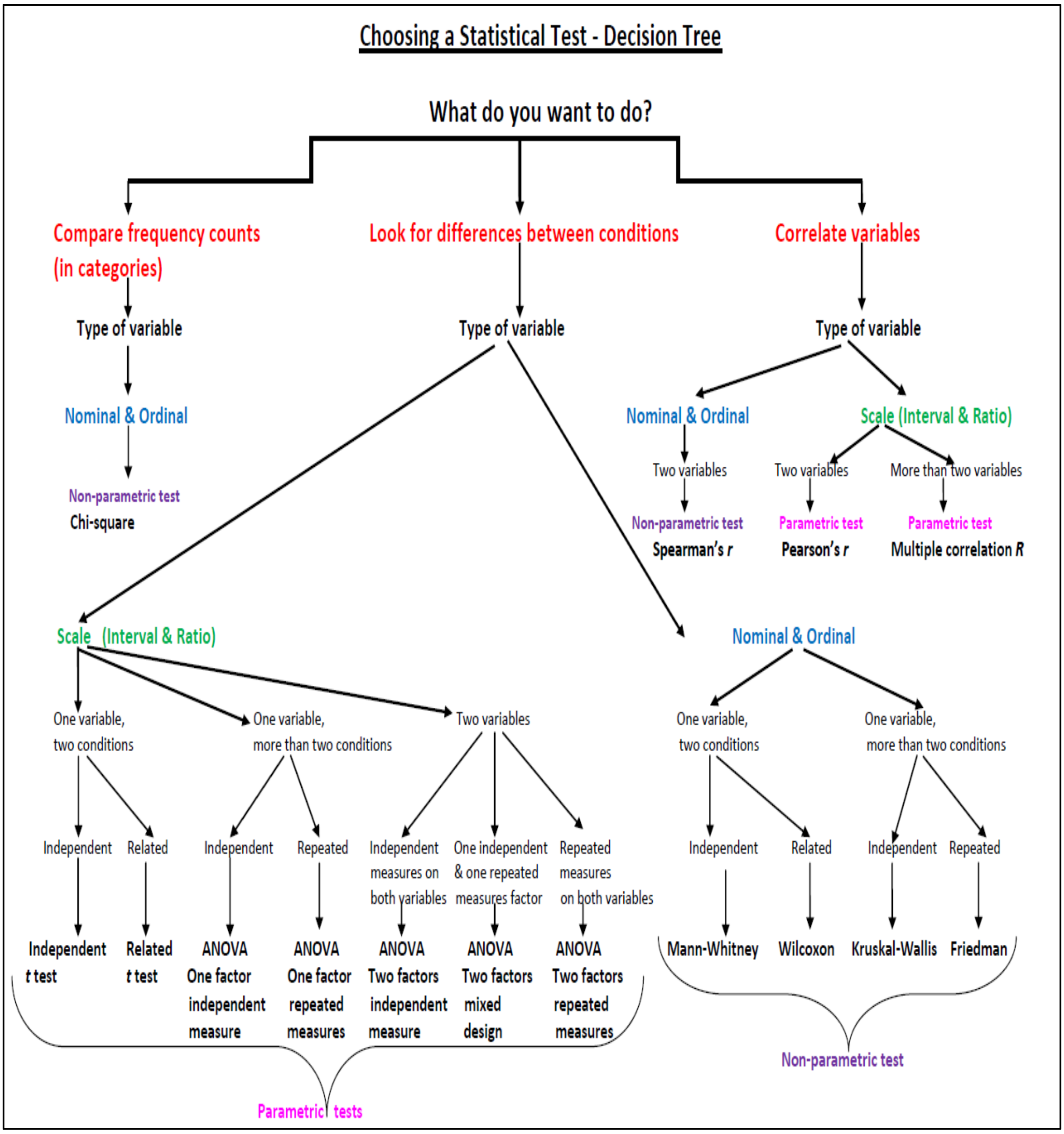

Figure 3: Choosing a Statistical Test – Decision Tree (Field, 2013; Dancey & Reidy, 2004)

List of Tables:

Table 1: Pearson's r correlation coefficient sign vs. type of correlation

Person's correlation coefficient sign	Type of Correlation	Description
Positive	Positive correlation	↑X ➔ ↑Y
Negative	Negative correlation	↑X ➔ ↓Y

Table 2: Pearson’s r correlation coefficient magnitude vs. strength of correlation

Pearson’s correlation coefficient magnitude	Correlation strength
$0 \leq \varrho < 0.3$	Weak
$\leq 0.3 \leq \varrho < 0.5$	Medium
$0.5 \leq \varrho < 0.8$	Strong
$0.8 \leq \varrho \leq 1$	Very strong

Table 3: Correlation between number of alternatives and minimum percentage change in the lowest criterion weight required to change the outcome of AHP

Correlations

		Number of Alternatives	Percentage Change
Number of Alternatives	Pearson Correlation	1	**-.230****
	Sig. (2-tailed)		.000
	N	1053	1053
Percentage Change	Pearson Correlation	**-.230****	1
	Sig. (2-tailed)	**.000**	
	N	1053	1053

**. Correlation is significant at the 0.01 level (2-tailed).

Table 4: Correlation between number of alternatives and minimum percentage change in the lowest criterion weight required to change the outcome of PROMETHEE II

Correlations

		Number of Alternatives	Percentage Change
Number of Alternatives	Pearson Correlation	1	**-.171****
	Sig. (2-tailed)		.000
	N	1118	1118
Percentage Change	Pearson Correlation	**-.171****	1
	Sig. (2-tailed)	**.000**	
	N	1118	1118

**. Correlation is significant at the 0.01 level (2-tailed).

Table 5: Correlation between number of criteria and minimum percentage change in the lowest criterion weight required to change the outcome of AHP

Correlations			
		Number of Criteria	Percentage Change
Number of Criteria	Pearson Correlation	1	**.233****
	Sig. (2-tailed)		.000
	N	1053	1053
Percentage Change	Pearson Correlation	**.233****	1
	Sig. (2-tailed)	**.000**	
	N	1053	1053
**. Correlation is significant at the 0.01 level (2-tailed).			

Table 6: Correlation between number of criteria and minimum percentage change in the lowest criterion weight required to change the outcome of PROMETHEE II

Correlations			
		Number of Criteria	Percentage Change
Number of Criteria	Pearson Correlation	1	**.080****
	Sig. (2-tailed)		.008
	N	1118	1118
Percentage Change	Pearson Correlation	**.080****	1
	Sig. (2-tailed)	**.008**	
	N	1118	1118
**. Correlation is significant at the 0.01 level (2-tailed).			